

A TEXT-BOOK OF LOGIC

A TEXT-BOOK OF LOGIC

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TO
MY FATHER

CORRIGENDA

- Page 11 line 15. *For* Scientarium *read* Scientiarum.
Page 20 line 20. *For* it *read* is.
Page 64 line 5. *For* seems *read* seem.
Page 133 line 2. *For* secure a new logical product *read*
constitute a term abstract.
Page 160 line 28. *For* obviaus *read* obvious.
Page 167 line 11. *For* cunctis *read* cunctus.
Page 223 line 21. *For* in the next *read* and later.
Page 234 line 31. *For* One of the *read* The alternative.
Page 343 line 25. *For* predicate term *read* major premise.
Page 386 line 6. *For* or more *read* at least.
Page 388 line 19. *For* need *read* needs.

Page 258. For 3 and 4 in table at bottom of page insert
the following:

- | | | |
|---------------------------|------|-----------------|
| 3. Particular Affirmative | Si P | Some S is P |
| 4. Particular Negative | So P | Some S is not P |

PREFACE

The time has not arrived when it is possible or desirable to write a text-book of logic for elementary students which departs very considerably from the course that has been defined by university requirements in various parts of the world. Both in Europe and in America instruction in this subject has to a large extent been controlled by what may be called the classical tradition of which Aristotle and Mill are the two fountain heads. In some quarters this fact has operated as a hardship, and has led to open, if rather one-sided, criticism of the subject's educational usefulness, on the one hand, and has directed attention to and aroused interest in the real and supposed defects of the traditional logical doctrine, on the other. The position both of those who maintain the conservative attitude, and of those who would substitute for the old some new kind of logic, is capable of defence up to a certain point, and there is no doubt that the stirring of the academic waters on this subject will result at some time in a larger freedom in the choice of the material and methods of logical instruction, and to that extent will reduce the possibility of attributing to the subject the imperfections of temper and understanding of those who teach it.

I hold no brief for traditionalism in logic, and while I sympathise with and, according to my

ability, appreciate the value of much that the advocates of the newer logics have contributed to our knowledge of the methods by which the human mind reaches truth, I have never been able to dispossess myself of the belief that the older logic in some measure had enabled me to understand what the newer logic had to say. Were I an artist, it would be my desire to paint the ocean, just because, I imagine, it is so constantly changing and withal so eternally the same. A similar interest may stimulate one to observe, in all the ways that modern logic is making familiar to us, the wonderful changefulness and practical resourcefulness of the mind's approach to truth; but if one were to identify logic with the observation and description of these shifting phenomena, he would be in the position of the man for whom the ocean was no more than the waves that rose and fell and broke upon its bosom. The human mind has a determinate structure, and it is with that structure, so far as it is displayed in the field of knowledge, that logic aims to make us acquainted. And I do not believe that it has been reserved for our own times to utter the only true words on this subject, nor that we can afford to neglect the teaching of the ancients. Aristotle defined logic and formulated many of its methods in a way that has given life to the subject for over two thousand years, and much that is modern takes its point of departure, either by way of criticism or refining, from what he wrote in the beginning of the science's existence. It is not to be understood, of course,

that the traditional doctrine stands in no need of critical examination. The reverse is the fact. The enormous increase of knowledge in the modern era makes it necessary. But how are criticism and revision possible if the subject is, as some seem to think, wholly illusory, or if we remain in absolute ignorance of the doctrine upon which criticism and revision are to be directed?

It seems, therefore, that for some time to come logic will continue, in the main, to be traditional, and that books of logic will have to conform to the requirements that are determined by that fact. But there are all sorts and degrees of conformity, and I confess to a liking for that kind and degree which is not inconsistent with the exercise of the largest liberty of one's academic conscience. It would not be surprising, consequently, if it were discovered that I had allowed the present situation in logic to sit lightly upon me, and that, where it seemed desirable, I had departed in method and doctrine from traditional views. It is I believe in some such way as this, rather than in assuming a critical, not to say captious, attitude toward the whole of traditional logic that the best interests both of the subject and of those who study it can be advanced.

It is hardly necessary to say that the present book as a whole is the product of experience in teaching logic to elementary and advanced students of the subject. No one, I am convinced, should attempt to write a text-book in any subject, and certainly not in logic, who has not learned in the

practical way of teaching it where its main difficulties are found, and who has not had an opportunity afforded him of devising methods of overcoming them. For, in the first place, a text-book must be written for the student, that is, for one who is presumed not to know anything about the subject of which it treats and, for him, the greatest service that it can render is to stimulate an interest in the problems with which it deals. But also it should be an instrument in the hands of the teacher which facilitates the task of teaching, and adapts itself to the purposes that he may hold before himself and the class. However far my book may be from completely fulfilling these objects, I hope it has entirely escaped the capital offence of substituting itself, in the regard and thought of the student, for the teacher, and that it can not be accused of helping to degrade the teaching function to the mere level of hearing a recitation. I trust, therefore, that there are many sections that will be found to require the generative touch of the teacher, and that what in his hours of preparation may appear to the student as a valley of dry bones will be vitalized by the teacher into living forms in the hours spent in the lecture room.

I have given to the judgment a prominence in the order of topics which I think is required for a true comprehension of the problem of logic, and have distinguished between it and the proposition for the purpose of indicating the class of questions which any attempt to express our judgments in

words quite naturally suggests. If I am not mistaken, current controversy in philosophy would sometimes have been simplified, if indeed the occasion for it were not entirely removed, if it had been distinctly recognised that the judgment is not the proposition, and that the problems of each are quite distinct. A close relationship must, of course, be recognised between judgments and propositions, but I am not without hope that the separate treatment of these topics will meet with the approval of my colleagues, although it results in placing at the very beginning what must be regarded as one of the difficult chapters of the book. I do not believe that the judgment chapter is insuperably difficult; it is not more difficult, for example, than is the neurological material to which the psychologist introduces his students at the very outset of their studies. And I may say that the effort has been made to present the subject in a manner as simple as is consistent with the real complexity and difficulty of the problem. With a simplification which falsifies a topic in the interests of easing the task of student or teacher I have as little sympathy as I have with the reverse method of creating or magnifying difficulties for the good of the learner's academic soul. But we must cultivate in ourselves and in those we teach the ability to recognise a difficulty when and where it exists, and to face it with determination, and this requires from teacher or pupil neither apology nor praise.

With respect to other features of the book, I may, perhaps, be expected to say something in justification of the relatively detailed account that has been given of the fallacies. There is, in fact, little to say, and that little I trust will appear quite superfluous to any one who has read the chapters in question. Both the mode of treatment and the amount of space given to the subject have been determined by the belief that the problem of fallacies has not received as a rule the recognition that it deserves, and that it has not been correlated to the main logical doctrines in a sufficiently organic way. I do not wish to be understood to mean that the chapters given here are at all adequate in these respects to the importance I conceive the subject to have. I have, however, sought to avoid leaving the impression that the subject has no practical or positive significance, and have endeavoured to keep the whole discussion in close relation to the body of logical doctrine developed in the preceding chapters. To leave the student with some degree of respect for the topic as a whole seems to me to be decidedly worth attempting, especially when the student is one who is making acquaintance with the subject for the first time. It will also be found, I think, that the two chapters in which the fallacies are discussed lend themselves readily to the purposes of a review of the main logical problems, and that the new point of view from which these problems are considered, the point of view, namely, of the sources of error that lurk in the processes of logical thinking,

will not only relieve the review from the banalities that usually attach to a twice performed task, but may be found to make the problems themselves more articulate and significant. At any rate, if we begin our logical studies with the object of finding out what is meant by truth, and what are some of the forms in which truth exists, one would think that a discussion of error or at least an examination of some of the forms in which error exists, would serve to emphasise the importance and the character of logical problems. And, I confess, it seems to me that no amount of technical skill in the manipulation of technical formulæ can compensate for any failure to perceive that the problems of logic originate in the effort to think and to express ourselves clearly and correctly, and that, therefore, the science that is devoted to the study of the principles of clear and correct thinking is one in which any person that has any pretense to education should have an interest. And in this connection I may add that the questions on the text appended to each chapter are designed as much to stimulate such an interest as to test a student's acquaintance with the details of the discussion; and if any apology for printing them or for making them so numerous is needed, I can only reply that my own students say that they have found them useful.

I confess to a certain compunction in offering to the public a book on logic which fails to discuss in a systematic way the so-called laws of thought, although I have never felt the necessity of apolo-

gising to myself for failing to expound to my elementary students so recondite a subject. I really have never found that any attempt to interest beginners in logic in this problem was satisfactory either from my own or from their point of view. The book, therefore, does in this particular conform to my practice, and the justification that may be offered is that it is pedagogically and scientifically preferable for the student to discover some problems for himself in the course of his studies than to force them in a more or less artificial manner upon his attention. There are places in the text where the subject here in question is quite near the surface, and if it happens to break through, the wise teacher will surely not fail to take advantage of his opportunity.

With respect to the general outcome of such a course in logic as is outlined in this book, it is enough to say that I should be exceedingly disappointed if the way had not been prepared for an intelligent study of the larger treatises on the subject, and if the door had not been opened, though only a little way, through which the student might pass to a study of the perennial problems of philosophy itself. About the former point, it is only necessary to say that the lists of advanced references at the end of the chapters, limited as they necessarily had to be, are intended to invite individual students and whole classes to read more widely and on a more elevated plane discussions of the problems which are considered in the text, and

to which reference is also made in the first lists of books. An experiment on the possibility of profitably using books of an advanced character might be made by the student himself in connection with those parts of the text which fail to interest or profit him. I shall not be surprised if very often the presumably more difficult reference turns out in the end to be the simpler statement. If any one should doubt the value of logic as an introductory approach to the problems of philosophy, which is the second point mentioned above, I can only say that so long as those problems continue to depend, to the extent that they actually do, upon logical construction, there is absolutely no possibility of understanding what the problems are, much less of contributing to their solution, if one has not acquired a mastery of the instruments of logical thinking. But not only is this true in respect of philosophy narrowly interpreted; an acquaintance with logical methods is also important if we who are not philosophers in the technical sense of the word are to take an intelligent attitude toward our own lives. Toward such an end, I hope some contribution has been made by the mode in which the logical problem has been conceived, and by the manner in which it has been worked out.

For those who are acquainted with the literature of the subject, it is not necessary for me to name the sources of my chief indebtedness, and for those who are not, such a proceeding would savour of a vain parade. I shall, therefore, content myself

with the statement that I have learned from the masters of many schools, and have retained a grateful affection for them all. As I am not conscious of being a partisan, I presume that even those who have taught me most would hardly recognise me as one of their disciples. The public, consequently, will be in no doubt when it comes to apportioning praise or blame for what is here set down, although it is evident, so that he who runs may read, that it is not all made up out of my own head. I have quoted when necessary, I have also paraphrased, I have refused neither suggestion nor illustration when any of these methods furthered the object I have constantly held before me in writing these pages. I want, therefore, to thank all who by their published writings have rendered themselves liable to this sort of literary piracy. I am also indebted to colleagues for particular services which it is a pleasure to acknowledge. Professor Rudolph Pintner has read many of the chapters in the first half of the book, and made suggestions which have improved the form over that in which they were originally cast. Mr. A. P. Weiss made the drawings for all the figures, and also read some of the sections on induction. Professor David R. Major has read all the chapters in manuscript and in proof. I feel that his criticisms and suggestions have been invaluable. It is certainly not for the want of friendly and intelligent advice that I may sometimes have erred in aim and execution. The final stages of the proofs

have been entirely in Dr. Major's hands. I take this opportunity to acknowledge assistance that has been generously rendered.

Eltham, Kent.

A. E. D.

Feb. 25, 1915.

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CHAPTER I

THE PROBLEM OF LOGIC

The Starting Point. It is customary to begin the exposition of a branch of knowledge with a statement and explanation of its definition. This is a useful mode of procedure when it can be presumed that those for whom one writes have previously acquired an elementary knowledge of the subject to which the definition relates. But since no such presumption is made in regard to those who may be expected to read this book, we shall postpone for the present the task of formulating a definition of logic. Meanwhile, we shall discuss a number of topics, all of which throw light upon some feature of the problem of logic, and each of which will contain suggestions as to the nature of the subject to the exposition of which this book is devoted. And we shall find it convenient to begin our discussion of these topics by asking what is the subject-matter of which logic, in all its various stages and branches, is the study.

Each Science has a Different Subject-Matter. It will, perhaps, help to emphasize the importance of this inquiry if we recall, first, that a science presupposes the existence of a special kind of material, called its subject-matter; and, second, that each science has a different subject-matter. For example, in geology we learn about the structure of the earth's

surface; in physiology, about the functions of living organisms. Physics is a study of bodies in motion; and geometry, of figures in space. In these, and in similar cases, the subject-matter of the science is the material which the scientist observes and describes.

The Thought Element in Knowledge. If we consider further how the several sciences are constituted, it soon becomes evident that the material selected for study is only one condition of the existence of a science. The scientist must also think about his material before his knowledge concerning it can have the characteristics of scientific knowledge. He must, for one thing, arrange, or, as it is said, classify, the material that he has collected. It is not, however, the purpose of this section to describe the methods that the scientist employs in studying his material. At present, we wish to call the student's attention to the fact that the attainment of any kind of knowledge is impossible without an active exercise of the thinking processes, and to warn him that the passive flow of images and ideas through consciousness must not be mistaken for thinking. It is true that without images and ideas there can be no thought; but thinking consists in comparing objects with one another, in differentiating the like from the unlike, in combining them into more complex wholes, in relating in many and diverse ways these wholes to each other, etc. Thinking, in other words, is a specialised sort of mental activity, an activity that taxes to the utmost, and

frequently brings into play, all the abilities with which the human mind is endowed. It is the supreme task to which the many have been called; but if we regard it lightly, or presume that it can be accomplished without toil, or if we erect our own incapacity or indolence into a reason for the uselessness of the endeavor, we must abandon the hope of joining the company of the few who are chosen. It is, therefore, with good reason that logic directs attention to the function of thought in human knowledge, for thinking is the one way, the only royal road, to the goal of an educated life. To think about the objects of one's experience is, then, necessary if knowledge is to exist; but thinking, it must also be borne in mind, is "not a passive suffering of something, but a doing of something with" these objects. Or, as Wundt has expressed it, "jedes denken ein Wollen."

Not only, then, must we think if we are to attain scientific knowledge; we must also think in order to attain knowledge that, in the strict sense, cannot be described as scientific. Illustrations of the truth of this statement may be found in history, economics, jurisprudence, as, indeed, it may be in all those other studies which, like these, are interested in the ways in which the various needs of men are met in a social organisation. It would seem, then, that wherever we have knowledge, whether it be in the form of the natural or the historical sciences, as these others may be called, we have a witness to the ways in which the demands for thought have been met. In

brief, unless we think about the material that observation brings, knowledge cannot exist; and, further, it is what we think that converts this material into definite species of knowledge.

The following illustration may throw additional light upon the statements of this section, and emphasise the importance of thought for all forms of human knowledge. We all know what a picture puzzle is, — a collection of odd-shaped and variously coloured pieces of cardboard which, when put together in their proper relations, form a complete and consistent whole. All of us, moreover, have experienced difficulty in putting the pieces of cardboard together so that the hidden pattern or picture is made evident. If, now, we study a case like this, we may observe that the difficulty we had in bringing out the pattern was not due to the lack of anything in our material, — all the pieces, we shall suppose, were before us, and were capable of being fitted to one another in an appropriate way. But the task of putting them together in the proper way was by no means easy. Where was the source of our difficulty? We cannot attribute it to our material; and most of us would agree that it was due to our inability to see which parts should come next and next and next. If anyone chanced to observe us at our task, he probably would have heard us exclaim, 'How stupid I am!' 'What's the matter with me!' 'Why can't I see it!' That is to say, he would have heard us blaming ourselves, and finding fault with the things we were doing to the material with which

we were working. What bearing, one may ask, does this have upon the question before us? Just this, that if we are to solve the simplest problem, thought must be brought to bear upon the material which constitutes the subject-matter of that problem. When we do not succeed in doing this, and when, for any reason, we are baffled in our endeavour to make out the nature and relations of the material we are studying, a special emphasis is thrown upon the thought factor, the one which, under more favorable conditions, is liable to be lost sight of entirely. In this we are like children who look about their world delightfully oblivious of the fact that the eyes provide them with the light of all their seeing. It is only when the eyes fail that one becomes conscious of their existence. Similarly, when knowledge fails an unsuspected factor of knowledge gains prominence, and to this factor, to which attention is thus drawn, we give the name — thought.

What we mean by Knowledge. The student who has read carefully the foregoing paragraphs will understand what we mean when we say, — we are always *thinking* about *something*. The italicised words call attention to the two factors in knowledge. There is the *object*, — the million-starred heavens, or the one solitary maiden, on the one side; and, on the other, our *thoughts* about this object. Both these elements — object and thought — are necessary to knowledge. Objects that no one thinks about are only the materials of knowledge, and thoughts that are not thoughts - about something in par-

ticular can hardly be called thoughts at all. It is only when our thoughts are thoughts-about-objects that we give them the name of knowledge. This statement holds so generally that by knowledge of any sort whatsoever — poetry or legend or history or science or just every-day opinion — we may be understood to mean our thoughts-about-objects. Accordingly, from the standpoint of our analysis, knowledge=object+thought. In later sections, we shall have more to say about these two factors of knowledge.

Knowledge is Systematic. If we consider our thoughts-about objects with some care, we shall observe that they tend constantly to become fixed, orderly, and systematic. This is not an accident; it is rather of the nature of a demand that we make upon ourselves and our fellows, so much so that we dismiss with a shrug of the shoulders the thoughts of any one who fails to meet it with a fair degree of success. We may say, then, that we are all alike in having ordered thoughts; we differ only with respect to the degree of success that we attain in introducing that particular kind of order that makes our thoughts most effective. For example, there is a great difference in the coherence of the thoughts of an insane or feeble-minded person and those of the average normal person. In the former case, we observe both a frequent change of the subject of his thought, and an obvious looseness of connection between one thought and another, even when these refer to the same subject. Sometimes the only con-

nection that can be observed between one thought and another is the similarity in sound of the words in which the thoughts are expressed. On the other hand, we expect to find the average normal person able to think in such a way that the connection between one thought and another follows the line of the essential connections of one part and another of the subject about which he thinks. When this is the case, we call the person's thinking 'systematic', — systematic, that is, in the sense that the order of his thoughts is warranted by the nature of the subject to which his thoughts refer.

Now it may be interesting and profitable to note what are the conditions of systematic thinking. There are three that, from the logical standpoint, require mention. First, there is the difference, native and acquired, in the ability of individuals systematically to develop by thinking any subject in which they may be interested. We recognise the fact that one person is more highly endowed in this respect than another in such every-day expressions as: 'He's just smart'; 'All there is to say is, he's got brains.' But we also recognise the fact that we are all capable of improvement, however excellent or mediocre our native endowment may be. The great national organisations for the education of the young are a recognition of this fact. But, of course, all that we can hope to do by education is to diminish the difference between one individual and another; we do not expect to overcome the advantage that lies on the side of the person who adds to a high

native endowment a careful and industrious training. The second condition of systematic thinking is found in the fact that not all the subjects about which we think seem capable of being systematised with the same degree of completeness. For example, we do not find in the social sciences the same degree of coördination that we find in the natural sciences; and, in a general way, we may attribute this to the greater complexity of the subject-matter of the former as compared with the latter.

It may be observed, thirdly, that not every occasion that calls for thought is met best with the strictest and most thorough exercise of our thinking ability. If we compare, for example, the thoughts of the man of business with those of the economist, or the thoughts of the politician with those of the political scientist, we shall see that they differ, not necessarily in proportion to the abilities of the individuals to think, nor directly with respect to the difficulties of the subject-matter about which they think; but the occasion that sets the thinker his particular task sets also the limits within which the task is to be carried out. The same thing is true up and down the line of our every-day observation and experience. The man, for example, who carried over into social life the passion for precision, accuracy, and completeness that properly belongs to a scientific publication would be voted a bore, just as surely as the man who worked in his laboratory with the leisurely and lax methods of a man of the world would be pronounced a failure. Circumstances alter

cases; and logic observes that in any of the departments of human life where thought is called for, it is only the degree of systematic thinking appropriate to the circumstances that it is ever necessary to attain.

Logic and the Knowledges. In our discussion of what knowledge is we have seen that knowledge may exist in many different forms, varieties, departments, or branches. For example, engineering, psychology, bacteriology, literature, biology, etc., are so many different forms of the one thing we call knowledge. For this reason, it would be better, simpler, and more concrete if we were to accustom ourselves to speak of these subjects as 'knowledges' whenever we wished to refer to the plurality of the departments of knowledge. We have a precedent for this usage in that we speak more often of the sciences than we do of science. By knowledges, therefore, we mean the several varieties, forms, or kinds into which knowledge has been observed to fall. Now we may observe that, historically, most of the other subjects of knowledge existed before logic was formulated into a science. Mathematics, astronomy, physics, zoology, political and social philosophy, for example, are must older branches of knowledge than logic. And if to this we add the fact that logic came into existence as a result of a study of the methods of these sciences, we shall see that it was the existence of the knowledges that made logic possible rather than the existence of logic that made them possible. Hence, it would seem to be true that while

every form of knowledge may be cultivated and may flourish independently of logic, logic cannot even begin to be unless some of the forms of knowledge have already established themselves.

The Subject-Matter of Logic. What, now, we may ask, is the relation of logic to the two factors of which all knowledge is comprised? And, first, what is its relation to the subject-matter of the other sciences? The question needs only to be asked to be answered; for it is obvious, in the light of what has already been said, that no two departments of knowledge study the same material. Logic, therefore, cannot study the same subject-matter as any other science without losing its identity by becoming absorbed in that science. For example, if logic were to study the origin, growth, and function of political institutions, the result would be, not a new knowledge, but an old one under a new name. Political science and logic would then be the same things. In like manner, if logic were the study of plants it would be identical with botany. Further illustration is not necessary. The subject-matter of logic is not identical with that of any one of the other knowledges. What, secondly, is its relation to the thought factor which, as we have seen, is present in all knowledge? Our answer is that it is this thought factor in the various knowledges that logic makes the direct object of its study. But logic's relation to this thought element is different from that of any of the other sciences. In the several knowledges, thought is the means by which the

qualities and relations of objects are described and explained. Ordinarily, we are no more aware of its presence, than we are of the pane of glass through which we look out upon the landscape that lies outside our window. But if, for any reason, we become curious to know how thought does its work, to find out whether it follows any method, we should then set thought before our minds as a distinct object of study, and the resulting knowledge would be called logic. That is to say, logic arises when the thought factor which is present in all knowledge is set before the mind as a subject of investigation. Thought then becomes the material of a special study, and the name of that study is logic.

Logic as *Scientia Scientarium*. We have just seen that logic is the science that is interested in the way in which thought does its work in the several fields of knowledge. Now we have previously seen that we are not everywhere equally successful in systematising our thoughts-about-things. Some branches of knowledge are more loosely, and some are more closely, organised. This fact we may attribute in part to the greater complexity of the subject-matter of some of the knowledges. History, for example, is a less exact science than is physics just in proportion to the greater complexity of the material that it studies, and to the difficulty of bringing its material under controlled observation. Now, in this connection, the question arises — Do the several departments of knowledge equally well provide guidance for the formulation of logical doctrine? If some

branches of knowledge are better articulated than others, should we not expect that the former would illustrate the methods of thinking more perfectly than those branches that have not taken on the same degree of coördination; and, further, if we can find any group of knowledges that stands pre-eminent in this respect, should we not save time and trouble by making it the basis of our study of logical doctrine? Such questions have been asked, and it has been customary to answer them in the affirmative. Thus logicians, from the days of Aristotle to the present time, have expounded the subject of logic with their eyes on the methods adopted by the better articulated sciences; and, by some, the study of logic has been confined to an examination of the methods that have proved useful in reaching the truth in such sciences. From this point of view, logic has been looked upon as a science that lies hidden in the various sciences, very much as the pattern lies hidden in a picture puzzle. All that needs to be done, according to this view, is to bring logic into the light that it may direct the labors of future workers in the field of the sciences. It is in this sense that logic is spoken of as *scientia scientiarum*: it is the science of sciences in the sense that it underlies and gives direction to all the work that science undertakes to do.

The Humanistic Influence in Logic. That view of logic which identifies it with the study of scientific method has always been considered a little partial and extreme, and it is doubtful if Aristotle, who

was the first and greatest of the formulators of logical doctrine, and whose authority has been appealed to in support of this position, would subscribe to it. It is probable, as Minto has shown, that in organising his logic, Aristotle had in mind, as much the formulation of the rules of dispute in the social life and court procedure of his times, as the systematisation of the methods of thought in the body of the scientific knowledge of his day. However that may be, it is certain that our own times are witnessing a marked humanistic reaction against a stiff and formal view of logic, and modern logicians are looking to the less exact forms of knowledge for enlightenment as to the way in which the business of thought is carried on; indeed, their study now extends to the ways in which difficulties arise and are overcome in our everyday life. This extension of the field within which the logician pursues his studies cannot be taken as an evidence of indifference to a strictly logical doctrine; rather must it be viewed in the light of what is taking place in other departments of knowledge where an investigation of the simpler, more primitive, forms has been found to elucidate much that was obscure in the more complex forms of the subject-matter studied by these departments. In zoology, for example, interest in the simpler organisms has flooded with light our understanding of the more complex ones, and in psychology the study of the mental processes of young children and of the feeble minded has added to our understanding of the adult, normal mind. Hence it

is felt that if we consider those methods of thought which, in some of the sciences and in the ordinary business of life, have not taken on a strictly rigid form, we shall be able to understand the nature of thinking in a more satisfactory manner than if we confined our observation to those sciences in which the highest degree of precision in methods had been attained.

Relation of Logic to Psychology. We have used the term 'thought' in the foregoing pages in a somewhat broad way. It has meant, in general, the mental factor which is never absent from the knowledge experience. Now the question may arise whether in claiming thought, in this sense, as the special subject-matter of logic we have not confused logic with psychology. For what is psychology if it is not a study of mental processes?

Now there are two ways in which we may distinguish between logic and psychology. In the first place, we may remark that there is a quantitative difference between the two. Grant, for the time being, that the subject-matter of both these sciences falls within the field that we usually designate as mental, it may, nevertheless, be said that not all that is mental is included in the scope of logical inquiry. Logic is interested mainly, if not exclusively, in what are called the cognitive processes; that is, in those processes in and through which we either affirm or deny something. But cognition in the strict sense, cannot claim to be more than a very small part of one's mental life. The latter, and so

the field of psychological inquiry, includes, in addition to knowledge or cognition, other kinds of mental processes, e. g., feelings, emotions, volitions. The whole field of the conscious life lies open to the investigation of psychology; but logic is interested chiefly in that part of it that is called cognition, and in the other parts only so far as they help us to understand what human knowledge is.

In the second place, we may remark that although logic and psychology both study the cognitive processes, they do not study them from the same point of view. Psychology is satisfied when it has described and explained the mental changes that take place in a cognitive process. Logic is concerned, not with the natural history of cognition, but with cognition as an instrument by means of which the world of reality—minds and things—is understood. That is, the images and ideas which make up any cognitive process, and which psychology endeavours merely to describe and explain, are, from another point of view, taken to represent objects in the real world; and it seems to be the function of these images and ideas to place those who have them in communication or contact with objects which exist in some sort of independence of these images and ideas. It is this character of images and ideas as affording us a knowledge of a world of objects that is the standpoint of logic, and which enables us to distinguish the logical from the psychological method of dealing with the cognitive processes. Logic is not interested to know what images and ideas are present in con-

sciousness, but whether the images and ideas that we entertain are true, that is, afford us accurate knowledge of the objects which they represent and to which they point. Or, to put it more succinctly, logic studies the conditions of the truth or falsity of our ideas, a problem which, for psychology, is quite indifferent or non-existent.

Logic as an Art. It is sometimes said that logic is an art as well as a science. The meaning of this statement can be made clear, if we bear in mind that the term 'art' is used in two senses. In the first and most common meaning, it refers to the developed skill that one acquires through practice of a set of more or less mechanised movements. If, for example, you go into a machine shop and watch the workmen at their tasks, you will find them engaged in different activities, and displaying varying degrees of skill. It is this ability to do a given task with some degree of proficiency that we call the workman's art. Art is, then, a specialised ability to do. On the other hand, there is an ability, much more specialised than the mechanic's, which consists in the perception of how a task should be done, but which does not necessarily carry with it an ability to perform it. The engineer, for example, is able to plan and superintend building enterprises, although he does not have the ability of the workmen upon whom he must rely if these enterprises are to be translated into steel and brick and stone. The skilled workman and the engineer display two different orders of practical ability, and we may

speak of each one of them as possessing and practicing an art.

Now, when logic is spoken of as an art, it is well to be clear whether we are using the term art in the first or in the second of the two meanings just distinguished; whether we are thinking of logic as giving to its students the ability to think correctly, or whether we are thinking of it as giving its students the ability to detect the correctness and incorrectness of arguments. We are inclined to the opinion that, in the case of logic, these two meanings cannot be separated, and that in learning the regulative principles of logic we are at the same time acquiring the ability to order our thoughts in a logical manner. The reason for this opinion, briefly stated, is that the study of logic is and must remain unprofitable to anyone who, while he is studying it, does not practice the methods of logical thinking. The meaning of this statement will become clearer as we proceed, but it should be taken by the student as a suggestion that the way to gain a knowledge of this subject is to think each step logically as it is unfolded in the successive chapters. For example, we shall presently be speaking about terms, and we shall call such things as, 'the Sultan of Turkey,' 'good,' ' H_2SO_4 ,' 'man,'—terms. Now, it is obviously impossible for anyone to memorise, or even to make, a list of the things that logic calls terms. The only alternative, therefore, is for the student to acquire the ability to think — term, to understand what a

term is. When he has done this, he will have to employ this acquired ability in all his subsequent study, so that it never becomes rusty through disuse. The same remark applies to all the topics of which logic treats. In this sense, therefore, the study of logic is at the same time practice in the methods of logic; and it is because we cannot acquire a knowledge of the correct methods of thought without at each step practicing these methods that logic may be said to be an art in both of the senses defined in the beginning of this section.

What Logic is. We may now, with a summary of the foregoing discussion in mind, undertake a statement of what logic is. We have seen (1) that logic presupposes the existence of knowledge in one or more of its several forms, and that it has sustained this relation to the other knowledges from the very beginning. Aristotle, as we said, was prompted to write his treatises on logic from the desire that, along with the improvement in the sciences and arts, there should go the consciousness of the method by which, in these fields, conclusions were being reached. Logic, therefore, was not an invention of new instruments of research and knowledge, but the formulation and systematisation of those that in use had already proved their effectiveness. Consequently, we said (2) that when logic sets about its task, it distinguishes between the subject-matter and the thought element in the knowledges, and erects the latter into a separate object of study. *Logic, therefore, is a study of the relation*

of thought to human knowledge. But, in this connection, we had to point out (3) that the interest of logic in thought is different from that which the several knowledges have in it. The knowledges aim to reach conclusions that increase our understanding of the nature of the objects they select for study. It is true that this increased understanding is reached by thinking; but, so long as results are reached, the scientist is no more concerned with thought than is a farmer with the engine that drives the machine that threshes his grain. This is not the case with logic. It studies thought with a view to finding out its methods of work. From this point of view, therefore, we may say that *logic is a science of the methods of correct thinking.* But (4) if logic is to be guided by the best examples, it will naturally be interested in the methods of the natural sciences. The influence of this interest of logic in the natural sciences has been to restrict unduly the scope of logical inquiry, and thereby to give to logic an importance that originally did not belong to it. The restriction is seen in the definition of *logic as a study of scientific method*, and the new importance it thus assumes is expressed in its claim to be the *scientia scientiarum*, — the science that makes all the other sciences possible and real. In this connection, we pointed out (5) that this view could not claim the support of the earliest writers on the subject, and that in our own times a positive reaction against it has set in. The form that the reaction has taken is to extend the field of logical observation so as to in-

clude not only the more highly specialised sciences, such as physics and chemistry, but also those that are less fully developed, such as the biological, social, and historical sciences. From this standpoint, *logic is a study of the methods of human knowledge* of whatever kind the knowledge may happen to be. In the section on psychology and logic we saw (6) that thought may be studied in two quite different ways. We may study thought, for example, as a series of consciousnesses made up of various elements and possessing various attributes. This is psychology. If, however, we study the function of thought in affording us a knowledge of the real world, we are led to a very different result, and the kind of knowledge thus reached will be logical rather than psychological in character. For logic, that is to say, thought has a meaning over and above what thought is as an item in a stream of consciousness. What thought is or is like is one thing, and what thought means and does it another. It is from the latter point of view that logic studies it. If, therefore, meaning, as one writer says, is a "product of thought in its relation to reality, or of reality in relation to thought," *logic may be defined as a science of the development of meaning*, or, more briefly, as the *science of meaning*.

Only one more point need be raised, namely, in what sense logic is a science at all. In the last section we saw that logic is an art in the sense that engineering is an art, but we also saw that as such

it is an art in a somewhat special sense. It is, if we may say so, an intellectual art, an art that consists in knowing how certain things should be done. In this respect, logic, to use Minto's illustration of the Aristotelian logic, may be likened to "Cavendish on Whist," — a book that states the rules and illustrates the methods of playing the game. We have done ample justice to this feature of logic, and the student is not likely to lose sight of its importance as we proceed. But we now ask whether it is also, in any sense, a science; in the sense, for example, that physics and chemistry are sciences. Now we ought to be clear that these sciences do not exist for the purpose of providing rules that students may follow in performing laboratory experiments. If this were the whole extent of the scientific character and purpose of these subjects, they would differ in no respect from the practical sciences in which we learn how certain things are done. If, however, we consider these sciences more carefully we shall see that they aim to formulate principles or laws, and that it is only as the experiments enable us to ascertain these principles that they have their perfect work. So we may say with respect to logic. It has its experimental side, and it formulates rules and states the methods of correct thinking; but it goes beyond this and tries to express the laws or the principles to which every concrete case of thinking actually conforms. In the light of this fact, we may conveniently bring this chapter to an end with the state-

ment that *logic is a study of the methods of correct thinking and of the principles on which these methods are based.*

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QUESTIONS

1. How do the various sciences differ from one another? Illustrate.
2. What function has thought in the several sciences, and what reason can you give for your answer?
3. What do you understand by knowledge, and into what two factors may all knowledge be analysed?
4. What is meant by calling knowledge systematic?
5. Upon what three factors does the systematisation of knowledge depend?
6. Is all knowledge equally systematic? Illustrate.
7. What is meant by 'knowledges'?
8. "It is the knowledges that make logic possible." Explain.

9. What is the subject-matter of logic? What is the relation of this subject-matter to the other sciences?
10. What is meant by calling logic *scientia scientiarum*?
11. Describe the humanistic reaction in logic, and how does it affect our view of the subject?
12. In what two ways do logic and psychology differ from one another?
13. What two meanings may be given to the term art?
14. Is logic an art as well as a science? Explain.
15. Summarise the various definitions of logic in this chapter, and state clearly the points of view from which these definitions are made.

EXERCISES

1. Point out the subject-matter in the case of any six subjects with which you may be acquainted.
2. Show the relation that thought has to each of the subjects chosen for your answer to the previous question.
3. Cite concrete instances of the way that thought becomes a problem in the pursuit of various kinds of knowledge.
4. Illustrate how observation of the ordinary modes of conversation may advance our knowledge of the logical processes.
5. Use concrete examples to show the limitations to which logic would be subject if it confined itself to the study of the scientific mind.
6. Give instances of the practical value that is claimed for an acquaintance with logical doctrine.

CHAPTER II

THE JUDGMENT

The Meaning of Thought. Before carrying our study of the logical character of the thought processes, begun in the last chapter, into greater detail, we wish to call attention to two simple and obvious considerations. In the first place, we may observe that the objects that come before us in the course of our experience present a great variety of characteristics. The object of which we are aware may be part of the material world, — the starry heavens; it may be part of the common human life that today we share with our fellows, — the rush of the ambulance in its race with death; it may be part of the inner individual life, — the secret and incommunicable thing that a man sees when he sits down face to face with himself. In short, the object may be anything from “the choir of heaven to the furniture of earth;” but whatever it is, it is an object because, and in so far as, we have experience of it. We shall use the term object, then, to denote that of which we have experience, without respect to the kind of object it is, or the kind of experience in and through which it may be said to exist. But this leads to a second remark. There are, as we intimated just now, a variety of ways in which we experience, or react to, or are aware of objects. The terms perception, memory, imagination, feeling, volition, are

the names of some of the more familiar ways of reacting to objects. Thus we may perceive an object, or remember it, or imagine it, or be affected pleasantly or unpleasantly by it, or do something because of it. All these are, of course, familiar experiences, and we do not need to be psychologists to be aware of the fact that the object assumes a somewhat different character for the different activities through which it is experienced. That is to say, the object as we perceive it, or remember it, or imagine it, or are pleasantly or unpleasantly affected by it, or do something because of it, has a slightly altered aspect in these several modes of reacting to it.

Now it will probably help us to understand the problems of this chapter if, at the outset, we observe that the recognition of this implication of objects in all or most of our conscious processes has led to a certain looseness in our use of the term thought. By some, especially among the psychologists, thought is the name that has been employed to denote the various kinds of mental processes, — perception, imagination, feeling, desire, emotion, etc. James, for example, speaks of the ‘stream of thought,’ a phrase in which the term thought, as he tells us, is used for “every form of consciousness indiscriminately.” In a somewhat more restricted sense, the term thought or cognition is used, in distinction from feeling and conation or striving, with which it is intimately associated, as the name for one of the ultimate modes of consciousness. It appears in

this sense in the writings of Stout and Ladd. When employed with this meaning, it denotes all those mental activities, such as perception, memory, conception, etc., in which we are, in one way or another, concerned with the nature of objects. "It covers," as Stout says, "all modes and degrees of being aware of or cognisant of an object." A still more specialised use of the term is that which limits its application to those processes whereby we designate objects merely as the objects that are, at the present moment, under consideration. To think, to use James' definition of conception, with which the term thought in this sense is practically synonymous, is to "identify a numerically distinct and permanent subject of discourse;" to mark it off from other objects, and to make it the bearer of whatever attributes or relations that our 'thoughts-about' it may show that it actually possesses.

The three meanings just distinguished of the term thought are doubtless closely connected from the psychological point of view. For if psychology be regarded as "the science of the processes whereby an individual becomes aware of a world of objects and adjusts his actions accordingly," and it is so regarded by Stout, it is perfectly obvious that the several mental processes must be viewed, in themselves and in their relations to one another, as modes of conscious activity which further, more or less directly, that acquaintance with objects which all the three statements of the meaning of thought alike emphasise. We need not consider whether,

and, if so, in what sense all the fundamental mental processes serve the purpose of establishing and improving our acquaintance with things; whether, for example, the so-called affective processes are correctly interpreted in this way. It will be enough if we recognise that certain processes can be grouped together, and can be regarded as having for their distinctive function the discriminating and relating of the objects of experience. If we approach the problem of thought from this point of view, we see that the term thought, in the second and third meanings distinguished above, have a peculiarly close relation. According to the second meaning, thought is a continuous process which, beginning in perception, culminates in what the psychologist calls judgment and reasoning. Without denying that thought is dependent upon the more elementary processes of perception, the third meaning limits the application of the term to the later processes in which, more particularly, we acquire what was described in the previous chapter as 'knowledge-about' objects. For the one, thinking is the name for a process in which perception, memory, imagination, conception and judgment may be found to function; for the other, it is a name by which conception and judgment are specifically signified. In whichever of these two contrasted senses the term is used, each implies, what is undoubtedly the fact, that there is no unique faculty of thought, that thought is a complex process in which a number of mental activities combine to give us a knowledge of objects, and that, while some

of these activities enable us to identify distinct and permanent subjects of discourse, others carry us beyond this to the knowledge of a system of existing things in which numerous terms and relations are discernible.

Judgment and Thought. Now when we consider the subject of thought from the standpoint of logic, it is in the third of the meanings already distinguished that, in the main, we shall use the term. Thought will thus signify the processes of judgment and reasoning, those processes in which the cognitive process as a whole culminates. Logic, that is to say, is particularly interested in the *terminus ad quem* of the thought-process, in the goal toward which our desire for knowledge points; and it studies the process of reaching the goal only when a knowledge of such process enables it to determine the nature and grounds of the validity that is claimed for thought. Thinking thus comes to be regarded by logic as having a character which is liable to escape notice so long as we keep strictly to the psychological point of view, — the character of assertion. And in order to give prominence to this feature of thought, logicians are more apt to speak of judgment than they are of thought. Thinking, as we have seen, is a mental process in which the psychologist distinguishes a number of constitutive processes, — perception, memory, etc. Judgment, as it is used by the logician, regards the thought process as being concerned with the specification of a content, the assertion of which is logically the

characteristic thing about a judgment. The content of the thought process is taken up by the judgment and constituted the subject-matter of assertion. Thus the experience, say, of red calls from the psychologist the statement that, like all sensational experiences red cannot be described, but must be experienced if any one is to know what the word signifies, that it is produced by the action, on the rods and cones of the retina, of ether waves of a certain length, and that these changes are continued, by way of the optic nerve, in a manner that is little understood, to the occipital lobe which is the cortical centre of the visual consciousness. For the logician, the particular patch of colour called red merely exists, and the acknowledgment of it as existing, by the individual who experiences it, is what is meant by assertion or judgment. The red is the content of the processes which are described by the psychologist, and it is this content that is asserted as the subject-matter of the judgment. Whatever, then, exists as content of thought, may become the subject-matter of a judgment; and we may, therefore, define a judgment, broadly, as an assertion of the qualities and relations of the object of thought.

A word or two may be necessary at this point to guard against a possible misunderstanding of what is meant by assertion as the central fact of the logical judgment. In current usage, to assert means to express in words the thoughts and opinions that we entertain about things. It is commonly employed as the equivalent of 'to say something.'

In logic, assertion has nothing to do with speech. We may, of course, express what we assert, but we can assert without putting into words, or into any other set of social symbols, the qualities and relations of the object of thought. The question we are now considering, moreover, has nothing to do with the problem whether, as individuals, we can think without words. It is a simpler matter than that; and all that we say is that no one, merely because he judges, is bound, in order to give his judgment the character of assertion, to put into language the truth or fact that his judgment embodies. On the contrary, by assertion we mean the attitude of mind in which, for that mind itself, there consciously exists such an acquaintance with an object that, in itself and its relations, it is distinguished from other objects with which it occurs in a common field of knowledge. For example, I may be looking over a collection of curiosities that a friend has accumulated from the four corners of the earth. I look at the objects one after the other, and find that each is a strange and unintelligible thing, and, as I put it back into its place, it is soon lost in the mass of the other strange and unintelligible things. If, however, I should find in the collection an object that is familiar, there would be, under the circumstances supposed, a mental tang and 'feel' toward it that would make it, as I put it, too, back in its place, stand out from the rest, — its identity would not get lost in the multitude of strange things. What makes the difference in this

case is my mental attitude toward it, — a mental attitude that, in this instance, is called recognitive judgment. In logic, the mental attitude in which we find the characteristic feature of the judgment is described as assertion. Whenever we judge we assert, and whenever we assert we put marks upon the objects that come before us by which they attain unique distinction.

Other Mental Attitudes Akin to Judgment. It may help to emphasise the assertive character of the judgment if we distinguish certain other attitudes which, like assertion, have received, in the development of language, definite verbal forms. Judgments, it should be observed, are always expressed in the indicative mood, and what they assert is always true or false. On the other hand, interrogatives, imperatives, optatives, and exclamations do not assert, and therefore are not judgments, but express either an inquiry, a command, a wish, or an emotion. Of the wishes that optatives express we can say that they are reasonable or unreasonable. The commands of imperatives can only be obeyed or disobeyed. Our interrogatives formulate inquiries and call for an answer. Exclamations are word gestures that express our emotional attitudes toward situations. But while interrogatives, imperatives, optatives, and exclamations are not judgments, they imply attitudes of mind which, if asserted, would be judgments. The rhetorical question, 'Is thy servant a dog?' doubtless implies an assertion, and is equivalent to 'Thy servant is not a dog.' But not

only the rhetorical, but any question may be interpreted in a similar way. For to ask a question would be meaningless unless we thereby implied that something can be asserted of its subject, and it is to this something that the question, in a general way, points. If, for example, I ask, 'Is that a fire?' I imply that it is a fire, but not in the way that would warrant me in asserting it as a fact. With imperatives the case is different. 'Come,' 'Do this,' cannot be said to be either true or false, and they imply assertion only if imperatives are indicated actions that have reference to a situation which they either help or hinder. Imperatives, that is, get their meaning from a context that exists in the mind of the person issuing the command, a context which must rely for its development upon the action of the person to whom the command is addressed. The implied judgment, therefore, may be thrown into the hypothetical form thus: 'If you do the thing commanded, such and such things will happen.' Commands, that is, are abbreviated antecedents of hypothetical judgments. Optatives express a desire for some state or condition which is thought of as possible, though not, at the moment the wish is entertained, as actual. They imply the assertion of definite objects as the objects of desire. In this way we must interpret, for example, 'Would that I were dead!' It implies that 'to be dead at this moment' is really a desired object. Such exclamations as 'Absurd!' 'Impossible!' are predicates of implied judgments, and by them, besides expressing

our personal attitudes, we are also characterising situations.

Judgment and Groundless Statements. Before proceeding to develop other features of the judgment, as this has been defined in the previous sections, there is a class of statements about which we are called upon to say something if only for the sake of clearing our exposition of the subject under discussion from unnecessary ambiguity. We have seen that assertion is the main characteristic of judgment, and that what a judgment asserts is some quality or relation of the objects about which we think. It would seem quite obvious, therefore, that unless we think about some object or other, there can be, for us, no such thing as judgment. Judgment implies thought, and thought implies something thought about.

In contrast to judgment, as thus understood, we are confronted with a class of statements which simulate judgments, but which fall short of being true judgments because they are not based on the thinking of the person who makes them. We may cite in illustration not only the parrot-like statements of the class-room recitation, but the ill-informed and often misleading expressions of opinion that many of us allow ourselves to make with confidence on all sorts of subjects about which we know relevantly nothing. The certainty that attaches to such statements has little or nothing to do with their subject-matter; it usually rests upon the

confidence we have in 'authority' of some kind or other. For instance, students rarely call in question the statements they find in their text-books, and consequently they are ready to state as true anything they read in them for no better reason than that it is 'in the book.' Indeed, most of us entertain, and express to others, beliefs that are based solely upon statements we have read in some magazine or newspaper, or have heard in a public address or in private conversation. To communicate, in this way, the opinions of others is doubtless a valuable social function, but the statements in which another's views are given currency should not be confused with judgments in the sense in which this term has been defined. We may, for want of a better word, distinguish these statements as groundless assertions, for what enables us to distinguish them from true judgments is the fact that, in respect of the latter, we are always more or less conscious of the reasons for what the judgment asserts. Our judgments are, as it is said, grounded judgments, and the grounds of the judgment are the reasons that thought can give for the particulars asserted by the judgment. For, as we have seen, the asserted particulars are always what thought reveals as true of the subject thought about.

The distinction that we have drawn in this section between groundless statements and judgment corresponds, in a general way, to that which appears in the history of Greek speculation as the distinction between 'opinion' and 'knowledge.'

Practical and Theoretical Aspects of Thought. Whether we use the term thought in the broadest of the meanings already distinguished, or in one of its more restricted senses, it is undoubtedly true that we cannot fully describe the experience that the term denotes without recognising the fact that, along with those differentiations which the object of thought undergoes, for example, in perception, in ideal representation, and in ideational thought, tendencies of a practical sort are aroused which not only accompany the thinking process, but, in part, give it the character and direction which it undoubtedly possesses. Thus the psychologists have been accustomed to recognise that instinctive impulses are present in perception which tend to place the percipient subject in relation to external objects; that in ideal representation active tendencies of the kind called desires are directed toward objects which, not being actually present, are held before consciousness as an end to be realised; and that in ideational thought the process of generalisation formulates ideals which are sometimes of so complex a character that, while they may become the objects of individual desire, they are attainable only as the result of social coöperation. Whether, therefore, in its simpler or more complex forms, thought seems to have, not only an aspect which is directly concerned with the definition of an object, a theoretical aspect, but also a practical aspect, a certain conative tendency whose function it is to direct the process

of thinking and to give its content a definite character and objective.

Relevant Assertion. From the point of view we have now reached, we may indicate certain features of the logical judgment which correspond, in a general way, to this distinction in the nature of thinking itself. The function of judgment we have said is assertion. We have now to point out that whenever assertion satisfies the logical conditions of truth, and is not mere or groundless assertion, it is seen to have two characteristics both of which are included under the single term 'relevant.' More explicitly stated, judgments, in the logical sense of that term, are always relevant assertions, and their relevancy is based upon, and is the logical expression of, the theoretical and practical aspects of the thought process which were indicated above. Judgments, in other words, always refer to some object, and fulfill some need of the person asserting the judgment. Whenever they do either or both of these things, our judgments are said to be relevant assertions. Briefly stated, judgments are assertions that are relevant (1) to the motives that operate to produce them, and (2) to the material by which they are called forth and to which they refer.

To consider the first meaning of relevancy, that which refers to the motives that operate to produce our judgments, we may say that this is concerned with the reasons why any one judges at all. If, then, it were asked why any one judges, the answer would be that one cannot 'get along' without it. In

a broad way, the life of thought is based in and grows out of our practical needs. As an illustration of what this statement means, we may refer to the development of agriculture in the history of civilisation. Before agriculture had become one of the institutions of human society, men no doubt were confronted with the problem of food supply, and they met this problem in a way that was fairly satisfactory until the accumulation of various forms of wealth made a predatory and wandering life both irksome and unprofitable. When, for instance, the domestication of animals had become an accomplished fact, this not only made a wandering life more difficult, but it also exposed the primitive community to greater loss through drought, long journeys, and marauding attacks from hostile and mercenary tribes. But unless fruits and cereals could be domesticated, as animals had been, there was before the primitive community the necessity of seeking food where it grew, whatever the risks and losses that might be incurred. Now it takes but a little imagination to see that the great difficulties that beset tribes advancing from a wandering to a settled mode of life would act as a stimulus to their thought, and would engage their thought in the direction of these difficulties themselves. It may seem to us who have grown up under settled conditions of life a simple matter to perceive the problem and the solution of which it was capable. But before the food supply of man and beast could be raised, within a restricted area, by man's efforts

from seeds, there was required exactly that development of the thought processes which the maintenance of the institution of agriculture calls for at the present day. What, therefore, we may say is that the principle of this illustration, namely, that our needs stimulate the judging activity and direct it toward an understanding of whatever threatens the satisfaction of these needs, is capable of indefinite extension; and that the incentives to knowledge become more numerous and diverse as, with the growth in complexity of our social life, obstacles are multiplied, and needs increase in number or change their form. Thus, over the whole course of our experience, thought not only interprets the environment, but determines the particular adjustments that, under change of environment, are necessary if the varying needs of life are to be met.

The relevancy of our judgments to the varying needs by which they are called forth may be further illustrated, and, for this purpose, let us suppose that we have a hundred books, and that our problem is what we are going to do with them. It is very probable, first, that we shall want to have them all in one place. Within the space allotted to them, we are likely to arrange them under some scheme of classification, — authors, subjects, or what not. Now why do we do this? Is it needless? By no means. We are simply obeying in our manner of dealing with the books a fundamental need for order. But what particular order we impose upon the books will depend upon the particular form that

the demand assumes. Thus, in the first instance, we bring all the books together in one place; place is the principle of our classification. In the second instance, we have carried the matter further, and have arranged the books, let us say, according to subjects, — subject-matter is a more specialised form of the need for order which operates throughout all our classifications. But if it is asked why any order is necessary, we can only say that when classified objects are more easily handled and used. The point of the illustration is that, when we have to deal with a hundred books, we are bound to think about the books in terms of our interest in them. Thus the first classification according to place might be satisfactory to the housekeeper whose interest in the books is æsthetic, — as articles of furniture in a room. The classification according to author would satisfy the interests of the man whose manner of speaking about books is, ‘Oh, yes, I have Dickens, Eliot, Thackeray, etc.’ The scholar’s interest is most likely to be met by the arrangement according to subjects. In each case, what we think about the books, — whether as articles of furniture, or as the works of such and such authors, or as dealing with given subjects, — depends upon the interest we have in them. And what is true in this particular instance is true in every case of judgment. Our judgments, in a word, are assertions that are relevant to the interests out of which they are born.

The foregoing statement and illustrations of the conditions under which the judging activity comes

into play indicate clearly enough what is meant by the practical nature of all our thinking. But because the word practical is, in ordinary speech, so intimately associated with the getting of material goods, or with the removal of material hindrances, the exact meaning of the statement that all judgment is conditioned by the presence of practical needs is liable to be given too narrow an interpretation. By a practical need, it is true, we mean one that is concerned with getting an anticipated good, or with removing a present hindrance to further goods. But the goods in question need not be material, and in many cases are not material. We have no wish to deny that man is an economic animal, and that what we call property is based in and grows out of his need for 'things.' But it is no less true that man is a social animal, and that the various organisations in which he is associated, — the family, the shop, the club, etc., — have their foundation in the needs which prompt men to associate with their kind. The social is, no less than the economic, a practical need. A similar line of remark applies also to our science, art, and religion. There are, doubtless, intellectual, emotional, and volitional needs, complex in their nature, and variously combined in the instances just given, which impel men to seek their satisfaction in the formulation of the objects to which these needs point. Over the whole range of life, then, situations are constantly arising in which unsatisfied demands impel us to think, and to think with reference to these demands. Nowhere

can this practical aspect of the judging activity find more or more pertinent illustration than in a study of the ways in which the human race has advanced in the arts of civilised life. But to bring the truth of this view home to ourselves, we need only to ask if we, as individuals, ever think when the situations in which we find ourselves are entirely satisfactory. Even a student, very often, will not think until a failure stares him in the face.

Turning now to the second meaning of relevancy, we may say that judgments are assertions about the material by which they are called forth and to which they refer. In the development of knowledge, our interests never become so one-sided that our judgments are freed from the control of the material through which these interests are to receive their satisfaction. If, for instance, we were conscious of nothing but compelling interests, be those interests psychologically never so definite, or if we were moved by a vague catholicity of mind that went out to everything in general, but took hold on nothing in particular, judgment would be impossible. There are states of mind that correspond more or less closely to those just described, states that are definite as to interests but indefinite, vague, or empty as to their objects; and others that are vague and indefinite in both respects; but no one would think of calling these states cognition, or the reactions they may involve judgments. What we aim at in the judgment is to characterise an

object in such a way that it becomes better known. The study of the judgment, therefore, is a study of the relation of thought to its object. Now, when one attempts to study the relation in which thought stands to its object, one is particularly prone to two kinds of error. In the first place, he is apt to think of the mind as a passive mirror that reflects indifferently whatever is held up before it. That is to say, he is liable to overlook the presence in all knowledge of the interest factor of which we have already spoken. If, however, he escapes this pitfall, he is prone to go to the other extreme and think of the mind as a magic mirror that transforms out of all likeness to their own nature the objects that pass in front of it. That is to say, he is liable to overlook or underestimate the material factor to which attention is now being directed. Interests, as we have seen, do make our judgments significant; but if they are to reach truth they must also be brought under the control of objective fact. No judgment can be regarded as having reached truth which does not open to us the nature of the object about which it undertakes to inform us. The fact, therefore, about the judgment is that, within the sphere of the interests that stimulate the judging process, the character of our assertions is limited by the nature of the material or subject with which we are dealing. For example, in writing this section of logic, I am prompted by the desire to write in such a way that the student who reads shall be able, with ordinary

attention, to understand the doctrine. But what the doctrine is does not depend upon this interest; what I say must be true of the subject we are discussing. However, this must be said, that what I include and what I omit, what form the sentences take, and what illustrations are used, are determined by the purpose and needs the section is intended to serve. I cannot say anything I please about the judgment, but I can examine it to see how far it lends itself to the satisfaction of the interests that lead me to write about it. We may take any other illustration we please, and the same thing will be found true. The difference, for example, between history and romance, other things being equal, is the difference between the way the historian and the novelist feel themselves bound by the character and order of the occurrences which they narrate. The historian's purpose is to describe a complex social situation as it actually existed, and to develop the consequences, in the way of cause and effect, of the situation thus depicted. The novelist's treatment of the same material is much freer from the control of the objective events within which, as in a framework, his romance is made to move. But it is only in the former of these two cases that truth is the aim; the relative unimportance of this aim in the novel accounts for the looser handling of the historical material. We may, therefore, conclude that interest or purpose sustains and directs the whole process in which our judgments are formed, and that what the judgment asserts, since all judg-

ments claim to be true, must derive its character from the material which constitutes the subject-matter of the process of judgment.

The Interest Factor in Knowledge. In the preceding section, we have seen that judgments are assertions that are relevant to the interests or purpose of the person who judges, and also to the material or body of facts which, in the judgment, receive a particular characterisation. Now it will serve to further emphasise this distinction, if we call attention more particularly to the way in which these factors of knowledge are related to one another. We may in general distinguish two cases. There is first the case in which our interest in the objects which we are led to observe is extrinsic to the objects themselves, and, secondly, the case in which it is primarily intrinsic.

To consider the former case, we may remark that we are often led to observe and make assertions about objects, not because of any particular interest in the objects themselves, but because of the relation in which these objects stand to other objects in which we are interested. Let us take as an example my observation of the shape, colour, and arrangement of the roses in the vase that stands upon the table at which I write. It is true, of course, that I may be induced to observe them, and to study their particular characteristics by reason of an interest that is rooted in my knowledge of floriculture. But, on the other hand, the attention that I give to the flowers may be due, for example,

to my interest in the person who picked them and placed them where, as I now write, they become the object of my perception. Or, if this is not the fact, my interest may grow out of my liking for a patch of colour of any kind, or for the particular combination of colours that the flowers present. In the former of these suppositions, the interest is social, and in the latter, it is æsthetic. In instances like these, we may say that the object sustains an interest that is extrinsic to itself, and gives to the interest a longer life than it would otherwise have. If, from this point of view, we speak of the flowers at all we should say that they were pretty, — a judgment it is true, but one which expresses one's feelings for the flowers rather than one which tells anything about the flowers themselves. Such judgments are usually called value judgments, — judgments which express the value or worth of the objects for the individual expressing the judgment. We express multitudes of such judgments in the course of our lives, and of a great deal that is noblest and best in our experience we can speak in no other way. The student will find that most of the judgments that he expresses on morals, art, and religion are what are called value judgments; judgments, that is to say, that are stimulated by interests that are extrinsic to the objects about which he judges.

If, now, we consider knowledge in the more scientific meaning of the term, we shall find that here the interest factor is intrinsic, that it is

grounded in the object in a way we have seen not to be the fact in the class of cases just studied. Scientific interest, in other words, is an interest in objects for their own sake. Of course, it is sometimes said that interest or purpose is altogether absent from our scientific activities, and that it is this fact that distinguishes science from all other kinds of knowledge. If all that has been written about the disinterested nature of scientific study were true, science would have been destroyed, before this, at the hands of its friends. The fact is, however, as our study of the nature of knowledge has shown, there is no knowledge which does not spring from some form of human interest. It is not true, therefore, that what we mean by science, in contradistinction to what is not science, can be stated in terms of its freedom from interests which direct the course and prescribe the goal that science aims to reach. What is true, on the negative side, is that certain kinds of interests are detrimental to the existence of science. But to deny the usefulness of certain kinds of interest, is certainly a very different matter from denying all interest whatever. For science, as was pointed out by Plato and Aristotle, would have no existence were it not for that curiosity and wonder which is awakened very early in all of us by the changing appearances in the course of nature. It is this curiosity which, when specialised, is the source of scientific interest, and which, in its developed form, is an interest which centres in the objects which it prompts us to study.

In other words, the interest which is the motive of scientific effort appears in the form of a desire to comprehend the nature and relations of the objects of our experience. The better we understand these objects, the more alive does our interest in them become; and the more alert this interest, the more keen do we become in our study of these objects. The interest and the object of scientific work are thus keyed to each other in a way that makes them mutually supporting; and the further we study their relations, the clearer does it become, as was intimated above, that they are different aspects of a single process. Scientific interest, then, is an interest in objects for their own sake; it is an interest that prompts us to ascertain all that can be truly known about these objects.

The Object of Judgment. If we now pass from the study of the relations of interest and subject-matter upon which we have been dwelling, and regard the judgment as an assertion about its subject-matter, certain other features of the judgment come into view. In our previous discussions, we have had occasion to specify what is meant by the subject-matter of the judgment, and this, it will be remembered, was found to be identical with what, from the psychological point of view, is denominated the content of thought. Thought and judgment are thought and judgment about some thing. Without, therefore, the challenge of some thing neither thought nor judgment takes place. But while thought and judgment imply the existence of a

subject, neither, in the strict sense of these terms, is identical with the existence of its subject. There seems to be, therefore, some other feature of the judgment which distinguishes it from mere awareness, and it is this feature that we have now to consider.

We may perhaps bring the question of discussion distinctly forward by the aid of an illustration. Let it, therefore, be supposed that I am gazing out of the window while thinking about the distinction, within the logical judgment, that it is now my desire to state. I turn the subject about which I am thinking this way and that, experimenting with it until my interest in it begins to wane. What I find, under the supposed conditions, is that the more interest in the original subject wanes, the more insistently do factors from other parts of my environment obtrude on consciousness; and by and by I find myself wholly engaged with some of these, — the sloping hill side, the frozen lake, and the lively company of girls and boys skating upon its surface. In such an experience, what one observes is that not only has there been a change from one subject to another, from a logical problem to a physical object, but also that changes of another kind have accompanied those already indicated. The two consecutive contents of the thought processes, that is to say, have been followed by changes in the character of what, in each instance, is asserted. I do not find, in other words, that what I am thinking when engaged with the logical problem fits in with,

or can be carried over to, the scene in the distance which has now become the content of my thought. Each content has its own appropriate set of assertions. To assert of its subject significant qualities and relations is what we understand to be the object of the logical judgment. It will serve to emphasise the distinction between the subject and the object of judgment if we recall that Bradley discriminates in the judgment between what he calls the 'that' and the 'what.' By the 'that' of a judgment we may mean the content or material of a thought process, that is, the subject of the judgment as we have used that term. By the 'what' may be understood the complex asserted by the judgment. For instance, the joyful scene upon the frozen lake, of an illustration already used, is the subject-matter of any judgment that I may make about it. In Bradley's terminology, it is the 'that' of the judgment. If, however, I assert that there is danger from the ice because of a thaw, and the children should be warned, this would be the object or the 'what' of the judgment, the complex whole that it is the business of the judgment to assert.

More recently a similar distinction has been drawn by the Austrian psychologist, Meinong. Meinong insists that we must not confuse in any act of judgment that concerning which we judge, and what we judge about it. The distinction that is thus drawn for all judgments is that with which we are familiar in the case particularly of legal

judgments. Let us say that a person is accused of a crime. In such a case, we may ask, What is it that the judge and jury are expected to judge? In one sense, the object of their judgment is the prisoner, and in another, it is, let us say, that he is not guilty of the offense with which he is charged. There seems, then, to be two objects of judgment, and, as this inevitably leads to confusion, Meinong thinks it important that they should be clearly distinguished. For that purpose, he proposes to use the term 'object' to denote what we have described as the subject-matter of the judgment, that is, in the case supposed, the prisoner; and the term 'objective' to denote what we have called the object, that is, in the case supposed, the verdict of the judge and jury. Now Meinong is perfectly right in saying that the object peculiar to the judgment is what he calls the 'objective,' that the judgment exists for the purpose of asserting something about a definitely apprehended subject-matter, and he is also right in calling attention to the importance of fixing this distinction in an appropriate terminology, because, in the absence of such terminology, of the tendency to confuse the two kinds of objects.

The Universe of Discourse. In view of the course that our discussion has taken up to this point, the logical judgment may be defined as an assertion of, or of something about, the objects of our experience. We may, for example, assert that so and so is the object of which, at the time the assertion is made, we are thinking, or we may assert certain other

things, besides its existence, of such an object. In each of these cases there is judgment because the object is apprehended as having a particular character, of existence or something else. The judgment, consequently, always implies a relational content of some sort, that is, an object qualified in a particular way, and it is such relational contents, as we have seen, that it is the specific aim of the judgment to ascertain and assert. If, for example, the words 'dog,' 'automobile,' 'zeppelin,' are not mere sounds, but significant terms, they must mean something more than they express; either that something not specified is 'dog,' 'automobile,' 'zeppelin;' or that the objects denoted by these terms exist; or that they have characteristics which distinguish them from all other objects; or that they stand in an indefinite number of relations to each other, and to numerous other objects. Each of these alternatives illustrates the peculiar character of the judgment, since each of them specifies a content of thought in which relations of some sort are involved. The special object of the judgment, as we have seen, is to assert just such relational contents as these.

The question may now be raised whether this is the whole story of the judgment, whether, that is, the asserted content embodies the whole meaning of an act of judgment. To this a negative reply must be given. Such an answer is necessitated in part by the fact, as we have seen, that our concrete acts of judgment are not satisfied with the assertion of any content, but only with those that are ger-

mane to the motives by which the acts of judgment are controlled. But the negative answer is rendered necessary also by the fact that the subject qualified in the judgment, distinguished as the logical subject, is only a part of the actual subject, distinguished as the real subject, which the judgment, taken as a whole, is understood to qualify. If this is true, a judgment asserts only part of what it means, and we must understand that the unasserted part of its content is related to the judgment as the judgment's presupposition. On this view, every judgment must be presumed to involve or presuppose, besides and as part of what it asserts, a wider range of fact in the light of which the particulars which form the content of the judgment's assertion are to be interpreted. The range of fact presupposed by any of our judgments is described in logic as a universe of discourse.

The meaning and truth of this statement will be readily seen in a simple illustration. Any proposition which contains, for example, the term 'style' is at once recognised to mean one thing if the topic of conversation is current modes of dress, and that when the topic of conversation is literature it means something quite different. Thus, if we take the familiar saying, *Le style c'est l'homme*, in abstraction from all contexts, its meaning is certainly ambiguous; and the only hope of removing this ambiguity is to restore the saying to a context in relation to which it can be significantly asserted. Or, to borrow an illustration from Major in discussing a closely

related topic from the psychological point of view, we may say that "if one is engaged with astronomical matters, the word 'star' means a heavenly body. . . . But if the general topic of our discourse is theatres or ball games or decorations, the meaning . . . of 'star' will likely be very different." From these and similar illustrations it is evident that the meaning of what is asserted in any judgment is rendered particular as much by virtue of what it presupposes, that is by its universe of discourse, as by what it definitely asserts. We may say, consequently, that it is the intention of every judgment to acknowledge the truth of what it asserts in the sphere of what the judgment presupposes, that is, in some definitely implied universe of discourse. To apply this view to another case, we may say that the statement, 'Mr. Wilson was elected by the popular vote of the people,' asserts a fact that qualifies in a unique way the political history of the United States, and not a fact that sets Mr. Wilson, by virtue of his election to the presidential office, apart from his fellow citizens. Mr. Wilson's citizenship is unaffected by his election, and, therefore, it cannot be in respect of this that our judgment is significant, but the election does determine in a particular way the political history of the Republic, and this, therefore, must, in logical terms, be the universe of discourse within which our judgment of the election must be understood to fall.

It follows from this view of the judgment that the universe of discourse implied by any judgment

exists always in a relatively indeterminate and incomplete form. This statement does not mean, although it is sometimes true, that particular judgments indicate only vaguely the universe within which their meaning is asserted. It means rather that, in the particular which the judgment asserts, the universe is found to be either not determined at all, or not to be determined with an explicitness that renders it serviceable for an immediately controlling purpose. Whatever degree of determinateness the general field or subject presupposed by any of our judgments may have, that field or subject can be used as a universe only if it is thought capable of receiving, through judgment, a more concrete and determined form. But it also follows from the view we are developing that judgments themselves are incomplete and point beyond themselves. They are, as it were, fragments torn from their contexts; but, even so conceived, they carry with them, as a rule, evidence of their connection with the contexts from which they have been torn. Unless this were so, single judgments would always remain ambiguous. But if this were the case, if, that is, there existed even a considerable number of judgments from which we are unable to remove their ambiguity, because they do not point with sufficient definiteness to an interpreting context, the fact that, apart from such contexts, they remain ambiguous is in itself evidence that they lack something which is characteristic of significant assertions. The meaning, therefore, of any judgment is

the meaning that it comes to have by virtue of its pointing relevantly to something which is not identical with the relational content asserted by it. In so far then as a judgment has to be supplemented by what its context supplies, so far is it incomplete; and if this incompleteness is to be overcome, it must be by what the judgment enables us to add through its presuppositions.

If, in view of this discussion, we now define the judgment, more fully than we have done hitherto, as an assertion of, or of something about, the reality presupposed by the judgment, we are immediately confronted with a question concerning the kinds of reality presupposed by our various judgments. In respect to this question, it has been maintained, for example by Bradley, that the real subject of which the thought-contents asserted by all our judgments are predicated is the totality of really existing things which we call the universe. Whatever any judgment asserts, just that, so it is maintained, is asserted of reality in the sense of the system of inter-related existences known as the universe. Reality, according to this view, is always that about which we judge, and our particular judgments assign to it, as its subject, the concrete details which such judgments assert. On this view, no question as to what is the universe presupposed by any judgment could arise, or if it did, it would be answered before it was asked.

Now, in considering this view, we do not need to inquire whether in the end we should be obliged

to take some such position as has just been stated, although there seem to be difficulties in the way of its adoption; what rather is more to our present purpose is to remark that such a statement does not represent the actual circumstances in which many of our concrete acts of judgment are made. For instance, the series of judgments in which the ancient mythologies state the relations of the gods and goddesses to one another, and to natural events and human history, cannot be said to be made of the universe of really existing objects in the sense that the series of judgments which constitute the body of one of the natural sciences can be said to be made of that universe. If, further, we consider the judgments in which we express social, ethical, and æsthetical facts and relations, judgments, as we have seen, which are not descriptive in the sense that scientific judgments are descriptive, but which refer human behaviour and natural phenomena to a norm or standard, it is difficult to maintain that these are made with immediate reference to the universe which sums up in itself all that may be called real. Whether the facts and relations that constitute the subject-matter of mythology, of sociology, ethics, and æsthetics can be asserted of the universe of which the facts and relations of the natural sciences are asserted, is not the question that confronts us; the question rather is whether, in the acts of judgment which give definite form to our knowledge of these subjects, they are so asserted, or whether they are asserted of other

universes than the one which is existentially real. If we admit, as we must, the latter alternative, if, that is, we admit the existence of a number of universes, other than the real, within which judgments may fall, the question whether the real universe includes all these, as is sometimes maintained, is still left open, and we do not need to close it, for the reason, if for no other, that the problem involved belongs to metaphysics rather than to logic. Logic is interested in the question only for the purpose of emphasising the importance of ascertaining the universe in relation to which, as a matter of fact, particular judgments are asserted as the condition of correctly interpreting such judgments. Thus if we were confined to the view we are criticising, the assertion that 'Fishes are animals which get their oxygen from the water in which they live through gill slits,' and 'The Royal dragon of China has five claws' would have to be taken as qualifying a single universe; but surely no one not blinded by a theory could fail to observe that in the former case we are asserting in the world of zoological fact, and in the latter in the world of Chinese heraldic design. The view, therefore, that seems most serviceable, and the one which keeps closest to the facts of our ordinary experience, is the view that emphasises the importance of bringing particular judgments into relation with their immediate contexts whatever these may be, and warns us against resolving the distinctions by which these contexts are characterised into a single context about which the only

intelligible thing that can be said is that, in some sense, it is real or exists. In this connection it may be remarked that for logic the important question is in what sense the reality or truth of an assertion is to be understood.

Truth, Probability and Error. Truth, probability and error, as employed in logic, are terms which describe characteristics that belong to assertions, they are never employed, as they sometimes are in popular speech, to designate qualities of the objects about which the assertions are made. From the standpoint of logic, objects exist, and they are only the assertions which are made about particular existing things that are said to be true, probable, or erroneous. Truth, probability, and error, then, can never be predicated of things, but only of our judgments of, or about, things.

Now, since all judgments claim to be true, we have to inquire what we mean by truth, and how there can be such things as probability and error. With regard to the meaning of truth, it is pretty generally agreed that the notion is so nearly ultimate that we find it difficult, if not impossible, to offer a definition which does not involve the notion itself as one of the terms. But if truth cannot be defined, we can describe, in a more or less satisfactory manner, what we mean when we use the word. We may say, then, that an assertion is true when it assigns a character to the elements and relations of a subject-matter which are found to be verified when this subject-matter is investi-

gated by appropriate methods. Thus the judgment, 'The fire is hot,' is true, if, when I put myself into sufficiently close proximity to the fire referred to, I experience the sensation called heat. Truth, then, is not a quality of judgments merely as such; it is a quality that belongs to judgments *ad hoc*, that is, as assertions of definitely specified contents. When, in other words, the ideas conveyed by a judgment are such that, in the universe presupposed by the judgment, there are facts which exist in the manner it asserts, the judgment is said to be true. Truth, that is to say, is always a claim that is set up on behalf of a judgment's object, in behalf, that is, of what Meinong called the judgment's 'objective.'

In view of this general statement, it may be asked whether any judgment is true, or whether judgments are at best only probable. The answer to this question obviously depends upon whether we ever need to go beyond a judgment's assertion to ascertain whether the facts and relations asserted by it are as they are asserted. When, for example, we assert that 'The sun is the source of the world's heat,' it is obvious that the verification of the assertion must be sought in astronomical physics, that, in other words, the judgment is true conditionally upon our being able to verify, in the appropriate universe, the facts and relations asserted in the judgment. But not all our judgments are, in this sense, assertions about a given subject; judgments are also assertions of a given subject. If, for example, we say, 'The subject about which we are

thinking is the sun as the source of the world's heat,' it is obvious that, whatever physics may have to say about this particular subject, it can have nothing to say that throws doubt upon the fact that, at the time the assertion is made, we are actually thinking about this particular subject. We select an illustrative example from the world of physical objects because arguments for the absolute contingency of truth have sometimes been framed on the supposition that contingency can be removed only when all the factors of the judgment are merely states of one's own mind. It is often said, for instance, that the certainty that belongs to mathematics is rendered possible by the fact that the fundamental notions of mathematics are concepts and not really existing things. Whether this view of mathematics is correct or not, it is certain that the judgments which assert these fundamental notions are no more indubitable than are those judgments which assert some object in the physical world as the object of thought's activity. It would seem to follow, therefore, that all judgments which are assertions of particular objects as the objects of thought are true in the sense that it is not possible for those who entertain them to be in any doubt that those are the objects with which thought is engaged.

Judgments which make assertions about, and are not merely assertions of, their subject-matter are in a different position from those just considered. The chief difference is that in their case we have to go outside the judgment's assertion for the veri-

fication of the truth of what such judgments assert. The grounds, that is to say, on which the truth of these judgments rest are to be found in the universe to which what the judgment asserts is known to belong. The fact to which attention is in this way being directed is that the assertions we make about the objects of our experience do not derive their truth from the fact that they are asserted, but from the fact that there can be shown to exist in a given universe facts and relations of the kind that our judgments assert. This interpretation of the truth of our judgments should not come as a surprise to those who have followed the discussions of this and the preceding chapters, for the demand for a test of truth is after all merely a challenge to refer one's judgments back to the conditions which, in the first place, made them possible. In this way we are enabled to show that, when all the motives that, in the first place, conditioned the judgment are operative, the assertion contained in it is just the assertion that that situation compels us to make. Thus when the chemist's assertion that the liquid contained in a particular bottle is sulphuric acid is called in question, he does not attempt to prove the truth of his judgment by an emphatic restatement of it, nor by any form or number of reiterations; what he does, if he wishes to remove the doubt that the question has raised, is to apply the tests for this substance which, in the first place, convinced him that it was sulphuric acid and not something else. We may conclude, therefore, with reference

to the class of judgments under consideration that their truth is conditional upon our ability to apply tests of a kind suitable to the character of the subject-matter about which the assertions are made, for it is only when tested that we are able to compare what is asserted with what happens in situations similar to those in which the judgments in question originated. In the absence of such tests, judgments of this class can only be said to have more or less probability.

There are situations, however, in which we are either unable or unwilling to verify our judgments, and are thus precluded from determining the particular degree of credibility that properly belongs to them. In these cases, the claim to truth which all judgments maintain on their own behalf can be acquiesced in as little as it can be dissented from. There belongs, consequently, to the judgments in question the probability that attaches to any unverified assertion from whatever cause it may arise.

It is because so large a number of our social judgments are incapable of the strict proof that can be insisted upon when the subject of the judgment is some part of the material universe that, as a class, they have generally been regarded as possessing a lower degree of certainty than that which seems to belong to some of the established conclusions of the natural sciences. We cannot revive in the form of actually existing situations, for example, the circumstances which determined our actions of a week ago, and we cannot, therefore,

verify the judgments of which these actions are the practical expression. That at the time indicated and in relation to the situation then existing we did judge so and so, is as certain as any judgment can be; but that the judgment at which we then arrived was true is, in the nature of the case, beyond the possibility of strict proof. It would certainly be a very gratuitous form of scepticism to repose no confidence whatever in any social judgment, because all such judgments labour under this disadvantage; and to suggest its possibility is sufficient to expose the absurdity of such a course. For it may be affirmed that the disadvantage in question is not confined to the social judgments we are now considering. It may be urged that some of the judgments that are made about the material universe are incapable of the experimental proof to which our exposition has pointed as the test of truth. Our belief that the sum total of physical things is a universe, is, that is, a sphere of material existences whose changes are all subject to law, is not beyond question; and when the belief is challenged, as it has been recently, we cannot put the material universe itself in evidence on either side of the question. Such a belief, and the judgment in which the belief is asserted, can have no other kind of certainty than that which belongs to the social judgments of which we have in particular been speaking. Current belief undoubtedly regards the moon as a sphere, but, on account of the particular configuration of the solar system, it can never be a practically

demonstrated fact that the side which is always turned away from our earth is not flat. Ether, ions, centres of force, and atoms themselves are all subject to the same kind of doubt, whenever any one, for what seems to him sufficient reasons, is inclined to call them in question. In view of these instances, it is only fair to conclude that judgments about any sort of material must be regarded merely as probable whenever we are unable to show that facts and relations of the kind asserted do exist in the universe that is presupposed by these judgments.

Again, judgments must be classed as probable whenever we are unwilling, for a reason that seems sufficient to us, to apply the test of truth. When, for example, one refuses a cup of coffee of one's hostess with, 'No, thank you; coffee always means a sleepless night,' the judgment by which we excuse or explain our refusal has no doubt a certain probability derived from past experience, but it is one that, for prudential reasons, we are unwilling to verify in the particular circumstances which are the occasion of its assertion. Sleepless nights, we may admit, have followed the drinking of coffee in the past, and it is anticipated that a similar result will follow on the present occasion. In a case like this, which is typical of many others that involve future consequences rather than past acts, we should be going beyond the bounds of necessity if we acceded without reservation the certainty that the judgment claims on its own behalf. We feel, perhaps, that the judgment in question has a high

degree of probability, but whether it deserves the confidence, say, that is commonly reposed in the prediction of a solar eclipse, is a matter upon which it is not necessary to dogmatise. It is more to the point to observe that the class of judgments that we are discussing is conditional, and, consequently, that it is only if we suppose the present situation is in all essential respects similar to those in which the consequence has followed in the past, that we have sufficient reason for believing that a like consequence will follow on the present occasion. But it is exactly this supposition of which the judgment itself is unable to give assurance, and which therefore renders the judgment less than certain until it is submitted to the process of proof.

We may now pass to the meaning of error. And, in the first place, error must be distinguished from mere irrelevance. Irrelevance may exist in either of two forms. It may exist, first, as an incompatibility in the content of what a judgment asserts. If, for example, Shylock's bond had been drawn for a pound of justice, and not for a pound of flesh, there would have been an irrelevance of the kind in question, because the avoirdupois scale cannot in any sense be made the measure of justice. The universe suggested by the assertion, 'I will have a pound of justice,' is one in which the content, on account of its internal contradiction, cannot possibly get realised. Irrelevance in this form, then, arises whenever we predicate of a subject qualities

and relations which cannot be realised in the universe to which the subject is thought to belong. Instances of this occur whenever we attribute to purely natural events moral or æsthetic significance, or when we explain any of our ideals in terms of mechanical causation. Secondly, irrelevance may exist as an incompatibility between what is asserted and the purpose by which the assertion is controlled. What in these instances is asserted may be true, as, for example, 'Mont Blanc is snow-capped all the year round,' but no one would think of making or entertaining the assertion in a discussion on the value of Rotterdam as a Dutch port of entry to the German Rhine. Failure to understand the limitations that a guiding purpose or interest imposes on what may and what may not be significantly asserted about a given subject-matter is one of the most fruitful sources of confusion in discussions of all sorts, and it is to this form of irrelevancy that we look for the origin of those half-truths which do more mischief in the world than frank and unblushing error. The comment is worth while in this connection because it is to the avoidance of such irrelevance that the student may look as one of the means of hastening his academic and practical efficiency. In either of the two forms which we have now distinguished, then, irrelevance is not to be identified with error.

Error, in contrast with the second of the two forms of irrelevance just distinguished, is an assertion which is relevant to a purpose, and, in contrast

to the first, is an assertion whose contents are compatible. Unless it possessed both these marks of a true judgment, error could not possibly exist. To emphasise the former contrast, we may say that error is ascribed to any judgment which, if true, would satisfy the purpose for which the judgment is made. There follows from this statement a consideration which is important for all our judgments, namely, that the range of assertion about any subject-matter is determined at the outset by the character of the purpose which stimulates and directs the judging process. In other words, to expect our judgments to be true is a perfectly reasonable expectation, but to expect that they will assert the whole truth is to convict our expectation of being either unmeaning or false. The latter expectation is indefensible for the same reason that the former is maintained, namely, that we do not as a rule, nor can any reason be advanced why we should, encumber our judgments by asserting non-significant features of the subject-matter to which these judgments refer. And if it is asked, 'What is and what is not significant?' we can only reply that that all depends upon what the judgments is wanted for. The truth of this statement can be readily seen in what occurs when all interest in a question vanishes. In such a situation nothing whatever can get asserted. But when an assertion is made, it must be made, not merely of something, but for some purpose, and it is the character of the purpose that determines what feature of that 'something'

is selected for assertion. There is, for example, no doubt that the grass is green and, let us say, that it is long, but it is only the latter assertion that is pertinent when one has to decide what instructions to give the gardener about the day's work. What one thinks about any object, say a house, depends upon whether we are considering it architecturally, domestically, or as an investment. These interests, no doubt, overlap, but they are sufficiently distinct to make quite irrelevant certain assertions that might be made from one of these points of view when, as a matter of fact, we are considering it from one of the other points of view. But we do not call judgments false which bear no relation to the interest by which the judging process is initiated. Error as well as truth exist only in those judgments which claim to fulfill the purpose of the individual by whom they are asserted.

The second contrast stated above calls attention to the fact that the contents of erroneous judgments are, as we said, compatible. That is to say the judgment must assign its content to a sphere in which it is possible for that content to be realised. The absence of incongruity between the elements of what is asserted is a characteristic of error as it is of truth. When, for example, we judge that an object is elliptical when in reality it is round, we are in error, but the error does not consist in assigning it a shape, but in attributing to it the wrong shape. The wrong shape, however, is an antecedently possible shape, and it is upon this ante-

cedent possibility that the erroneous character of the judgment rests. If instead of asserting that the object was elliptical we had asserted that it was blue, we should not regard that as an error; we should regard it either as a misunderstanding of the motive of the judgment or as nonsense.

We are now in a position to state what error is. The problem of error is greatly simplified when it is observed that no judgment which fails to assert those features of its subject-matter which are pertinent to the purpose in hand could by any possibility be regarded as true or erroneous. They are, as we saw, merely irrelevant. It is only when judgments offer themselves as the means of satisfying a purpose that they are ever entertained. There is, therefore, from the standpoint of the interest involved, little chance for error. All that we can say about a judgment, from this point of view, is that it does more or less completely fulfil a present purpose. It is, therefore, to the character of what is asserted that we must look finally for the detection of error. And here, as in the corresponding case of truth, we are confronted with a simple question of fact. Is it a fact, we must enquire, that there exist in the universe presupposed by the judgment qualities and relations of the kind asserted in the judgment? If there are, we call the judgment true; if there are not, we call the judgment erroneous. Error, then, may be defined as some discrepancy between assertion and fact.

We have been speaking in this section of error as a failure of the judgment to assert the significant qualities and relations of the object upon which thought is directed. The term is also used to denote those mistakes of judgment which depend, for example, upon inference. In this sense it is preferable to speak of fallacies. A discussion of this subject will be found in the concluding chapters of this book.

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QUESTIONS

1. What general meaning may be given to the term object?
2. In what several ways may we be acquainted with objects?

3. Is the character of an object affected by the way in which it is apprehended?
4. What do you understand by objects being implicated in all mental processes?
5. Distinguish three senses in which the term 'thought' has been used, and state how they are related to each other.
6. To which of the meanings already distinguished is thought, from the logical point of view, most closely related?
7. What particular aspect of thought does logic emphasise in its use of the term judgment?
8. What is meant by saying that the subject-matter of judgment is the content of a thought process? Illustrate.
9. Explain: 'Judgment is an assertion of the qualities and relations of the objects of thought.'
10. Is language necessary to give thought the character of assertion? Explain.
11. State clearly what you understand by assertion. Illustrate.
12. Show in what ways judgments are implied, but not expressed, in interrogatives, imperatives, optatives, and exclamations.
13. Describe what is meant by groundless statements, and in what respect do they differ from judgments?
14. Define and illustrate the practical and theoretical aspects of thought processes.
15. 'Judgments may be defined as relevant assertions.' How does this view of the judgment agree with and differ from that stated in Question 9?
16. In what two respects may all judgments be said to be relevant?
17. Explain what is meant by saying that judgment is based in and grows out of our practical needs. Illustrate.

18. Define what you understand by 'practical,' and show how unsatisfied needs of the sort here in question condition the formation of judgments.
19. How is your answer to the preceding question affected by the fact that judgments are assertions about a subject-matter of some sort?
20. To what two kinds of error is one exposed in trying to show the relation of thought to its object?
21. Explain and illustrate what is meant by saying that the interest that conditions a judgment may be extrinsic.
22. What do you understand by a value judgment?
23. Describe the nature of scientific interest, and explain why it may be called, in contrast to other kinds of interest, intrinsic.
24. Is judgment to be distinguished from awareness of an object? Explain.
25. What difference, if any, do you recognise between the subject and the object of judgment, and illustrate by referring to Bradley's distinction between the 'that' and the 'what.'
26. Explain what Meinong means by the 'object' and the 'objective' of logical judgments.
27. How may Meinong's distinction be correlated with the view taken in the text?
28. 'Judgment may be defined as an assertion of, or of something about, the objects of our experience.' Explain this statement, and correlate it with the definitions of judgment in Questions 9 and 15.
29. Why is the definition of judgment given in the preceding question not completely satisfactory?
30. What do you understand by a universe of discourse? Illustrate.
31. Why must any universe of discourse have a more or less indeterminate character?
32. Why must single judgments be regarded as incomplete, and, therefore, as being to some extent ambiguous?
33. How may this ambiguity be overcome?

34. 'Judgment may be defined as an assertion of, or of something about, the reality presupposed by the judgment.' Explain and compare with other definitions given in Questions 9, 15, 28.
35. What, according to Bradley, is the universe presupposed by all our judgments?
36. What objections can you urge against Bradley's view?
37. Is there a plurality of universes? Justify your answer and illustrate.
38. Of what are truth, probability and error predicated in logic?
39. In what respect does the logical differ from the popular use of these terms? Illustrate.
40. What is the logical meaning of truth? Illustrate.
41. Of what class of logical judgments can you say that they are certainly true? Illustrate.
42. Can the truth of any of our judgments about a given subject-matter be maintained? State reasons for your answer, and illustrate.
43. What two conditions compel us to regard certain of our judgments as probable, not true?
44. Why do we generally regard judgments about social material merely as probable?
45. Do similar reasons apply to any of our judgments about the material world? Explain and illustrate.
46. How does unwillingness to verify an assertion affect its claim to be true?
47. In what two forms may irrelevance exist? Explain and illustrate each.
48. How do you distinguish irrelevance from error?
49. Why do we not call those judgments which fail to fulfil a purpose erroneous? Illustrate.
50. What is meant by saying that in order to be erroneous judgments must assert contents that are compatible? Illustrate.

51. Wherein consists the nature of error?
52. Can you frame a definition of error?
53. In what respect does error differ from fallacy?

EXERCISES

1. In what sense is the word, thinking, used in the following statements:
 - (1) All human beings are thinking beings.
 - (2) There is no truth or error but thinking makes it so.
2. State at least two judgments about each of the following subjects:
 - (1) Mathematics as a required subject for college graduation.
 - (2) Inter-collegiate football as a desirable form of student activity.
3. What are the grammatical forms of the following, and show in what sense they may be said to involve judgments:
 - (1) O wad some power the giftie gie us
To see ourselves as others see us.
 - (2) How oft the sight of means to do ill deeds
Makes deeds ill done.
 - (3) Can one desire too much of a good thing?
 - (4) Are things what they seem?
Or is visions about?
 - (5)

Give me that man

That is not passion's slave, and I will wear him
in my heart's core, ay, in my heart of hearts,
As I do thee.
4. Give six illustrations of what are called groundless statements.
5. State six judgments on subjects of your own choosing that show the influence of purpose on the judging activity.
6. Give six illustrations of the way that judgments are controlled by the nature of the subject-matter about which one thinks.

7. Take three instances of judgment from your answer to Question 5, and the same number from your answer to Question 6, and show in each case the relation of interest to subject-matter.
8. Take the remaining instances from your answers to Questions 5 and 6, and state what is the subject and the object in each case.
9. What can you say about 'water' in each of the following universes of discourse: domestic life; chemistry; physics; art; physiology.
10. Examine the following judgments, and state whether, and under what conditions, you regard them as true, irrelevant, probable, or erroneous:
 - (1) Everything comes if a man will only wait.
 - (2) For a man's house is his castle.
 - (3) The many still must labour for the one.
 - (4) Thought is free.
 - (5) Virtue is its own reward.

CHAPTER III

THE PROPOSITION

Judgment and the Proposition. In text-books of logic, it is usual to discuss the judgment and the proposition in the same chapter, and to emphasise the points of similarity between them. In the present chapter, we are to consider the proposition by itself, because in this way we shall be able not only to distinguish between it and the judgment, but to group together certain problems that arise out of the nature of the proposition which are more or less distinct from those of the judgment which were discussed in the previous chapter.

In its most general meaning, a proposition may be defined as the verbal expression of a judgment. As we proceed, this statement will be made more explicit; but it will suffice for our present purpose if it enables us to understand the importance of putting into verbal form the truths that are asserted in our judgments. The necessity of finding a suitable medium for expressing our judgments is forced upon us when it is realised that explicit statements of our meanings are, sometimes, if not frequently, important factors in our search for truth. We often find, for example, that ideas with which we had supposed ourselves perfectly familiar suddenly take on a penumbral haze when we are required to use these ideas in working out new problems, — for instance,

in solving an 'original' in geometry or in finding an 'unknown' in chemistry; or when we try to communicate these ideas to other people, — for instance, in a class recitation. In both these cases, we have before us the task of setting the ideas in question in a different context, and it is the new context into which they are to be fitted that reveals to us defects in the ideas themselves of which we were hitherto unaware. In order to succeed in the problem that is thus presented, it often becomes necessary to give verbal expression to our ideas, and our search for the suitable form is not infrequently found to react upon the ideas themselves to make them more exact. It may also be noticed that when we have once expressed our ideas in accurate verbal forms, we are able to analyse these ideas, and thus to adjust them to other ideas with which they have various relations. For it must be admitted that our interest in ideas is not merely to know that they are true, but, concurrently with this, to use them in enlarging the boundaries and directions of our knowledge. And it contributes to both these objects to secure for our ideas an adequate verbal expression. The statement of our judgments in the form of propositions both clarifies the meaning and renders the application of our ideas possible.

We may bring out the difference between the judgment and the proposition if we refer to the way that each originates in our ordinary experience. In the foregoing chapter, we have had occasion to indicate in a general way the conditions under which our

judgments arise. Judgments were there described as completed solutions of problems set by the situations in which one finds himself from time to time. That is to say, judgments arise out of whatever is problematical, undetermined, baffling, whether this falls in the sphere of perception, or in the sphere of the social, scientific, ethical, æsthetical, or religious life. Whatever sets a barrier to the satisfaction of wishes, whatever hinders, arrests, or makes impossible the carrying out of cherished plans, challenges and stimulates the life of thought, and forces us into a judgment. Judgments arise out of our needs for knowledge, and this need is stimulated when the situation which instigates the need is of such a character that our way to the successful conduct of the theoretical or practical life is blocked. When we have unravelled the tangle, and see the total situation in its bearing upon our arrested pursuits so that we either modify or change our aims, or they are given an open field, we may be said to have judged. It is another story when we ask what are the conditions that determine the rise of propositions. In the proposition we are not seeking to extricate ourselves from an embarrassment, because no embarrassment is present, and we are not seeking knowledge, because that already exists. A proposition, as we have already said, is the verbal statement of what we already know; it is a statement in which something is either affirmed or denied of something else. It is, consequently, not an instrument of the search for knowledge, but of its communication.

When we put our judgments into propositional form, we are making them socially available, and presenting them for acceptance or rejection.

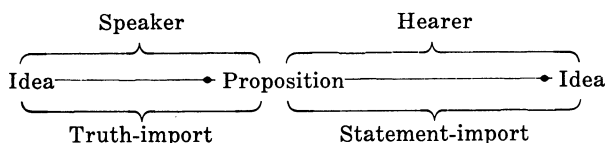
The Import of Propositions. A proposition may be interpreted from either or both of two points of view: from the point of view of the person speaking, and from the point of view of the person spoken to. In each of these cases, the proposition has a different import or meaning. In the former, where the person speaking is expressing a judgment concerning a subject about which he has reached a conclusion, the proposition means what the judgment which it expresses means, and must be interpreted as a judgment. That is to say, the truth of the proposition is also the truth of its corresponding judgment, and vice versa. The meaning of the proposition, interpreted from the point of view of the speaker, is designated in logic as the truth-import of the proposition. In the other case, when we consider the proposition from the standpoint of the hearer of it, the proposition and the judgment which it expresses are not the same, and the truth they express may not be identical. The reason why the proposition may mean one thing to the hearer and another to the speaker must be looked for in the differences of the relations that the hearer and speaker have to the knowledge that the proposition embodies. Or, to put the matter more definitely, the speaker's relations are with the knowledge which the proposition expresses, while the hearer's are with the terms of the proposition itself. When we interpret a propo-

sition from the standpoint of the person spoken to, we have what is called in logic the statement-import of the proposition. The statement-import is, consequently, a structural interpretation of the proposition; it is the meaning that a proposition has when it is considered from the standpoint of its factors or terms.

We may look at this distinction between the statement-import and the truth-import of propositions a little more in detail. It is obvious that the question before the speaker is one of selecting the words that will adequately convey his meaning. When one wishes to make a judgment socially available, one has to express the judgment in words, or in some other recognised means of communication. When, on the other hand, we listen to what is said, or read what is written, the question before us is to ascertain the meaning that the words were originally intended to convey. This is not always as simple as it may sometimes seem. Because, for example, the words in which we are addressed are familiar, it does not follow that the idea that the words express is at all understood. The common misunderstandings of ordinary life, and the more serious difficulties that we meet with in the course of our studies, are due very often to our failure to interpret aright the meaning, not to our inability to understand the words, of the propositions that are laid before us. In such a case, the proposition means for us something that it does not mean for the speaker; it has

for us, that is to say, a statement-import but no truth-import.

It will help our understanding of this statement if we throw the essential factors of our problem into the form of a diagram:



If, reading the above diagram, our interpretation of the proposition is made from the standpoint of the speaker, we start with the idea or judgment that has to be expressed in the proposition; the proposition is then the verbal expression of the judgment or idea from which we set out. For the speaker, the proposition does express his judgment, and so interpreted it embodies the truth of his judgment. If, on the other hand, our interpretation is made from the standpoint of the hearer, we start with the proposition, the verbal expression of the speaker's judgment, and the words of the proposition have to be interpreted so as to render an idea or judgment possible. But what is the guarantee in such a case that the meaning derived from the proposition will be the meaning expressed by it? The idea in the hearer's mind may be different from the idea in the speaker's mind. It is this idea, the idea or meaning that the hearer of a statement gets from what is said

to him, that we designate the statement-import of the proposition. The truth-import and the statement-import of a proposition, therefore, may not be the same.

But, it may be asked, does not our interpretation of a proposition, even when it is taken from the standpoint of the hearer, aim at an identity of meaning between its truth-import and the statement-import? This is doubtless true. But we should miss a great deal in our understanding of the life of thought if we did not see the possibility of the same statement meaning different things for speaker and hearer. It is this fact that we have been emphasizing. And it is as desirable to ascertain how this difference may arise as to see how it may be overcome. For if we understand the origin of such differences, we shall know how to avoid or overcome them. In either case, we are concerned with the context of the idea or judgment that we are trying to understand; and it is obvious that when the context is different for speaker and hearer, the idea the proposition conveys must, in some measure, be different for both. For, as we have seen, the contexts in which our ideas occur give part of the meaning that ideas may have. If, therefore, you change the contexts you change also in part the ideas themselves. Thus, whenever we fail to get the meaning of what is said to us by reason of our failure to understand the universe of discourse within which the thought of the speaker is moving, we do not naturally ask for a repetition of the sentence or

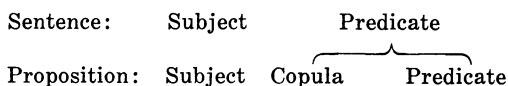
proposition, but require, for its comprehension, a larger section of the context, and, in some cases, the revival of the whole universe within which the truth of the judgment is, by the speaker, presumed to exist. In other words, it is the fragmentariness of all our judgments, to which reference has been made, that is the ground of the difference between the truth-import and the statement-import of propositions; a fragmentariness that can be overcome only if it is supplemented by a larger portion of the interpreting context than is frequently implied by the single proposition.

We may further elucidate the important distinction just made between the truth-import and the statement-import of propositions if we consider it as a special instance of the difference, first drawn by the English historian, Grote, and made current by the American psychologist, James, between "knowledge of acquaintance" and "knowledge about." An illustration sometimes used to indicate the contrast between these two kinds of knowledge is the difference between the normal child's experience of light, and the experience of light that a child, blind from birth, has through the reports of seeing adults. The knowledge of the two children, without doubt, is at different removes from the real character of light. That is to say, the contrast between "knowledge of acquaintance" and "knowledge about" calls attention to the fact that the knowledge that any one has is sometimes immediate and direct, and sometimes mediate and indirect. Now, the dis-

inction between the truth-import and the statement-import of propositions seems to be contingent upon whether propositions express either the one or the other of the above-mentioned kinds of knowledge. We are all aware that much that we ordinarily call knowledge is based upon nothing more certain than our ability to understand the words in which this knowledge has been communicated, while a much smaller fraction of our knowledge rests on direct acquaintance with the facts, and a study of the reasons by which such knowledge is supported. In the one case, propositions embody judgments, and are to be interpreted as judgments; in the other case, propositions are statements to be understood, whether they introduce us or not to the facts out of which the judgments which they express have come. To understand what is told us is a first step in knowledge; but it is only a first step, and should lead us beyond itself to a more intimate and personal acquaintance with the truths and facts that are the material of advancing knowledge. In other words, we must proceed from the statement-import to the truth-import of the propositions which convey to us so large a proportion of the material upon which the educative processes as a whole depend.

Formal Analysis of the Proposition. The proposition was defined above as the verbal expression of a judgment; it is the logical instrument by which what is true for judgment gets a simple and unequivocal expression. But not every verbal expression of a judgment is a proposition; it may be, for

example, a sentence. A sentence may be a proposition, as, for example, 'The book is on the table;' but this is not necessarily the case, as, for example, 'A friend of mine has gone abroad.' From the formal side, the essential difference between the sentence and the proposition lies in the number of the factors present in each. A sentence always requires the presence of a subject and a predicate; the proposition cannot exist without a subject and predicate, which are called terms, and also a copula. If we compare the sentence and the proposition in these respects, we shall see that the predicate of the sentence is, by logic, broken up into two parts, the copula and the predicate of the proposition, and that the subject remains the same. The following diagram represents these relations to the eye:



The doctrine of formal logic depends upon the structural difference between the proposition and the sentence as thus outlined. In the following sections, therefore, we shall dwell on the meaning that logic assigns to the several parts of the proposition.

Terms. The subjects and predicates of propositions are called terms. Now, since the proposition is a statement in which, as we have seen, something is predicated — affirmed or denied — of something else, we may define the predicate term as that which is predicated, and the subject term as that of which

the predication is made. If, as is usual, we use the symbol S for the subject term, and the symbol P for the predicate term, the structural proposition may be written S—P. In such a proposition, P is said to be predicated of S. We may observe that the subject term of a proposition is always written first, and the predicate term last. The reason for this seems to be that since the proposition is determined by the interests of communication, it is important, at first, to call attention to that about which we wish to speak. If the student remembers this he will avoid the difficulty that is sometimes experienced in distinguishing between S and P. For example, if we wished to express in propositional form the statement, 'Uneasy lies the head that wears a crown,' it would not be accurate to say that 'Uneasy' is the subject. The rule that may be followed, whenever we are in doubt as to what our terms are, is to ask: What am I speaking about? The answer to this question will give us the subject term. The predicate term may be found by asking, What am I saying of this? If we apply these tests to the example just cited, we shall find that we are speaking about 'The head that wears a crown,' and this, consequently, is the subject term of the proposition; and that what we are saying of this is that it is 'uneasy,' and this, consequently, is the predicate of the same proposition.

It should be remarked that we are frequently at a loss to know which is the subject, and which the predicate, term of given statements. The reason

for this seems to be that in order to know what any statement means we require sometimes to have before us a larger section of the whole context of thought than the particular statement itself provides. This takes us back to what we said above about 'the universe of discourse,' and the function it serves in giving to our particular judgments the meaning they are intended to have. Propositions express judgments which are torn loose from their contexts, and they sometimes do, and sometimes do not, retain sufficient of the original context to make our interpretation of them certain. The example that was quoted above illustrates the former case; we were doubtless talking about 'crowned heads.' A more difficult case, taken from Jevons, is the following: 'Life is held dear by every man.' In this statement, are we talking about 'life,' or about 'things that are held dear,' or about 'every man'? Grammatically, 'life' is the object of the verb 'holds dear,' and cannot, therefore, be the subject. The meaning of the statement, consequently, is: 'Every man holds life dear.' We are speaking in the universe of values; and, in the statement, we are naming one of the objects of value. If, however, to take an illustration from Sidgwick, we say that 'Newman wrote the Grammar of Assent,' we have no means of determining whether 'Newman' or 'The Grammar of Assent' is the subject, because, as Sidgwick says, "the statement is regarded in isolation from all context. Regarded in this abstract way, it may equally well be called a statement about Newman,

or about the book, or again about both the man and the book." It would seem, then, that back of all particular statements there lies a universe of fact or idea which flows over into these statements to give them the determinate meaning they may properly have.

Names and Words. Another question, closely connected with the foregoing, arises when we ask whether terms can exist outside the propositions in which they are found. We may ask, for example, whether 'The head that wears a crown,' which, as we saw, was the subject term of the proposition which was studied above, is always a term, and in whatever connection we may meet the phrase. Or, we may inquire, is it a term only when it occurs in a proposition? To this question we reply that terms are terms only in relation to the proposition in which they occur. For example, we may write down the following:

Silver	Conductor of electricity	Comets
Plants	Without weight	Roses

But neither this, nor any similar list, is a list of terms. In logical terminology, this is a list of Names. By a name, therefore, we understand a word, or a collection of words, which has a complete meaning, and which may be used as the subject or predicate of a proposition. In the following, 'Silver,' 'Without weight,' 'Plants,' etc., names from the above list, are terms:

Silver is a conductor of electricity.

Comets are without weight.

Roses are plants.

Now it may seem that the distinction between terms and names is over-drawn, and that it does not correspond to any real difference. This is not the case. For observe that although the words in the list of names, and the words that appear in the propositions are the same, the meaning in the latter case is much more precise than in the former; it has changed from being vague, general, and loose, to being specific, particular, and fixed. In so far as our propositions express judgments, this is the result that we should expect to find. For the sake of emphasis, the difference between a name and a term may be stated in the following way:

A Name is a word, or a collection of words, with a complete meaning, which may be thought of as the S or P of a proposition.

A Term is a word, or collection of words, with a complete meaning, which is thought of as the S or P of a proposition.

Both terms and names are ordinarily expressed in words, but it is not because they are words, but because they have meaning that they are so called. Terms and names are significant words. We are not confined, however, to the study of words from the standpoint of their significance or meaning. Grammar, for example, studies words with respect to the uses they have in sentences. Substantives,

verbs, prepositions, adjectives, adverbs, etc., are the names of various classes of words that are recognised by grammar when their use is made the basis of classification. Logic also finds it convenient to classify words with respect to their uses as terms in the proposition, disregarding, as does grammar, the meaning that particular words may have. But the logical classification is simpler than the grammatical, because the logical requirements are narrower and more precise than the grammatical. Words, for logic, are arranged in two main groups, according as they are or are not able to stand by themselves as S or P in a proposition. Words which by themselves are capable of being thought of as S or P of a proposition are called *Categorematic words*. Words, on the other hand, which require the assistance of other words before they can be thought of as the S or P of a proposition are called *Syncategorematic words*. 'Metal,' 'laws,' 'elements,' 'custom,' are *categorematic words*, since they may stand by themselves as terms in a proposition. For example:

All metals are elements.

Some laws arise from custom.

The words 'of,' 'the,' 'always,' 'a,' in the following statements, are *syncategorematic words*, because by themselves they cannot be thought as S or P in a proposition.

Mistakes are not always a proof of ignorance.
Bismarck is the man of iron.
A burnt child dreads the fire.

The only apparent exception to the statement of what should be understood by a syncategorematic word is illustrated by such instances as the following: 'Good is an adjective,' 'Of is a preposition,' But it should be observed that it is not the use of the word in a logical proposition, but the word as an example of a group of words in grammar that is the subject term of such propositions. They are for logic categorematic words.

If we examine the logical classification of words, we shall see that the terms of propositions in which categorematic words occur are single-worded terms, while the terms of propositions in which syncategorematic words occur must necessarily be many-worded terms. On the basis, therefore, of the kinds of words that enter into them, we may classify terms as either single-worded or many-worded terms.

The Copula. The most distinctive element of the proposition is the copula. It is always expressed by 'is' or 'is not,' 'are' or 'are not,' depending on whether the subject term of the proposition is singular or plural, and whether the judgment is affirmative or negative. From the standpoint of the hearer, the proposition tends to fall apart into its terms; and it is the function of the copula to hold them together in the concrete unity of a single idea,

and thus to preserve in the proposition a quality that is characteristic of the judgment itself. The copula must not be looked upon as a coupler between the subject and predicate terms, nor is there any separate and distinct element in the judgment corresponding to it, and coming between the idea of the subject and that of the predicate. Its function is to express the act of judgment by which the S and P are really held together in the way that the proposition expresses.

The copula has been a much debated subject of logic. It will help us to a better understanding of it if we consider some of the views that have been advanced concerning it. We may distinguish two such views, the relational and the existential, and these we shall touch upon briefly in order.

The Relational View. This view is based upon the assumption that each term of the proposition has a distinct meaning, and it holds that the copula brings these terms together in the unity of a single statement. If we adopt this view, we are required to admit that our concepts or terms are independent of our judgments, and that is to deny that judgment is the starting-point of knowledge. The discussions of the logicians of the Middle Ages over Universals were concerned with this subject, and they were divided in their opinion as to whether concepts existed apart from our knowledge of them. The views developed by the Schoolmen, as the great thinkers of the Middle Ages were called, are three, and these we may enumerate and describe briefly. The real-

istic view, held by Anselm and Aquinas, was that universals or concepts have an existence before and independently of the particular objects through which we become aware of them. Briefly expressed, this position was summarised in the Latin statement: *Universalia ante rem*. The view of the Nominalists was that universals or concepts owe their existence to the prior existence of the particular objects of our experience, or, as they said, *Universalia post rem*. This view was held, among others, by Roscellinus. A conceptualist view, which aimed to embody the truth of each of the former views, was developed by Abelard. This view called attention to the fact that universals or concepts have no meaning apart from the particular objects in which they are embodied, and that particular objects have no meaning apart from the universal or concept which gives them their significance. These ideas are expressed in the Latin statement: *Universalia in re*. Outside the official teaching of the Roman Church, which is realistic, some form of conceptualism quite generally prevails at the present day. At any rate, it would be agreed by most logicians that concepts are elements of our knowledge, and that it is our ability to judge, — to assert likenesses and differences of the objects of our experience, — that gives existence and life to the things we call concepts. Psychology and logic both teach that the objects of our experience are inextricably connected, and that our knowledge grows by distinguishing now one aspect and now another of the

inter-related world of men and things around us. If this is true, there is no object that is merely itself, or that exists in isolation from everything else. Every object with which we are in any way concerned is always becoming for us something else, showing some new quality or entering into some new relation. This state of affairs is expressed in logic by saying that there are no S's that are not P's, that the P predicated of any S is not a foreign thing that is added to it by the copula, but some quality that it already possesses, or some relation that it actually sustains. In other words, we never start with a bare S, with an S that is an unrelated unit, but with an SP that is a relational unity, and it is this fact that becomes explicit in the judgment, S is P. Unless S were already P, judgment would be impossible, and the proposition could not exist. The proposition makes explicit what is already present in knowledge, and since knowledge, whatever its form, is a qualification of an S by a P, there does not seem to be any separate meaning for the copula when knowledge is expressed in a proposition. That is to say, we do not need the copula for the establishment of the knowledge relation; but it is used in the proposition, as Mill asserts, as "a sign of predication." In other words, the copula must be understood as serving notice upon the person who undertakes to interpret the proposition that S and P are not separate and distinct things, but, for the purposes for which the proposition is expressed, one and inseparable. The proposition, that is, does

not tell us about any S, but only about the S that is P.

The Existential View. This view arises out of the fact that 'is,' besides serving as "a sign of predication," has also a meaning of its own, namely, existence. Thus, if we say that, 'Aristotle is the father of formal logic,' it may seem that we are not merely stating the relation of Aristotle to the science of logic, but are also asserting the existence of Aristotle. For if we do not believe in the reality of Aristotle, what significance, it may be asked, can belong to the statement that he is the founder of formal logic? To preserve the proposition, therefore, from evaporating into mere nonsense, we seem forced to admit that the reality of the subject term must be secured by the structural character of the proposition itself. Thus, if we consider the structural formula, 'S is P,' we must, from this point of view, interpret it as meaning, 'S is, and it is P.' That is to say, the significance of the predication, 'S is P,' depends upon our right to take the copula twice over, and in each case with a different meaning. In the first case, 'S is' is equivalent to 'S exists;' in the second, 'S is P,' 'is' is "a sign of predication," in the sense already explained. This, in brief, is the existential view of the copula.

It is evident that we can accept this view only if we take the copula in the same double way in all propositions. But this we can not do. There are some propositions whose terms forbid this interpretation of their copula. When, to take Mill's example,

we say that 'A centaur is a fiction of the poets,' existence is explicitly denied, but it is no less excluded when, explaining a name, we say, 'A couch is an article of furniture used for reclining.' Further, if we say that 'Shakespeare's Henry V is nobly drawn,' we are obviously moving in the sphere of fiction, as we are moving in the realm of mythology when we say that 'Zeus is king of all the gods.' In cases like the foregoing, the existential interpretation of the copula seems to be without meaning; but this is so not because 'is' has lost the meaning of existence, but because the terms of the proposition will not support such an interpretation. Thus we see that the effort to give the copula a distinctive sense, over and above what it has as "a sign of predication," breaks down before the controlling influence of the terms which constitute our propositions. As we have seen, propositions are statements of the relationship of terms, and the relations are determined not by any special meaning carried by the copula, but by the character of the terms that enter into propositional statements.

To meet these difficulties, it is sometimes pointed out that, for speaker and hearer alike, all motive for judgment is taken away unless we presuppose the reality of the subject-matter of our judgments. We do not express judgments, it is said, about subjects which we believe do not exist. This may be readily granted; but we should remark, at the same time, that it is one thing to admit that existence is presupposed by our judgments, and quite another

to affirm that reality is asserted by the copula of the propositions in which our judgments are expressed. Judgments may imply reality without asserting reality. Belief in the reality of Columbus, for example, is implied in the statement that 'Columbus is the discoverer of America;' but what the statement asserts is that he discovered America. And what is true of this case, is substantially true of every case of the judgment.

If, then, we must hold that belief in reality is presupposed by every judgment, it is obvious that we cannot mean the same thing by existence in every case of judgment. In the examples, 'Utopia is the ideal commonwealth as depicted by More,' and 'The sun is the centre of our solar system,' we can not be supposed to imply that 'The sun' and 'Utopia' belong to the same sphere of existence or reality. In other words, we must recognise different spheres of reality, and it is existence within one or other of these spheres that is implied in every judgment that we make. Thus, 'Utopia' belongs to the world of the imagination; 'The sun' to the world of physical objects. This is a doctrine of logical importance, for we may remark that ambiguity, misunderstanding, and error arise when what a judgment asserts as true within one sphere of reality is taken as true within another with respect to which the judgment has not been determined. If, for example, I say that 'I believe in the reality of Santa Claus,' I am, of course, asserting a belief which falls within

the world of ideas; but if you understand me to imply the corporeal and individual existence of Santa Claus, you only emphasise your misunderstanding of what I asserted if you challenge me to produce him, or to have him as my guest on Christmas Day.

We may, then, sum up this discussion in the following statements: (1) the copula does not express existence, but is only 'a sign of predication;' (2) reality of their subject-matter is presupposed — not expressed — by all our judgments; (3) there are different spheres of reality within which the meaning of our several judgments is to be determined; and (4) a failure to refer any judgment to its proper or intended sphere of reality leads to ambiguity, misunderstanding, and error.

Kinds of Propositions. There are three main types of propositions; the categorical, the disjunctive, and the hypothetical. In this section we shall limit ourselves to the task of characterising each kind of proposition, reserving for other chapters the discussion of the logical problems which are connected with these forms of statement. In the first place, however, it should be remembered that we are considering propositions in general, and that, however the categorical, the disjunctive, and the hypothetical propositions differ from each other, they all share the common characteristics of propositions. It is, we have seen, the business of propositions to express the affirmative or negative relations between the objects of our experience when

these relations have become explicit in judgment. This function of the proposition is called in logic 'predication.' In every proposition P is predicated of S, and as we have seen, the copula is the sign of such predication. With respect, therefore, to the three types of proposition we may say that they differ, if they differ at all, by reason of the difference in the forms of their predication. And this is the usual way of distinguishing them. Thus, in the categorical proposition we have simple predication; in the disjunctive proposition, alternative predication; and in the hypothetical proposition, conditional predication. The structural formulæ for these three kinds of proposition respectively are: 'S is P;' 'S is either P or Q or M;' 'If S is P, it is M.' We may also illustrate each kind by the following examples: 'The way of the transgressor is hard;' 'Today is either Monday or Tuesday or Wednesday or etc.;' 'If Cæsar was ambitious, he deserved to die.'

Relations of the Main Types of Propositions. Propositions have been distinguished as either categorical, disjunctive, or hypothetical. These different forms of proposition correspond to the different degrees of certainty or explicitness of the judgments they express. Now, since we know more about some subjects than we do about others, and our knowledge of some parts of some subjects is more exact than it is about other parts, we should expect that this fact would be reflected in the propositions in which we express our several judgments. In

other words, the difference in the forms of propositional statements corresponds to a difference in the explicitness of the knowledge asserted by our judgments. There is pretty general agreement among logicians as to the foregoing statement. There is not, however, the same harmony among them on the question of the precise order in which the several types of propositions should be arranged. We would suggest that they may be taken, in the order of their increasing exactness, in the following way: the disjunctive, the hypothetical, and the categorical. That is to say, the disjunctive and the categorical propositions, respectively, express the least and the most explicit forms of knowledge, and the hypothetical, to which class most of our judgments belong, will appear as a form of statement that expresses, in the majority of cases, the maximum attainable certainty in our quest for knowledge.

To consider each case separately, we may say of the disjunctive proposition that it expresses uncertainty as to the details of a subject-matter which is explicitly under consideration. For instance, we may not be in doubt that there will be students whose task it will be to read this paragraph, but we may not be certain how far they will understand it. If, in such a case, we tried to make the details of judgment as precise as possible, our proposition would then assume this form: 'The student who reads this paragraph will either understand it, or not understand it, or misunderstand it.' Such a statement, or the judgment presupposed by such a

statement, does not carry us far on the way to knowledge. It leaves us in doubt at the point where, if we are to have probable knowledge, uncertainty can not exist, namely, in respect to the details of the judgment's content. In other words, the disjunctive proposition starts with a belief in the reality of an S, but is uncertain as to what S is, that is, whether it is one or other of a number of alternatives which are possible of realisation in the sphere of existence or reality covered by S. All that, in such a case, we can say is that 'S is either A, or B, or C, or D.'

If now we consider the hypothetical proposition, we find that the defect of the disjunctive proposition has been overcome, and we are no longer in doubt as to what we are asked to believe. But we also notice that the greater explicitness of the relations expressed by the proposition has been secured by a corresponding loss. To consider the latter point first, we may observe that the hypothetical proposition does not imply, as does the disjunctive proposition, a definite belief in the existence of the sphere of reality within which the relations expressed by the proposition are understood to fall. In the case cited, for instance, the disjunctive proposition started with belief in the existence of 'The student who reads this paragraph.' In the hypothetical proposition the relations involved in this belief are used as a condition of the ensuing judgment. Thus, 'The student who reads this paragraph' becomes, in the hypothetical proposition, 'If a student reads this paragraph,'—a form of statement

which, of course, leaves us uncommitted as to whether there is any such student or not. So far, therefore, as the hypothetical proposition is concerned, belief is never definitely implied in the existence of the sphere of reality within which the relations asserted by the judgment find their possible realisation. This, however, is not an uncompensated limitation. For, to consider the other point mentioned above, the meaning expressed by the hypothetical proposition is specific; the details with which the judgment is concerned have become defined in such a way that the conditions on which belief rests form part of what the judgment asserts. Thus, fully expressed, the hypothetical proposition reads, 'If a student reads this paragraph, he will understand it.' In this case the internal structure, the organic relations, of the elements of thought are more highly specialised than they are in the disjunctive proposition, and what we are told is that an understanding of 'this paragraph' is definitely grounded in the sphere of belief implied, although indefinitely, by the proposition itself. If we may hold that knowledge grows by the definition of details, we shall have to say that the hypothetical proposition expresses a higher type of knowledge than the disjunctive proposition.

The categorical proposition, if it exists, carries us outside the boundaries of probability into the field of certainty. The categorical proposition combines certainty as to the existence of the sphere of reality which is characteristic of the disjunctive

proposition, and certainty as to the detailed relations of the matter of the judgment which is characteristic of the hypothetical judgment. Hence our proposition reads, 'The student who reads this paragraph, will understand it.' But in order to understand the categorical proposition, we should observe that our knowledge of any subject-matter is so rarely, if ever, complete, and that, in those cases where it is, the material of judgment is likely to be so insignificant, that, from the logical point of view, the categorical proposition must be held to express either an ideal limit of knowledge, the goal toward which knowledge is constantly advancing but never quite reaches; or that it is the least serviceable, because least significant, form for the expression of truth. We may, perhaps, roughly indicate the meaning of this remark by the commonplace observation that an examination of the forms of statement of the poorly educated reveals a predominance of categorical propositions; and, that, as we rise in the scale of educated intelligence, the other forms, and especially the hypothetical form, predominate. Dogmatic statements, in other words, are characteristic of the man who does not know, or of the man who knows things about which no one particularly cares. In either case, we should have to say that such statements, from the logical point of view, are pseudo-categoricals, that they express a belief which either goes beyond or falls below the range of our knowledge of significant reality. But this perversion of the categorical proposition by the ignorant,

need not obscure for us its proper logical position. It does express the form into which knowledge, when it is perfect, would naturally fall. The categorical proposition implies belief in the existence of a sphere of reality within which certain expressed relations are fulfilled. It, however, remains an ideal because, our knowledge being finite, we can never get beyond the possibility of doubt as to whether one or other of the conditions of the judgment has been completely satisfied.

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QUESTIONS

1. In what respect does the proposition differ from the judgment?
2. What is meant by the import of a proposition?

3. Distinguish between the truth-import and the statement-import of propositions.
4. What light does this distinction throw upon the interpretation of propositions?
5. Why do we always write the subject term of a proposition first?
6. How may we distinguish between the subject and predicate terms, and is it always possible to make this distinction? Explain.
7. What is the logical difference between terms, names, and words?
8. Give a logical classification of words.
9. What is meant by single-worded and many-worded terms?
10. What is meant by saying that the copula is the most distinctive element of the proposition?
11. State and criticise the relational view of the copula.
12. What are the realistic, nominalistic, and conceptualistic views of the concept?
13. What is the existential view of the copula, and what objections can be brought against it?
12. What do you understand by predication? Characterise the various kinds of predication, and show how these give rise to the several types of logical proposition.
15. What are the conditions under which we give a disjunctive expression to our judgments?
16. Discuss the nature of hypothetical propositions, and show their relation to disjunctive propositions.
17. Discuss the nature and limits of categorical propositions.

EXERCISES

1. Illustrate, from your own observation preferably, the possibility of confusing the truth-import with the statement-import of a proposition.
2. Write the following sentences in the form of propositions, indicating the subject and predicate terms in each:

- (1) Over the mountains poured the barbarian horde.
 - (2) The moonlighh plays fitfully upon yonder rippling stream.
 - (3) Happy is the man that findeth wisdom.
 - (4) All is not misfortune that seems at the moment unendurable.
3. Write six sentences in which you think it difficult or impossible to distinguish the subject and predicate terms.
 4. Make a list of twelve categorematic words.
 5. Write six statements which show the error of an existential interpretation of the copula, and explain the reason in each instance.
 6. Write disjunctive propositions about the following subjects:
 - (1) Your presence at a concert to be given, say, next week.
 - (2) The result of a particular political election.
 7. Write hypothetical propositions about the following subjects:
 - (1) Tomorrow's weather.
 - (2) The study of the biological sciences.

CHAPTER IV

TERMS

Proposition and Terms. In the last chapter, we defined a proposition as the verbal expression of a judgment in which something (P) is predicated of something else (S). We also analysed the proposition into its constituent factors — terms and the copula — and saw that the relation which these elements sustain to one another is determined by the meaning of the proposition taken as a whole. The proposition, that is to say, was considered as determining the existence and character of its terms, and not the existence and character of terms as determining the existence and character of the proposition. Terms are different aspects of the meaning of the proposition in which they occur. Expressed otherwise, they are functions of their propositions. The further discussion of terms which we begin in this chapter must be carried on in light of these general positions, and it must be considered as a more detailed consideration of certain important features of the proposition. The separate chapter headings are employed merely for pedagogic convenience.

Connotative and Denotative use of Terms. We are to consider at the outset a distinction in the use of terms upon which hinge many of the problems that will come before us in the succeeding pages. It is,

therefore, important to give special attention to the precise logical significance that the distinction has, and to get clearly before ourselves the scope of its application. The latter point will be discussed in the next section; we shall, therefore, confine ourselves here to a statement and illustration of the uses—connotative and denotative—to which terms, whatever their character, may logically be put.

It is part of our common knowledge that words are frequently used as labels by means of which we identify things, and distinguish these things from other things. Many illustrations will come to mind: 'the Rocky Mountains,' 'the Amazon River,' 'my old bay horse,' etc. There is hardly any end to the number of words which have this use of pointing to objects, and which put us in contact with particular things, or groups of things. Now it is this fact of our common speech that logic describes when it talks about the denotative use of terms. Terms are used to point out, refer to, or indicate objects. For instance, in the statement, 'When it comes to packing, books are a nuisance,' the term 'books' refers to the individuals of a class of objects, and it is, therefore, said to be used denotatively. In the same way, the term 'lion' in the proposition, 'Lions are carnivorous animals,' is used to point out this, that or the other particular lion, and the proposition means that any individual lion will be found to possess the specified characteristics. Since the term 'lion' points out the individual objects one has in

mind when the term is used it is said to be used denotatively.

We may also go to our ordinary employment of language to ascertain what logic understands by the connotative use of terms. Words, as we know, not only point out objects, but they also have meanings. When, for example, we hear a strange word spoken, or meet with it in our reading, we ask what the word means. If no one can tell us, we go to the dictionary for the information. Now we do this, because we have been trained to expect that the words we meet will arouse some sort of mental image, and because experience has taught us that, when they do not, we fail to understand what is said to us. That words should have a meaning is just as important as that they should indicate individual objects. Unless such words as 'violin,' 'art,' 'mathematics,' 'landscape,' 'ocean' aroused within us quite different images or ideas, we should be at a loss how to use them, or to understand them. Now logic indicates this aspect of words, their meaning aspect, when it speaks of the connotative use of terms. Terms, from this point of view, are used to call up the qualities or attributes of objects. When we use a term connotatively, it must be understood, consequently, that we are speaking about its dictionary meaning. But, it should also be observed, the particular dictionary meaning of any term can be determined only from the standpoint of the particular proposition in which the term occurs. For example, in Shakespeare's *Coriolanus* we read, 'She

will but disease our better mirth,' a passage in which the term 'disease' means 'disturb.' When Locke writes, 'Although great light be insufferable to our eyes, yet the highest degree of darkness does not disease them,' he is using the same word in the sense of 'cause suffering to.' When, further, in Thomson's *Summer* we read,

Then wasteful forth
Walks the dire power of pestilent disease,

'disease' is used in the more usual sense of 'a physical or physiological disorder.' This dependence of the meaning of terms on their propositions is of capital importance, and should be kept constantly in mind.

Relation of the Connotative and Denotative use of Terms. The difficulty that is usually experienced in understanding the doctrine of connotation and denotation is due to the fact that we do not have one group of words whose use is always connotative, and another group of words whose use is always denotative. Most, if not all, terms are capable of being used in either the one or other of these ways. But this is not all. The most important fact to remark is that in whichever of the ways mentioned a given term is employed, it is so employed with the other use more or less distinctly in mind. It would be quite a fair statement of the case to say that there is always a reference of the one to the other, and that in any particular instance the one implies the other. It is not difficult to see,

for example, that the identification of the individuals which are denoted by any term depends upon their possession of the qualities that are connoted by the same term. It is also clear that the qualities that a term connotes are the qualities of the individuals which the term denotes. Let us, for example, examine the subject term of the proposition, 'Man is a rational animal.' If the term 'man' is used denotatively, that is, to point out 'Tom, Dick, Harry, etc.,' the proposition must mean that 'Tom, Dick, Harry, etc.,' are the individuals in whom are found realised the qualities expressed by the predicate term. If the term 'man' is used connotatively, the proposition must mean that the qualities, 'rational animal,' will be found present in each individual, that is, in 'Tom, Dick, Harry, etc.' Let us take, as another case, 'Man is a being who voluntarily sets himself tasks.' If, in this proposition, the term 'man' is used to denote individual men, then our identification of the individuals who set themselves tasks depends upon our knowing what the qualities are which the term 'man' connotes; and if it is used to connote certain qualities, our understanding of the proposition depends upon the supposition that these qualities can be found in a number of discoverable individuals. These illustrations serve to emphasise the general rule that the connotative use of a term does not break away entirely from its denotative use, or vice versa; and the sole logical distinction between them is that, in the one, we lay the primary emphasis on the *individuals*

which possess the qualities, and, in the other, on the *qualities* possessed by the individuals.

Another inquiry which comes before us here is whether this relation between the connotative and the denotative use of terms is realised in all terms whatsoever. We can give an answer to this question only by anticipating certain features of the discussion of terms contained in the following pages. We may, however, at this point call attention to the generally accepted position that no term is ever so exclusively denotative as to lose all connotation whatsoever, and that no term can be said to have developed on the side of meaning, i. e., connotatively, so one-sidedly as to have lost all reference to objects. The discussion of this question has been brought to the study of proper names as to a test case; and it is thought that if the position can be maintained with respect to these terms, it will be possible to maintain it with respect to other kinds of terms. The question is not a theoretical one, and we shall gain nothing by considering it as such. It is a question of fact, and can be determined only by a study of each kind of terms as it comes before us in this and the following chapter. We may express the opinion, however, that it is quite possible that, in the course of time, the employment of particular terms may shift from a predominantly connotative to a predominantly denotative use, and there is no reason, except one of convenience, why all meaning should not drop away from some terms, and that they should stand in our vocabulary merely as sym-

bols of the existence of things. Whether there are any such terms cannot be decided now; it is more important to see that the possibility of this being the case does not obscure for us the more general relation which was expounded in the preceding paragraph.

A subject that is usually discussed in this connection relates to the changes that take place in the connotation and denotation of terms when either of these is increased or decreased. Under these conditions, the changes in the connotation and denotation of terms are said to vary inversely. More fully stated, when the connotation of a term is increased, its denotation is decreased, and vice versa. For example, the denotation of the term 'man' is 'the total population of the earth,' and its connotation, we may say, is 'rationality and the upright position.' If we increase the connotation by specifying 'white man,' the connotation of which term is 'white, rationality and upright position,' we decrease the denotation which then refers to 'the white population of the earth.' If a further increase in connotation is made by specifying 'European white man,' the denotation is decreased still more, and indicates 'the white population of Europe.' The process thus begun can be carried as far as interest or ingenuity allows. The general truth to which such illustrations point is that the denotation of a term is diminished as its connotation is made more definite by the addition of new attributes; and, conversely, that

when the connotation is decreased by dropping attributes the number of individuals to which the term applies is increased. This statement holds good as a general rule, but there is no numerical ratio between the increase or decrease of connotation and the accompanying decrease or increase of the denotation. Indeed, the denotation of a term may be increased greatly without any change in its connotation, as actually happened, for example, when the British government annexed some millions of Africans at the close of the Boer war without making thereby any change at all in the connotation of the term, 'British subject.'

Distribution of Terms. If we carry our study of the denotative use of terms a step further, we shall find that the purposes which prompt our statements sometimes require us to refer to a determinate number of the individuals denoted by the term, and that sometimes our purpose is satisfied if we indicate an indeterminate number of the individuals included under the term. Let us suppose, for example, that we are discussing the theory of kingship, and that I express the belief, after Hobbes, that the king can do no wrong. If this belief were thrown into a propositional form, it would become clear that I am thinking of all the individuals who have exercised, or at some time may exercise, the functions of kingship. What I mean is that 'No king can do any wrong,' that is, wrong doing cannot be predicated of any king, — of this, that, or of any other individual who is or may be a king. If such a state-

ment does not recommend itself to your judgment, and you express dissent from it, you are not required, by the conditions of the situation which determine your judgment, to concern yourself with each and every king; it will be sufficient, if wrong doing by kings has ever occurred, for you to limit your statement to any of the cases which contradict the theory you oppose. Expressed in a proposition, your judgment will be 'Some kings can do wrong,' or more definitely, 'Some kings have done wrong.' You are obviously speaking of a number of individuals, not definitely specified, who belong to the group denoted by the term 'king.' The distinction, thus indicated, in the denotation, width, or extent of terms, is known in logic as the 'distribution' of terms. By the distribution of terms we mean the consideration of terms from the standpoint of their quantity. Now, in respect of quantity, we are limited in logic by the contrast between a definite and an indefinite number, and the terms which denote these quantities are said to be distributed and undistributed respectively. A term is 'distributed' when, from the form of the proposition in which it occurs, it is known to refer to a determined number of individuals of the class denoted by the term; it is said to be 'undistributed' when, from the form of the proposition in which it occurs, it is known to refer to an undetermined number of the individuals of the class denoted by the term. The technical sign of a distributed term, in an affirmative proposition, is 'All,' and, in a negative proposition,

‘No.’ The technical sign of an undistributed term, in both affirmative and negative propositions, is ‘Some.’

We have just said that the difference between distributed and undistributed terms is that in undistributed terms the exact number of individuals referred to is left indefinite, as, for example, ‘Some birds are blue;’ while in distributed terms the number is made definite, as for example, ‘No man is free from the defects of his qualities,’ and ‘Half my goods I give to feed the poor.’ We shall have occasion to discuss this distinction again when we study the interpretation of propositions; but for the present, it is enough to remark that neither in the case of a distributed, nor in that of an undistributed, term can we say, from the form of the proposition, what the arithmetical number of the individuals denoted by the term is. It may, perhaps, help to make the matter clear if we recall that in the study of algebra we are taught to use the first letters of the alphabet, a, b, c, to represent the known quantities of a problem, and the last letters of the alphabet, x, y, z, to represent the unknown quantities. The mathematical difference between these two classes of quantity, the known and the unknown, is the difference between the distributed and the undistributed use of terms in logic. In the former case, the known objects are determinate; in the latter, the unknown objects are indeterminate.

Some other Characteristics of Terms. In the foregoing paragraphs, we have considered mainly those features of terms which are important from the point of view of their denotation. There are certain questions which arise from the standpoint of the connotation of terms to which we shall direct attention in the remainder of this and in the following chapter. The questions to which we refer must not be confused with those that will engage us in a later chapter, and which relate particularly to the problem of definition, that is, to the problem of how the meaning of terms receives logical formulation. What, rather, we have to consider at present is the fact that there are certain general characteristics of terms which do not depend upon the particular meaning that terms have, but upon the nature of the interest which stimulates our judging activity. The interests under which we are led to judge any subject-matter may require us to express what we wish to say either concretely, abstractly, absolutely, relatively, positively, or negatively. That is to say, our meaning may be either concrete, abstract, absolute, relative, positive, or negative. These distinctions are not coördinate, and they are not at all of equal importance; but the particular relations which they sustain to one another become apparent only when it is clear what each signifies. We shall, therefore, confine ourselves, in the main, to stating how each of these distinctions is to be understood, and we shall begin with the broadest

among them, the distinction between concrete and abstract meaning.

Concrete Meaning. One of the most familiar facts of our every day life is that we distinguish from one another the various objects that come under our observation. When, for example, we go into the city, we perceive the houses, the shops, the traffic on the streets, and the people on the sidewalks as quite different things. We also carry over into our academic studies the endeavor to make distinctions, and to regard different things as different. Thus, the student of zoology learns to discriminate between the blood vessels, the supporting tissue, and the characteristic cells of the specimens he observes under the microscope. Indeed, so general and important is this ability on our part that there is no interest that is not served by it, and no limit to which it may not be carried. For instance, we may mark off in this way not only the objects of the material world, but the qualities and relations of these objects, and set them before us as distinct objects of our thought. Thus, 'green,' 'heavy,' 'but,' 'cause,' may become for us objects as distinct as are the chairs and tables of our domestic life. Now whatever we set off as a distinct object of perception or thought is in logic considered as a concrete object, and the term that denotes such an object is called a concrete term.

It may be observed that the significance for logic of the term concrete depends upon the significance of the word 'thing' in the body of our common

knowledge. By a thing we ordinarily mean some part of the physical world which is perceived as distinct from other parts with which it coexists. Thus, the pictures on the walls of our room, the articles on display in the shop windows, the mountains and streams of the landscape, the cattle on a thousand hill, are all called 'things' whenever we think of them as distinct from other objects with which they are associated in the same common environment. But the word 'thing' is not confined to objects of the material world. We also designate as things whatever can be thought of as having a distinct existence of any kind. Thus, if I am trying to recall something in particular that has been told me, I may express my impatience at not being able to do so by saying, 'Why can't I remember the thing?' What I refer to here as a thing is an idea. A similar usage is found in the commandment against idolatry among the Hebrews when it is forbidden them to make a graven image "in the likeness of any thing in the heavens above," where the word 'heavens' has a quasi-spiritual meaning. Instances might be multiplied of the very wide and varied use to which we put the word thing in popular speech, but those given are enough to emphasise the connection between it and the logical meaning of the term concrete.

While, however, there is this general connection between the two words, the meaning of the term concrete is more precise than is the meaning of the word thing. It is true, for instance, that we do not

ordinarily refer to persons as things, but logic would have no hesitation in calling the terms, by which they were denoted, concrete terms. On the other hand, we should describe as a thing any object of which we could say only that it was in our consciousness, but unless we knew some of its attributes or relations, we should not be warranted logically in calling it a concrete object, or the word used to denote it a concrete term. These cases are suggestive of the truth that it is not mere existence, but significant existence that logic, in the main, denotes by the term concrete. Whenever an object is thought of as having a definite kind of existence, that is, as possessing certain qualities or standing in such and such relations, it is called a concrete object, and the words we use to denote such objects are concrete terms because they are used to connote these qualities and relations. There are, as we shall see, varying degrees of definiteness in the ways in which significant existence is asserted, and there are, consequently, several degrees of explicitness in the meaning of the terms used for denoting such existence. This matter we shall discuss in the following sections; it will be sufficient, meanwhile, if the broad meaning of the term concrete has been made clear.

Individual Meanings: Proper Names and Designations. The first class of concrete terms that we shall consider is that which denotes particular individual objects. To be able to point out individual objects, we must have in mind, as a general rule,

some attribute, or group of attributes, which is thought of as belonging to the one object we desire to single out, and to make the subject of further consideration. The names of such objects, when they are used as terms in a proposition, are definitely denotative and explicitly connotative; they point to their objects by means of the attribute or attributes which the term connotes. Such terms are called individual, or singular, terms.

Individual concrete terms are of two kinds: proper names and designations. A proper name, to consider that first, is the name given to an object — person, place, or thing — as a special means of distinguishing this object from all other objects. We give names to the members of our families, — Tom, Dick, Mary, Sue, etc., to continents, mountains, rivers, towns etc. We also give names to the horses in our stables and the cattle in our barns. There is no limit, except its usefulness, to the process of naming the objects of our acquaintance in order to distinguish them from others with which we do not wish them confused. In all such instances the names are proper names, and they serve to individualise the particular objects to which they belong. When such names are made the subject of predication they are singular or individual terms.

Now, about proper names, as thus defined, we may ask whether their designation of particular objects is made possible because they imply an attribute or attributes, that is, because proper names are connotative. In answer to this question, Mill,

for example, replied that proper names are non-connotative, they can be used only, he said, to signify an subject. This is, in many quarters, considered an extreme position, and it may be due to the failure to see that significance may belong to a subject not only because of the qualities that it possesses, but also because of the relations in which it stands. "The very fact," as Creighton points out, "that a proper name is given to an object implies that it possesses a certain marked individuality," and the intelligent use of the name surely implies a reference to the qualities or relations by which that individuality is defined. However, be that as it may, we may admit that the connotation of proper names is ordinarily quite general and often vague, without thereby refusing to grant them any significance whatever. If the application of such terms is unambiguous, it is quite natural that their meaning should drop into the background of consciousness; but if its application should become uncertain or be misunderstood, we should then feel the need of making the connotation of the term more definite.

Individual concrete meaning may be expressed, in the second place, by a designation. Logic regards as a designation any phrase that describes an object by its qualities or relations, when such phrase is used for the purpose of definitely pointing out this object. 'The assassin of Franz Ferdinand of Austria,' 'my winter overcoat,' 'the centre of the material universe,' 'my partner,' — all these are designations in the sense defined. It is clear that a desig-

nation depends upon the appropriateness of the description it embodies for the accurate identification of the object to which it points. That is to say, a designation must be explicitly connotative as well as definitely denotative.

The wide usefulness of this method of expressing our individual concrete meanings is very evident. It is true that, in ordinary life and in scientific pursuits, we need a number of terms which shall have the specialised function of designating particular individual objects, that is, terms that are definitely proper names. But neither in science nor in daily life is it necessary to assign proper names to all the objects that engage attention, and yet some method of referring to these objects is necessary if we are to develop the interests that prompt us to take note of them. It is under such circumstances, as a general thing, that the designation becomes of use. We should be on our guard, however, against supposing that the objects to which proper names are assigned are necessarily of more importance than those to which we refer by means of designations. While it is true that some of the objects which we denote by designations are relatively insignificant, this is not always the case. In science, for instance, we speak of 'the law of gravitation,' 'the law of diminishing returns,' 'centrifugal force,' etc.; and in daily life we use such phrases as 'the President of the United States,' 'the Secretary of State,' etc. These phrases are, according to the definition, designations; they point out

definite objects by means of a brief description of the objects' characteristics. They have, moreover, a higher importance than some of the objects to which proper names are given.

General Meaning. The second class of concrete terms, which we have now to consider, is characterised by the fact that the meaning conveyed by such terms is applicable to more than one object. Now, it is true that, if we were to list all the attributes of all the objects of our acquaintance, it would be found that no one object possessed in every respect precisely the same qualities as any other. Every object, in this sense, is a unique object; and, if there were no other interest in things than that which leads to the observation of their uniqueness, we should have to use, when speaking of such objects, either proper names or designations. It frequently happens, however, that our practical or theoretical interests require us to distinguish among the attributes of individual objects those that belong to only one individual, and those that are possessed in common by a number of individuals. On the basis of this distinction, we are able to group together the objects which possess a common set of attributes, and, conversely, to make the common attributes the basis of our inclusion of other individuals within particular groups. For example, we classify certain animals as quadrupeds, and certain others as bipeds, on the ground that, amid an indefinite number of differences, certain animals are alike in the fact that they walk on four feet, and that cer-

tain others are alike in the fact that they walk on two feet. Stated otherwise, in constructing such classes of objects, each of the individuals belonging to any one class is thought of as possessing exactly the same characteristics as any other individual of that class.

Now the names which embody a meaning that can be applied to any one of an indefinite number of individuals are known in logic as 'general' or 'common' terms. Of such terms we may say, in view of the foregoing discussion, that they are explicitly connotative, but indefinitely denotative. In other words, the attributes that a general term connotes belong equally to each and every individual of a group; but we cannot tell which one or more of these individuals is denoted, merely by knowing the connotation of the term. The characteristic fact about a general term is, then, that while its connotation or meaning is concrete, its application is always general; the meaning, that is to say, can be applied to an indefinite number of individuals. It should be observed, however, that the meaning of any such term does not exist apart from some or all of the individuals in which this meaning is concretely embodied. That is to say, it is the existence of the individuals which form the group that determines the meaning that any general term may have. Apart from these individuals, indefinitely denoted though they are, the general term would lose all its meaning. One reason for remarking this fact is that the interest, which leads us to observe the com-

mon characteristics of numbers of objects, is an interest that still recognizes the differences that exist among the objects which it groups together. When, therefore, these common characteristics are expressed in general terms, it is not to be supposed that the differences that distinguish one object from another are thereby denied. What our general terms imply is that the presence of individual differences is not inconsistent with the possession, by a number of objects, of certain characteristics in common.

Collective Meaning: Individual and General. We have traced the development of meaning from the singular concrete to the general concrete, and we have seen that, in this development, we have been concerned with some feature or features of the individual or individuals which our interests prompt us to study. For instance, our interest in the differences that exist between one object and another is embodied in the meaning that singular terms have; our interest in the likenesses which exist along with these individual differences is expressed in the meaning that general terms have. That is to say, the development of meaning which leads from the individual to the general term has been guided by the consideration that we must not do violence to the differences that exist between one individual and another while seeking for the common ground of their likeness. We have now to consider whether, neglecting the differences between individuals, we may start with their common likeness, and carry the develop-

ment a step further. Or, to state the inquiry more definitely, we may ask what alteration in our meaning is made if and when, in our study of individuals, we ignore their differences, and take note only of their likenesses. We have seen that we may take account of both likenesses and differences, and still be able to define a meaning which applies to each of a number of otherwise dissimilar individuals. What we have now to see is that if we pay attention only to likenesses, or, what amounts to the same thing, consider only similar individuals, we are able to define a meaning which applies to no one of the individuals, but can be intelligently used of all these individuals taken together as a whole. To illustrate, let us suppose that we bring together in one place a number of animals — lions, tigers, elephants, jaguars, ibexes, zebras, etc. Such a collection we should call a *menagerie*. If, again, we were to bring together a number of children for the purpose of educating them we should call such a collection a *school*. There are many such collective groups of individuals; and we can say that the meaning of the word by which each is indicated applies to none of the individuals which constitute the group, but only to the group which is constituted by the individuals. Thus, ‘a *menagerie*’ is the name for a particular group of animals, ‘a *school*’ is the name for a particular group of children under instruction; ‘the United States of America’ is the name for a particular group of political organisations, etc.

Now a name which embodies a meaning that can be applied to a number of similar individuals, taken together and thought of as a whole, is called in logic a collective term. It is obvious, from what has been said, that the connotation or meaning of collective terms is determined for us by the points of observed similarity between the individuals which constitute the groups which these terms denote. Thus, we call a particular group of individuals a nation when we think of all the individuals belonging to the group as citizens, — citizenship is the one attribute that all have in common. We call another particular group of individuals a legislature when we think of all the individuals belonging to the group as makers of laws, — making laws is the one common task in which all its members are engaged. The same thing, *mutatis mutandis*, will be found true of all collective terms whatever: the connotation of such terms embodies certain observed similarities between the individuals which belong to the groups which the terms denote.

When, on the other hand, we consider the denotation of collective terms, it is evident that the groups which such terms point out constitute a new kind of individual, and must be distinguished from the individuals which are denoted by the general term. The individuals which general terms, as a rule, denote are objects of our ordinary perception and thought; they are the things which, in the course of our daily life, we are accustomed to take as distinct objects; such things, we may say, as

each of us has been trained to observe in the course of the education that the community provides. On the other hand, the individuals which collective terms denote are individuals of a higher order, so to speak; they result from a definitely conscious effort on our part to bring together under a single view a number of diverse things, and to regard these things as one object. The objects which collective terms denote are more complex than those denoted by the general term, and should be carefully distinguished from the latter.

Collective concrete terms may be either individual or general. The individual collective term is used to point out a particular group of objects which we wish to distinguish from all other objects whatever. 'The Rough Riders,' 'the Gordon Highlanders,' 'the Allies,' (in the present war in Europe) are examples of collective terms used in this way. It may be observed that the individual collective term always contains the definite article, 'the,' as part of its verbal form. Thus, 'the committee,' 'the library,' 'the family' are collective terms used to point out particular distinctive groups. The denotation of these terms is definite. The general collective term is used to indicate a particular kind or class of grouped objects, and, therefore, may be applied to any particular group of objects which has the characteristics of the class which the term denotes. For example, when the military and naval forces of two or more nations act together for the purpose of de-

feating a common enemy, these nations are called allies. 'Allies' is a collective term, and, in this case, is general, since we apply the same name to each of the many combinations of this character which history records. Other illustrations of collective terms which have a general application are, 'family,' 'library,' 'committee.'

It is sometimes said that the individual collective can be changed into a general collective term by substituting the indefinite article, 'a,' for the definite article, 'the;' or by omitting the article without substitution. Thus, 'the allies,' which is an individual collective term, becomes a general collective term by omitting 'the;' 'the committee,' 'the family,' 'the library' become general by substituting 'a' for 'the' in each instance, — 'a committee,' 'a family,' 'a library.' Sometimes, however, the connotation of a collective term has become so highly specialised that it is capable of being applied to only a single object, and in that case it cannot be used, by any change whatever, as a general term. Examples of this are: 'the Congressional Library at Washington,' 'the family of King George IV,' 'the Committee on Foreign Relations in the Sixty-third Congress.'

Abstract Meaning. In the foregoing sections, we have studied how concrete meaning develops, and have seen that such meaning may assume one of three forms, or, as we may say, indicate one of three kinds of objects: — individual, general, or collective. We have said that, in these instances, the meaning is concrete because, in each of them,

the connection between attribute and object is particularly direct and intimate, although the connection is in differing degrees of intimacy. Any meaning is considered concrete which conveys the idea that 'these are the attributes which such or such an object has,' or that 'this is the object which has such or such attributes.' It has been shown that the object, in some instances, is a material object; in others, it is mental. But so long as the meaning we wish to express is concrete, we always suppose that we can pass from object to attribute, or from attribute to object, without a break, or that the connection between attribute and object is unequivocal. In other words, whenever we take an object in connection with its attributes, or think of attributes only in connection with their objects, our meaning is concrete. If, however, we were to consider attributes apart from objects, we should confront a new situation, and one which would involve a new set of problems, the consideration of which, in the end, would lead us to define a new kind of meaning which, in contrast to concrete, might appropriately be called 'abstract.' For by abstract meaning we commonly understand the idea of an attribute — quality or relation — thought of independently of the particular object or objects of which it is the attribute.

Now, when we look at the possibility of the existence of such meanings as this, we are confronted with the danger of meaning less than we actually say. For instance, some logicians have defined the

terms in which our abstract meanings are expressed — ‘abstract terms,’ as they are called — as those which apply to objects thought of apart from the whole to which they belong, or to attributes which are thought of apart from the objects of which they are the attributes. Thus, a ‘leaf’ thought of apart from the tree on which it grows is, according to this definition, an abstract term, and ‘green,’ if it is not thought of as the color of this particular leaf, or of any other particular green object, is likewise an abstract term. In our view, this is a very misleading statement of what abstract meanings and terms are. For instance, in the cases just mentioned, both ‘leaf’ and ‘green’ are concrete general terms. The one is the name of an object, the other of an attribute, and although these terms are not thought of as belonging to any one particular object, still they are thought of as belonging to any one of an indefinite number of particular objects; unless we think of them in this way we cannot think of them at all. But if this is what we are to understand by an abstract term, then abstract are not distinct from general terms, and have to be classed as such.

Although the view we have just criticised is apt to be misleading if it is taken as providing a definition of what abstract meaning is, it may nevertheless serve a useful purpose if it indicates how such a meaning is developed. It is true that in order to make our meanings abstract, we have to take attributes and objects apart from their mutual implication of each other; but merely to take objects or

attributes in isolation is not, in itself, sufficient to secure a new logical product. The difficulty of such a view as we are criticising can be seen if we observe what would follow from its acceptance. For if, according to this view, only a meaning were retained, the thing meant by it being ignored, we may ask about this meaning whether it is a detached meaning, — a meaning that is not the meaning of any object. If we answer this question in the affirmative, as we are required to do, we have opened the door to all the absurdities that belong, in the popular understanding of the word, to 'abstract' thought, — to thought, that is, that has nothing to do with any definable reality. To illustrate, if from a student who comes for the first time to the study of philosophy, we were to elicit by careful questioning what his uninstructed understanding of philosophy is, we should probably find that for him philosophy is a set of ideas more or less connected with one another, but having little or no connection with anything else; and if, by further questioning, we were to get him to state why he had undertaken its study, we should probably find that he looked upon it either as a good mental gymnastic, or as a field in which, since no one can ever be right, he at least can never be wrong, whatever opinions he may hold. Such views are held merely because it is supposed, by those who entertain them, that philosophy is a set of ideas dissociated from all objects whatever, that it is, in the popular understanding of the word, an 'abstract' subject. But

ideas which do not make some object more intelligible cannot be saved from the fate of oblivion by giving them a name; and we may neglect, as falling below the minimum of what knowledge requires, this understanding of the term abstract.

Having rejected the popular interpretation of the expression 'abstract idea,' we have to ask whether, in fact, such ideas exist, and if they do, under what conditions their meaning is developed. In answering these questions, we may remark that the situation with respect to abstract ideas is somewhat as follows: If, concerning these ideas, we say that they are true of the objects in connection with which they have been formulated already in our experience, then they are not abstract ideas at all; they are, as we have seen, general ideas; and if we say that they are ideas which mean nothing, we are talking nonsense. To illustrate, let us suppose that in the spring of the year, we are looking out upon the landscape, and that you remark, 'How green everything is!' Here 'green,' as you use it, is a general idea. If, however, in the midst of a conversation on monometallism, or of an unbroken silence, I were to say 'green,' you would naturally ask, 'Green what?' And if I replied, 'Green nothing, just green,' you would have a right to suspect my sanity. I can only rehabilitate myself in your regard by pointing out that 'green' is as possible an object of thought as 'trees' or 'grass;' and this is, without doubt, the course that, under such circumstances, one would take. Now, in this illustration

we have a clue to what happens when a concrete general idea has broken away from its moorings in the world of concrete fact: the meaning conveyed by the general idea becomes at once a new object of thought, which is capable of further consideration and characterisation. In brief, it is only when a general idea has, in this way, been dealt with as a subject of predication that we arrive at a true abstract idea. For instance, if we take our former illustration we may observe that although the grass, the oak, maple, sycamore, and elm trees, the winding river, etc., are all 'green,' they are not the same greens; they vary, as the psychologist tells us, in tint and saturation. Now if we ask, with respect to this fact, what we mean by calling things admittedly different by the same name, we shall have to say that there is something, whether we can state definitely what it is or not, which is common to all these objects, something which we express by the word 'greenness.' 'Greenness' is the abstract quality by virtue of which any object is called green, it is that which belongs as an attribute to all greens whatsoever. If, to take another illustration, we sort out the skeins of yarn for testing colour blindness, and ask what it is that leads us to consider objects which give so many different sensory impressions as belonging to a single group, we should doubtless, answer colour. 'Colour' (=colouredness) is the abstract quality by virtue of which objects are classified with regard to the common visual impressions that they make. Again, 'triangle' is an

abstract idea, because it implies the quality (triangularity) by virtue of which three-sided plane figures are grouped together. These illustrations indicate with sufficient clearness the answer that must be given to the question we are considering. Any idea which denotes certain attributes common to a group of concrete objects is abstract if, at the same time, it connotes the underlying ground of the similarity which exists between these attributes.

REFERENCES

SEE THE END OF THE NEXT CHAPTER

QUESTIONS

1. What is the relation of terms to propositions?
2. What is meant by the connotative and denotative use of terms? Illustrate.
3. Are terms ever used exclusively in a connotative or exclusively in a denotative sense? Explain your answer.
4. In case a term is both connotative and denotative, are the connotation and denotation always equally prominent? Explain.
5. How does any increase in the connotation of a term affect its denotation, and vice versa?
6. What is the relation of the distribution of terms to the connotative and denotative use of terms?
7. Explain what is meant by the distribution of terms, being careful to distinguish between distributed and undistributed terms.
8. What is the relation of the meaning of terms to the connotative and denotative use of terms?
9. What general characteristics of terms may be mentioned from the standpoint of connotation?

10. Upon what do these characteristics particularly depend?
11. How is the problem of definition distinguished?
12. Upon what fact of our common experience does concrete meaning depend?
13. In what respects does the term concrete differ from the word thing?
14. What do you understand by an individual or singular term?
15. Name the kinds of individual terms.
16. Define and illustrate proper names.
17. Are proper names used connotatively? Explain and illustrate your answer.
18. Define and illustrate designations.
19. Are designations used connotatively and denotatively? Explain.
20. What is the importance of designations?
21. Explain how meaning becomes general, and show that in doing so it remains concrete.
22. What is a general, or common, term? Illustrate.
23. Discuss the connotative and denotative use of general terms.
24. Explain how meaning becomes collective, and yet remains concrete.
25. What is a collective term? Illustrate.
26. Discuss the connotative and denotative use of collective terms.
27. What is an individual collective term? Illustrate.
28. What is a general collective term? Illustrate.
29. What is the relation between individual and general collective terms?
30. Does this relation always hold? Explain.
31. How does abstract, differ from concrete, meaning?
32. Distinguish carefully between abstract and general terms.
33. Discuss the connotative and denotative use of abstract terms.

EXERCISES

1. What are the connotation and denotation of the following terms:
Gentlemen my typewriter logic
George Washington gratitude the University
2. Show by an illustration that the denotation of a term may decrease if its connotation is increased.
3. Classify in a table all the terms discussed in this chapter, showing which are coördinate and which are subordinate.
4. Write a list of twelve concrete individual terms, and indicate which are proper names, and which, if any, are designations.
5. What kind of term is 'goodness' in each of the following statements?
 - (1) Goodness knows what we shall do.
 - (2) He did it out of sheer goodness.
6. Write a list of six general terms.
7. Derive, if possible, abstract terms from each of those that occur in your answer to the previous question.

CHAPTER V

TERMS (CONTINUED)

The Purpose of the Chapter. In the previous chapter we have traced the development of meaning from its concrete to its abstract form, and in doing so we have touched upon the main principles that control the logical use of terms. There are, however, a number of other interests that lead us to consider objects and their qualities in ways slightly different from those already described, and thus to give our doctrine of meaning, in each case, a slightly different emphasis. In this chapter we shall define and illustrate some of these more common interests, and point out the varying emphasis that our meanings come to have under their guidance.

The Basis of the Distinctions to be Studied. The distinctions in meaning that we are to study in this chapter, like those studied in the last, have a basis in common linguistic usage. The basis of the distinction of concrete and abstract meaning is the fact that we distinguish between an object and its qualities, — between the object that is the possessor of qualities, and the qualities possessed by the object. The distinctions studied in the present chapter are based on this broad contrast between objects and their qualities, but are not identical with it. When, for example, we have distinguished between an object and its qualities, so that we can think of the object merely as having existence, and can think of qualities as possible characteristics of objects without at the same time identifying the objects

of which they are the qualities, we can take a further step in regard to objects, and consider whether they do or do not stand in relation to other objects, and, in regard to qualities, whether they do or do not belong to certain objects. In the former case, we ask whether the relation that objects sustain to other objects affects the meaning that such objects have; and, in the latter, we ask whether the possession or non-possession of certain qualities by objects in any way modifies the significance that such objects have for us. In the first question, we are concerned with what are known, in logic, as absolute and relative terms; and in the second, with what are known as positive and negative terms.

Absolute and Relative Meaning. This distinction is one that concerns the existential character of objects, that is, objects considered with respect to whether they do or do not depend for the meaning they have upon the relations in which they stand to other objects. Stated otherwise, we are asking whether it is possible and useful to study objects independently of the connections they may have with other objects, and whether the terms we use to denote objects do or do not get part of their meaning from the relations which these objects sustain to other objects that fall outside the denotation of these terms themselves. The answer to this inquiry is quite obvious. Objects whose connection with other things is for the time being neglected may certainly be held before the mind as subjects of investigation. These objects may be quite simple,

as a chair, or they may be quite complex, as the solar system. But whenever a simple or complex object derives its importance for consciousness from the fact that we ignore the connections it may have with other existential things, it is said to be an independent object. Now when the names of such objects are used as terms in a proposition, they are called absolute terms. Thus, all individual terms, whether they denote single individuals, or single groups of objects, are, in the sense explained, absolute terms.

What we should understand by a relative term is more difficult to state. We shall first consider the most general aspect of this question. It is quite clear, in the first place, that since the whole problem of meaning rests, as we have seen, upon the connection between the connotative and denotative use of terms, that there can be no object entirely devoid of qualities, and no quality which is not the quality of some object. There is always a relation between these two aspects of a term's meaning. Further, it is clear that all our meanings, whether concrete or abstract, singular or general, are also relative in the sense that the objects denoted by such terms stand in relation to objects denoted by other terms. Used in the broad way, indicated by these illustrations, the word 'relative' does not point out a particular logical distinction, but describes a common characteristic of all human knowledge whatsoever.

The term relative is used in logic, in contrast to absolute, to indicate the fact that part of the meaning that a term has is derived from the exist-

ence of an object that the term itself does not denote. Stated another way, a relative term is one that implies the existence of some other object than the one that the term itself denotes. It is obvious that no concrete individual term can be relative in this sense. But some concrete general terms are. For example, the term 'father' is a concrete general term since it is a name which applies to an indefinite number of individuals in whom the relations implied by the name are present; it is also relative because these relations are such that they depend for their application not only upon the existence of the individual to whom the name is given, but also upon the existence of either a son or a daughter. A similar double relationship exists in all family connections: uncle, grandfather, niece, etc. We may observe the same thing in other than the family type of social organisation: political, — king-subject; economic, — debtor-creditor; juridical, — plaintiff-defendant; commercial, — employer-employee; domestic, — master-servant, etc. Whether this type of relationship is ever found outside of social organisations, or if found, whether they are common, has been questioned. We may point out that some logicians have been inclined to find the meaning of such a word as 'mountain' in the reference that it is said to contain to the plain from which it rises. There may be other words whose meanings are of this relative sort, such as, right-left, heavy-light, up-down, etc. In deciding whether any term is relative, it should be borne in mind that the only clue

that is of logical value, is the sense in which it is actually used in carefully guarded and intelligent speech. And by this test it appears that the words whose meanings are determined by the existence of other objects than those which such words denote are chiefly those that indicate the relations that we, as human beings, have with one another in some type of social organisation.

One other point remains to be mentioned. It is sometimes found that when one object derives part of its meaning from the existence of another, the latter also derives part of its meaning from the first. Not only, for example, does uncle or aunt imply nephew or niece, but nephew or niece implies uncle or aunt. That is to say, each term of the relation implies, and is implied by, the other. Relationships of this kind are called reciprocal. The terms between which such relationship holds are also called correlatives. Thus, when we regard the creditor-debtor relation from the standpoint of creditor, we say that 'debtor' is a correlative term, and when we regard it from the standpoint of debtor, we say that 'creditor' is a correlative term. By a correlative term, therefore, is meant that one of a pair of relative terms which is implied by the other.

Positive and Negative Meaning. The distinction between positive and negative meaning rests upon the answer we give to the question whether a given object has or has not a certain attribute or set of attributes. In a general way, it is true that every object has or has not the attributes which the word

we use to point it out implies; otherwise, it may be asked, how can we identify any subject of discourse? It may, however, be said that it is sometimes necessary to state emphatically that the objects we are speaking about have or have not certain attributes, and that when this is the case, the meaning of what we express is either positive or negative. A positive meaning, to consider that first, calls special attention to the fact that an object has the attributes which are implied by the term which is used to point it out; and it is necessary sometimes to emphasise this fact, because the meaning of what we affirm, without such emphasis, is liable to be misunderstood. Whenever the denotation of a term is unambiguous, or when its popular meaning does not render it equivocal, we do not need to call special attention to its precise connotation. For example, the term 'rent' is popularly understood as a fixed sum of money that is paid or received for the occupancy of a house or of land. It is primarily a denotative term, and many judgments may be expressed which are perfectly intelligible about the object, — the fixed sum of money — which this term denotes. We may say, for example, that 'Rent is a constant drain upon a poor man's wages,' 'To pay rent all one's life is an indication of improvidence,' etc. In all such instances, the sense of our assertions is carried easily by the denotative use of the term 'rent.' However, when what we assert is dependent, not upon the object's existence, but upon its meaning, we are in a different position. In that

case, the meaning of the term by which the object is denoted must be made precise 'if we are to have an intelligent understanding of what a proposition affirms. Thus, for the economist, the term rent is 'the remuneration that is derived from the concrete things that have the value of capital,' and this meaning of the word must be kept in mind in any discussion which involves the idea for which the term stands. In this instance 'rent' is used connotatively, and, so used, it limits whatever can be said intelligently on any subject that implies the idea. It is a positive term because it is used to indicate the fact that any object to which it is applied must have the qualities which the term connotes.

The Conditions of Negative Meaning. In discussing negative meanings, we shall inquire, in the first place, into the conditions under which such meanings arise. We call attention to three situations in which knowledge is forced to assume a negative form. First, whenever, for any reason, we fail to reach a positive conclusion on any subject of inquiry, we may map out the progress of the inquiry by a series of negations. For example, let us suppose that a physician is called to see a patient who has a serious illness. As the examination of the patient proceeds, a number of prominent and secondary symptoms are noted and the possibility of typhoid, tuberculosis, scarlet fever, etc., suggest themselves in turn, only to be rejected one after the other; and the physician concludes his visit with a confession, to himself if not to the family of the patient,

that he has reached no positive knowledge of the disease, or complication of diseases, from which the patient is suffering. The physician's knowledge, in such a case, is made up of a series of negative judgments; not typhoid, not scarlet fever, etc. Through what has been denied, that is, through what the physician calls his differential diagnosis, he has come nearer to, but has not succeeded in making, a positive diagnosis. Negations thus mark out the limits within which the positive characteristics of an object are to be found. No doubt, in new and complicated situations, business and professional men are often in the position where the only thing that is clear to them is what not to do or think. In such cases, we have negative but not positive meanings; but the former exist, not as something opposed to the latter, but as stages in our progress toward it. Our subject-matter refuses, one after the other, a number of suggested qualifications, and often we have to stop our investigation of the matter in hand before we have found the particular qualification which fits.

Secondly, negative meaning arises when we safeguard a positive meaning from misunderstanding or falsification. Let us take, as an example, the case where a new truth is presented for the first time for our understanding. We are more often in this relation to truth, as students, than in any other. The teacher, let us say, is expounding the psychological doctrine of perception. We follow him attentively as step by step he enumerates and explains

the various factors in the problem, and their relation to each other. Now the important question for us as students is whether we have really understood what the teacher has said. Class-room experience shows that a student's reaction to the subjects of his study comes most often in the form of a question, or a statement, which, from the teacher's standpoint, is the suggestion of a predicate or series of predicates, for the topic of discourse, — in the example, the psychological doctrine of perception; and the teacher's problem is to determine whether the proposed predicates are compatible with those by means of which his own exposition has been carried on. Now, since our first apprehensions of any new truth are liable to be uncertain, vague, or partial, our proposed predicates not infrequently endanger our grasp of the new truth, and have to be eliminated from the list of those attributes which, when taken together, constitute the positive meaning of the subject under consideration. Now, this elimination of incompatible predicates is performed by the teacher whenever he refuses to accept our statements or questions as elucidations of the subject-matter in hand. And his refusal is given in the form of a negation which means, from his point of view, that he is defending a positive meaning from misunderstanding or falsification.

Thirdly, negative meaning arises whenever, in carrying forward a special line of work, our problem is to see how far, and in what directions, our subject is capable of development. The starting

point, in a case like this, is the body of knowledge we have already acquired about our subject. We do not become possessed of the truth that lies beyond our grasp by a method that implies disloyalty to the truth that lies within it. The working criterion of the truth of any thing new must be the truth that we already possess. And this means that nothing becomes true which contradicts what is already true. Truth is a system, systematic and coherent, one part of which supports and supplements every other. However, this position should not be held in such a way that the limitations of the truth in hand are allowed to make us inhospitable to the constantly increasing and enlarging body of truth. Truth, it should be borne in mind, grows from less to more; and, in the process, not only do we come into possession of new truths, but the old truths are transformed, and sometimes the entire system of truth changes its character. A familiar instance of this is the change from the Ptolemaic to the Copernican astronomy. The particular use of the statement and illustration in this place is to call attention to the fact that such changes in the character and system of truth is effected by negation working within the field of established truth. For what is the recognition of truth's limitations but the admission that negation not only erects a barrier against outside error, but also against that hardening process in the heart of truth itself which, unchecked, steels it against the recognition of outside truth? When, therefore, we are endeavouring

to extend the boundaries of a particular subject, or when we have to consider the truth of any subject in its relations to other fields of knowledge with which it may be correlated, we are often forced into the position of either denying all outlying truths, or modifying, in the interest of the systematisation of truth, the knowledge that we already possess. Development of knowledge takes place only when we follow the latter course. But, we should remark, the modification of established truths, which is necessary if what is new is to find a place in the developing field of knowledge, is made possible by the elimination of what is erroneous in it; or, stated otherwise, the limitations of existent truth are brought to light by negation working within the area of our established beliefs.

The Logic of Negation. That there is such a thing as negative meaning is not open to serious question, and what the conditions are under which it arises we have already seen. The inquiry that confronts us now is, How are negative meanings to be interpreted? We may approach this question by remarking that negative meanings are developed out of the interests that lead us, in our inquiry into the nature of things, to affirmative conclusions. By this statement we mean that negation must be interpreted as a form of knowledge, and not merely as an assertion of the absence of knowledge, and that, as such, it satisfies, in the particular instances in which negative meanings arise, the motives that sustain all our cognitive endeavours. Stated other-

wise, we may say that since in all knowledge thought is, as we have seen, in some particular relation to its object, we must hold either that this is true of negative as well as of positive meanings, or that negative meanings fall outside of the field of logical inquiry. The only alternative to this position would be to deny the existence of negation altogether. But this we cannot do. And we cannot do it for the simple reason that if we did succeed in getting rid of negation in one form, it would break out in another, — a fact that is significant of the distinctive function of negation in our search after the truth of things. An illustration of the fact just referred to may be found in the endeavour to get rid of negative meanings by reducing all propositions to a common affirmative type. Thus, it is contended, the meaning of the statement, 'Some men are not Caucasians,' is not, 'Some men are-not Caucasians,' — a negative proposition, but, 'Some men are not-Caucasians,'—an affirmative proposition. But, if this is true, it is obvious that, in the latter case, the predicate term has been burdened with the negative that belonged, in the former case, to the copula. And, what is more important, this way of escape from the difficulties of negative statements seems to have rendered affirmative ones equivocal. For, it will be remembered, in all affirmation we are supposed to predicate a positive characteristic, quality, attribute or relation of a subject; but when we affirm by means of negative terms, we seem to be saying that the subject does not possess the attri-

butes connoted by the predicate of the proposition which expresses our meaning. So far then from escaping the problem of negation, this attempt to reduce all statements to an affirmative type forces upon us the question of how we are to interpret negative terms. To this question we shall now give attention.

Let us be perfectly clear what our problem is. When we say 'not-Caucasian,' 'not-typhoid,' etc., it is quite possible, as we have said, that we mean nothing more than that our knowledge is falling short of what knowledge, strictly regarded, requires. This, however, is a different thing from saying that we have attained the full measure of knowledge when all that we can affirm is that a subject has some quality or relation vaguely expressed by negative terms. When, for example, we say of an individual that he is not-Caucasian, or of a disease that it is not-typhoid, what, we should like to know, are the positive attributes, which these terms connote? And we can ask such a question just because we are dealing with an affirmative statement. Now one way of answering this question has been to say that, by the exclusion of a particular group of attributes, expressed by the negative term, we include within the meaning that the negative term must have all other attributes whatsoever. In other words, the negative term, or, as it is sometimes called, the indefinite, indesignate, or infinite term, is one which divides all existing things into two groups, in one of which there is placed definitely

the excluded meaning, and in the other all other existences and meanings. According to this view, it is the function of the negative term to denote the latter group; it is the name of a miscellaneous number of things, such as "accidence and adjectives and names of Jewish kings." Thus, as Bosanquet says, "not-Christian literally interpreted includes not only heathen humanity, but the fixed stars, the sea, and indeed, in Aristotle's words, 'every thing whether existent or non-existent' except Christian. It refers to no one sphere in preference to another, and thus says nothing definite enough to be intelligible." It is an example of what we may call bare negation, that is, of negation unlimited by any reference to a positive sphere of reality; and, because it includes so much, succeeds, in the end, in denoting nothing in particular. Now it may be doubted whether, outside the field of a purely formal logic, there are any such infinite terms as this; that is, whether bare negation, a negation which is not a stage or step in our acquisition or development of positive knowledge, is ever rooted in the purposes or motives which, in our relation to things, give birth to relevant assertion. But if these infinite terms do not express meanings that are the outgrowth of the motives that control our knowledge processes, if, that is, they have no relation to the purposes that guide and determine the life of judgment, they lose all interest for logic; they may have a psychological, but can have no logical, significance.

Another way of meeting the question we are considering is to point out that the meaning of any proposition must be sought within the universe of discourse which the proposition implies, and that, consequently, this places a limitation upon the meaning that the proposition's terms can have. If from this point of view we consider the statement, 'Some men are not-Caucasian,' it is obvious that we are speaking of men from the standpoint of their racial characteristics, and that the group of men which now interests us, — the 'Some men' of the proposition — do not have the characteristics of the Caucasian. But if we are making the statement at all intelligently, we mean more than this; we mean that they have the characteristics of either one or other of the remaining divisions of the human family looked at from the racial point of view. That is to say, the positive meaning of the negative term 'not-Caucasian' can be expressed by 'either Negro or Mongolian or American Indian.' Thus, the whole meaning of our statement is, 'Some men are not-Caucasian, but are either Negro or Mongolian or Indian.' Likewise the other statement, 'This disease is not-typhoid,' means that a pathological condition is present, but it is not typhoid and it is either scarlet fever or tuberculosis or diphtheria or one or other of an indefinite number of the ills to which the human body is subject.

Privative Meaning. Closely connected with negative terms are those called privative, terms which indicate the absence of an attribute naturally or

usually belonging to the object which such terms denote. 'Blind,' 'maimed,' 'orphaned,' 'deaf,' are examples. Because they are so closely connected, it may help us to understand both negative and privative terms if we indicate wherein they seem to differ from each other. The negative term, as we have seen, indicates a number of positive alternatives by means of a negative characterisation of the object which such term is made to qualify. It is, therefore, not a term that is negative in meaning at all; it is only its form that has this characteristic. The reverse seems to be true of privative terms, their meaning is negative, and what is negated is definitely the attribute or attributes which such terms connote. Thus, 'The boy is an orphan,' must be understood to mean that he has no parents, — 'the absence of parents' being what the term 'orphan' connotes. A further difference between these terms may be indicated. In the case of negative terms, we are able, through what is denied, to affirm certain alternative qualities, that is, to go beyond what is definitely stated by such terms. In the case of privative terms, this is not true. Of the boy who is an orphan, all that we can positively affirm is that he has no parents; we cannot say whether he has either a brother, or sister, or uncle or aunt. If of any one we can say, 'He is deaf,' we do not find in this statement any ground of assertion with respect to the presence, absence, or degree of acuteness of any of his other faculties. In view of these characteristics, privation must be interpreted as an ex-

treme case of negation; that is, of a negation which arises under conditions that make defect of quality a differentia of the meaning we intend to convey.

Disparate and Contrary Meaning. We may touch upon two other related distinctions which have negative significance. Whenever we can divide a class of objects into a number of mutually exclusive classes, as, for example, the class animal into lions, tigers, etc., these subordinate classes are said to be disparate. If, further, we can arrange the exclusive classes of a group of objects with respect to their possessing more or less of some quality, for example, organisms in a series from amoeba to man, and if we can say of two of these classes that, with respect to the common quality, they differ from each other in a greater degree than either does from any other, the relationship between these two most differing classes is said in logic to be that of contrariety. A contrary relation is one form of negative meaning, and indicates the greatest degree of exclusion, actual or possible, within a single genus. In other words, contraries are the extreme limits of the universe of discourse within which the meaning of any proposition falls.

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QUESTIONS.

1. How are the logical distinctions of absolute and relative, positive and negative terms, related to those discussed in the previous chapter?
2. Upon the answer to what question does the distinction of absolute and relative depend?
3. State and illustrate the meaning that belongs to absolute terms.
4. What senses of the term relative are excluded from our discussion of relative terms?
5. What is meant in logic by a relative term?
6. What class of experience provides most of our illustrations of relative terms?
7. What are correlative terms?
8. Upon the answer to what question does the distinction of positive and negative depend?
9. What is a positive term?
10. Under what conditions do we express our meaning positively?

11. State and explain the conditions under which our knowledge has to be expressed negatively.
12. Are the motives that lead to negative statements different from those that lead to affirmative ones? Explain.
13. How have some logicians tried to get rid of negative meaning?
14. Criticise this effort.
15. State and illustrate two views of the logical significance of negative terms.
16. What do you understand by privative meaning?
17. How are privative related to negative terms?
18. What do you understand by disparate and contrary meaning?

EXERCISES

1. Write three propositions to illustrate the absolute meaning of terms.
2. Discuss each absolute term in your answer to the previous question to show whether it is concrete or abstract. Write new propositions to illustrate your results.
3. Write new propositions to illustrate concrete and abstract terms.
4. Write three propositions to illustrate the relative meaning of terms.
5. Discuss each relative term in your answer to the previous question to show whether it is concrete, abstract, absolute. Illustrate.
6. Write three propositions to illustrate the positive meaning of terms.
7. Discuss each positive term in your answer to the previous question to show whether it is concrete, abstract, absolute, relative. Illustrate.
8. Write three propositions to illustrate the negative meaning of terms.
9. Discuss each negative term in your answer to the previous question to show whether it is concrete, abstract, absolute, relative, positive. Illustrate.

CHAPTER VI

DEFINITION AND THE PREDICABLES

The Questions of this Chapter. In our study of terms, in the two preceding chapters, we saw first, that a term may be used either to point out particular existing things or to convey some meaning. When a term is employed to point out particular existing things it is said to be used denotatively; when it is employed to convey some meaning, it is said to be used connotatively. Arising out of this distinction of the denotative and connotative use of terms, certain problems were forced upon us for consideration. For instance, our study of the denotative use of terms brought before us the problem of the distribution of terms, and our study of the connotative use of terms brought to our attention certain general characteristics of terms the study of which, as we may now say, involved the problem of the classification of terms. In our statement of this latter problem, we were led to recognize another problem, — the problem of definition, — and this problem was said to be concerned with the way in which the particular meaning of terms receives logical formulation. The facts and relations embodied in this statement of the logic of terms may be expressed in a tabular form as follows:

	<i>Standpoint</i>	<i>Problem</i>
Terms {	Denotation	Distribution of terms
	Connotation	{ Classification of terms Definition of terms

This table may be read as follows: Terms studied from the standpoint of their denotation give rise to the problem of the distribution of terms; studied from the standpoint of their connotation, they give rise either to the problem of the classification of terms or to the problem of the definition of terms.

Of the two problems connected with the connotative use of terms, the problem of the classification and of the definition of terms, the former may be said to be concerned with the question, What are the various kinds of terms that form the S and P of logical propositions? And the latter is concerned with the question, What, irrespective of the kinds that terms may be, are the meanings that terms may have? We have considered the first of these questions in the two previous chapters. The second question we shall study in the first part of the present chapter. In this part, we shall try to show how to make the particular meanings of the terms we employ definite, how to formulate them in such a way that no doubt remains as to the sense in which they are used. We hope, as the result of our study, to learn how to proceed when we are required to make the meaning of any of our terms explicit.

After we have concluded this study, we shall be in a position to ask and answer the question, What kinds of things can be predicated of a subject? The convenience of discussing this question here is that doubt has been cast upon the possibility of predicating of any subject its own definition. But it is more than a question of convenience; for, as we shall see, our whole view of the nature of definition is involved in the position we take with respect to the predicables. There are, then, two problems to be discussed in this chapter, the problem of definition and the problem of the predicables, and of these the first to be considered is the problem of definition.

Non-Ambiguity. We may approach the study of this question by calling attention to the fact that in the fields of science, more than anywhere else, our ideas must be accurate, and the language in which these ideas are expressed must definitely convey the meaning intended. That is to say, the natural fluidity of language must not be allowed to go in the sciences to the point of ambiguity. Whenever, that is, we wish to express our ideas in words, there is a limit to the indefiniteness that can be allowed to the forms of our speech, and if we really convey the sense of what we mean our words must be definite in what they connote.

This demand for precision in the use of words may be easily misunderstood, and it may lead to an effort after the impossible or undesirable. The average man's impatience with over-primness of speech

is a healthy protest against pedantry; but the same man's laxness in the use of words bears witness quite often to the vagueness of his ideas. These are the extremes against which it is desirable to guard: the extreme, on the one hand, of an accuracy of speech out of all proportion to the character and importance of the subject-matter of our thought; and the extreme, on the other hand, of being tyrannised over by words and of using them as substitutes for ideas. There is always an appropriate exactness that we should strive to realise in our use of language, and nothing more than this can be demanded of us by the strictest logic. We must aim to make our speech non-ambiguous, but non-ambiguity is relative to the purpose of the speaker and to the subject of his thought. Non-ambiguity changes with the change of purpose and subject, and the degree of exactness that is requisite on some occasions and with respect to some subjects might be altogether inadequate if these were different. There is no one standard of explicitness that we are required to attain; the most that can be demanded is that on all occasions, and with respect to all subjects, our language be made a fitting vehicle for the expression of the ideas we wish to convey.

We have called attention to these somewhat obvious considerations, because, through their neglect, the importance of the problems with which we are here concerned has frequently been obscured.

This, we think, is unfortunate; for whatever can help us to avoid misunderstanding, whatever can strengthen our reputation for saying what we mean and meaning what we say, cannot be of indifference to any one of us, in a world where intellectual integrity is coming to play an increasingly important part.

These considerations may also enable us to see that whenever we are in danger of being misunderstood, whether in science or literature or business, on account of the indefinite meanings of the words we employ, there is required some method by which the misunderstanding can be avoided, some method by which words can be moulded to the uses of ideas. Life is one, and the occasions when it is necessary to state explicitly the meaning of the words we use arise all along its way, and are not confined to, even if they are more frequent and more urgent in, the various departments into which our human knowledge has been organised. In other words, ambiguity may be as undesirable in the drawing-room or counting-house as in the laboratory, and whenever and wherever it is a hindrance, we naturally try to get rid of it. But before we consider how this may be done, a word may be added on ambiguity itself.

Ambiguity. The traditional doctrine of ambiguity may be briefly stated. Any term is ambiguous if it is given a meaning in a proposition which it does not have in the judgment which the proposition expresses. For example, 'He was a man who

always chose a mean line of action.' Terms that have more than one meaning, and which, therefore, lead to ambiguity of statement, are called ambiguous or equivocal; and, in contrast to these, those that have only one meaning are called unambiguous or univocal.

There are two respects in which this view of ambiguity does not seem to be satisfactory. First, it implies too formal or mechanical a view of logic. It leaves the impression that words are like counters in a game to be pushed here and there as the exigencies of the game demand, that no difference is made to the words themselves by their change of relation in different propositions. Second, it supposes that there are certain words which have a fixed meaning, and that the statements in which they occur are, therefore, free from ambiguity. This is difficult to maintain for any class of words, and if it were true of particular words there is no guarantee that it must remain so. The principle that should guide us in a discussion like this is that it is not words that make a language, but language that moulds words and fits them to the uses for which the language exists. For example, the word 'Democrat' has a definite historical meaning, as also has the word 'Republican'; but as names of political parties they no longer mean what they meant originally, and have become vague and indefinite through the growth within each party of sub-classes or varieties of democracy and republicanism, each of which would deny to the other the right to the name.

This process of splitting off from a perfectly clear centre of meaning is going on all the time in the living languages of the world, and is giving rise to conflicting meanings in the uses of words.

The problem of ambiguity may be viewed in a broader way still. Instead of considering it as having to do with the indefiniteness that arises from the vague or shifting meaning of words, we may view it as in some sort the central or fundamental problem of the science of logic itself. For logic, as we have seen, is a study of the methods by which meanings can get asserted, and how asserted meanings can get expressed in propositions. It is because meaning tends to elude us, and because words are not always, if they are ever, faithful representatives of ideas, because, that is to say, ideas and words are both equivocal, that we are compelled, if the aims of thinking are to be attained, to study the sources of this uncertainty, and to derive guidance from the knowledge that we thus obtain. Ambiguity, therefore, is a term that describes a natural condition of human thinking, as well as a natural indefiniteness of human speech. But, as we all know, vagueness in an idea and indefiniteness in speech, have many sources; and it will not be until we have reached the end of our studies that we shall have reviewed even the more important ones. Ambiguity in the meaning and application of words, therefore, is part of a much larger problem; but in this chapter we shall consider only the ambiguity that arises in connection with our use of words.

Translation. One of the most common means for the removal of ambiguity is translation. Thus, if what we say is not understood, or is in danger of being misunderstood, we often translate the same idea into another set of words in the hope that the ambiguity may be removed. This is a common practice both in literature and in ordinary conversation. When, for example, Shakespeare makes *Hamlet* say,

O, that this too too solid flesh would melt,
Thaw, and resolve itself into a dew;
Or that the Everlasting had not fixed
His canon 'gainst self-slaughter!

he is putting into *Hamlet's* mouth words that express a single idea twice over, namely, the longing for death as a means of escape from his troubles. If, again, a teacher should ask his class a question which fails to elicit the correct answer, he would probably restate the question in the light of the difficulties that the answers had thrown upon the class's understanding of the question as first stated. If further, to take an illustration already used, we were to say of a given individual, 'He was a man who always chose a mean line action,' and we were misunderstood to say that he was a man 'devoid of a generous disposition', we should probably substitute for the word 'mean,' the word 'moderate' in a re-statement of what we had intended, in the first place, to say. There is no doubt that this habit of translating into a variety of forms the ideas we wish to express is so inveterate that most of us talk more

than is necessary for understanding, and many of the books we ordinarily read are much longer than they need be. But the care we exercise in making our meaning clear is grounded in a healthy instinct; it is the respect we pay to the demand for unambiguous statements.

When Translation may be Employed. If we look at the class of cases illustrated in the preceding paragraph, we shall discover that the source of their ambiguity does not lie in the fact that our words have a vague fringe of meaning, but that the centre or core of meaning has been misunderstood. Whenever ambiguity arises from our unfamiliarity with the meaning of the words in which ideas are expressed, what is needed is that the same idea be translated into a set of words with the meaning of which we are familiar.

In translation, then, ambiguity arises from the fact that the central meaning of a word is not precisely determined, either because the word carries no meaning at all, or because it carries a meaning that renders the whole sentence indefinite. If, for example, you do not know what the word 'commandeer' means, you are not likely to understand any statement in which the word occurs, for example, 'My motor has been commandeered.' To meet such a situation it is obvious that some other word or explanatory statement would have to be substituted for the meaningless 'commandeered.' If, to take another case, a class in logic were given the task of stating the meaning of the assertion 'Non

omnis moriar (I shall not all die),’ and it proved difficult or impossible, the reason would be that ambiguity spreads over the whole sentence from the indefiniteness of the word ‘all,’ an indefiniteness that is not due, as in the former illustration, to the absence of meaning, but to the conflict of two meanings that are not clearly distinguished. In this example, the Latin word *omnis* has a distributed meaning, and if the Roman had wished to express the collective meaning of ‘all,’ he would have written *cunctis*. We have only, therefore, to point out the distributed use of the word ‘all’ to enable the student to see that the meaning of the statement is ‘some part of me is immortal.’

When Definition is Required. The conditions under which definition is required are different from those under which translation is most effective. Definition is called for when the work-a-day meaning of the word that requires defining is already known, but is becoming obscured by the fact that, through the similarity of its meaning to other words, its distinctive characteristics are in danger of being overlooked. For instance, the most obvious common characteristic of the violin, viola, ‘cello, and bass viol is their shape, and it is this that leads one to regard all of them. on first acquaintance, as, let us say, violins. Our knowledge, that is to say, emphasises the common likeness that exists between the several instruments named, and there is confusion of one thing with another because the differences which give to these objects a distinct place in the universe of musical instruments have

been overlooked. The confusion does not arise from what is included, but from what is omitted, and what is omitted, in this case, lies, outside the centre, at the boundaries of the words' meanings. To state it generally, the meanings of closely related words encroach upon one another when the boundaries that keep their meanings distinct are overlooked or broken down. Now it often happens that the sense of what we say depends to a large extent upon a word's ability to suggest just such differences as, in the illustration used, we persistently ignored. When this happens ambiguity is sure to result, an ambiguity, let it be observed, that is due to the confusion of closely related things. In such a case, the practically important problem is to find some method by which ambiguity of the kind here in question can be overcome when it exists, or be avoided when it does not.

The Method of Definition. In any case of ambiguity that can be overcome by definition, the indefiniteness of the word's meaning which occasions the ambiguity is due to our failure to mark off the boundaries of this meaning from those of other related meanings. Indefiniteness of this sort in the meanings of words may be illustrated if we ask ourselves, for example, what we understand by a balloon and an air-ship. The conspicuous central meaning of the two words is doubtless the fact that they are machines that travel in the air; but if they mean no more than this, we shall very soon be calling each by the name of the other, or we shall be calling all

machines that travel in the air by the name of the one or the other. It is in cases like this, as we have seen, that the definition of words is necessary. How, then, shall we go about the task of defining? Obviously, by the way of comparison, by the comparison of the things which our words denote. Thus, if we wish to make our meanings of the words balloon and air-ship precise, we compare an air-ship with a balloon, and this comparison will lead to the discovery that they differ from one another in the method of their locomotion: the air-ship is driven by an engine and the balloon is not. We shall, then, call anything an air-ship which travels in the air and is driven by an engine, and the word balloon will be reserved for any air-traveling machine which depends for its locomotion upon the air-currents. This method of definition is for all ordinary purposes the most practical, and it lends itself to the requirements of the specific occasions which make definition imperative. We get, that is to say, the meaning that suits the purpose of the statement. Constant practice in the comparison of what words denote will not only render our meanings more precise, but free us from that indefiniteness which comes from an exclusive familiarity with the dictionary meanings of words; for what a dictionary cannot do, and what comparison does, is to render the meanings of words appropriate to the particular contexts in which they occur. By the method of comparison we find the relevant meaning, the meaning that is required by the universe of discourse

within which our thought is moving, and by the purpose which controls our particular assertions.

Definition per genus et differentiam. By definition, as we have seen, we aim to substitute definiteness for indefiniteness in the meaning of our words, and thus to avoid ambiguities of statement. We have seen, also, that not all cases of ambiguity are overcome best by definition, but only, or particularly, those cases in which the vagueness of the boundaries of a word's meaning obscures the sense of what we wish to say. In definition, therefore, we seek to limit the range of a word's meaning, by giving an appropriate degree of definiteness to its fringe or outlying area.

Now it is quite obvious that we cannot limit the meaning of our words in the way required by definition if these words are used or understood by us with no meaning at all. We cannot talk about limiting the extent of our words' meanings unless these words are used, and are understood, in some, however vague, sense. Hence, as we said above, the central meaning of words is not in question in these cases of ambiguity; it is only the range of the word's meaning that is in doubt. It would seem, therefore, that the limitations that we put upon our meanings in defining them must have some relation to these meanings themselves, but the particular direction in which the limitation is made depends upon the purpose for which we are seeking the definition. To take a simpler illustration, if the purpose for which we need a definition of 'water' is domes-

tic, we may say that it is a liquid which is useful for cleansing; but if our interests are chemical, we must say that the liquid is composed of two molecules of hydrogen and one of oxygen. In both instances, we start with the central idea of water as a liquid, and in both we limit the range of this meaning in a direction which is determined by the purpose or interest that the definition is to serve. It would seem, consequently, that every definition has two parts, a central and a peripheral, and that these are different, though related, aspects of a word's explicit meaning.

Now the two parts of which, as we have seen, the meaning of words is comprised are called in logic the *genus* and the *differentia* respectively. By the *genus* we mean the clear centre, and by the *differentia* the vague periphery of a word's connotation. The connection between the two is obvious if we remember that both are specifications of a single meaning. Since, therefore, in defining a word's meaning we start with the recognition of its *genus*, the *differentia* cannot add to that meaning anything inconsistent with what the *genus* already connotes. That is to say, we must look within the range of meaning broadly marked out already by its *genus* for the marks that make the meaning of a word distinct. The *differentia* must lie within the *genus*, and not outside it, if we are to mark off from encroaching meanings the one that, by definition, we are trying to make distinct. For what lies outside any *genus* belongs to another *genus*, or is that

genus itself, and is not likely to threaten us with ambiguity; but the distinctions that lie within a *genus* are likely to be overlooked because of the common meaning that pervades all that belongs to the *genus*. For example, no one is in any danger of confusing a typewriter with the table on which it stands because each belongs, in our common acquaintance with it, to a different *genus*, the one to the *genus* 'objects for writing with,' and the other to the *genus* 'objects for placing things upon.' But if, to take another illustration, the poet or philosopher or scientist were to mistake the world of his imagination for the world of his actual experience, this would be because he had failed to observe the marks that keep the two worlds distinct. If, again, the student were to use the words 'university' and 'college' synonymously, this would be because he had failed to notice, within the meaning that both have in common, the specific difference by which the one is distinguished from the other in the same *genus*. From these statements and illustrations it is obvious not only how *genus* and *differentia* are related to each other in a logical definition, but also what the terms themselves signify. Thus, by a *genus* is meant a group of qualities belonging to a number of particular objects in common. The word is also used to denote any group of objects when these objects are thought of as possessing common qualities. By a *differentia*, on the other hand, is meant the specific difference by which a distinct place is given to an object within a *genus* by reason of its possessing a

quality, or group of qualities, that does not belong to any other member of its *genus*. Hence, in looking for the *differentia* of an object we must look for some quality, or group of qualities, that does not belong to any other object of the group to which this object belongs. When this quality or group of qualities is not clearly apprehended we are liable to confuse one thing with another.

Verbal and Real Definition. The distinction between real and verbal definition, or, as it is sometimes stated, between scientific and nominal definition, is as old as Aristotle, although in the case of verbal definition modern writers do not seem agreed as to how it should be understood. By a real or scientific definition is meant a definition that is based upon and gives a description of the essential nature of the object which the word to be defined denotes. For example, a 'liquid' is defined as that form of matter in which the molecules move with perfect freedom without changing their size. Verbal or nominal definitions, on the other hand, have received at least two different interpretations. The first, or Aristotelian, interpretation bids us regard as verbal definitions the current or popular meanings of words. For example, the definition of a house as 'a building in which people live' would be, on this interpretation, a verbal definition. The second interpretation would make any definition verbal when the word in question is used to indicate what anything is called. For example, the definition of the word 'dog' is verbal when it fails to convey any

information about the essential qualities of the object that the word denotes, and connotes only those qualities which make the use of the word appropriate.

Now the distinction of real and verbal definition, and the variations of the sense in which verbal definitions are understood, are interesting, from a logical point of view, because they raise the question whether in definition we are dealing with thoughts or things, with the nature of objects, or with our interests and purposes in their relation to objects. According to the view of logic which we are expounding, no answer can be given to this question which ignores the intimate connection of thoughts and things. In the pursuit of knowledge, we are not interested exclusively in either thoughts or things; knowledge, as we have shown, exists in the form of thoughts-about-things. If this is borne in mind, it will be evident that in all our efforts to avoid ambiguity we strive to bring our thoughts into relation with things, and things into relation with thoughts; and in definition it is sometimes the one and sometimes the other of these factors of knowledge that requires to be made explicit. In the light of this statement, the distinction of verbal and real definition is not an absolute but a relative one, and were it not for the fact that our words need to be wedded to fixed meanings, and meanings to be fixed and made definite by the use of words, the distinction would have no particular logical interest, and might vanish entirely. The distinction, however,

persists and is likely to persist, because our knowledge is always in different stages of development, and because with the improvement of knowledge old words receive more precise meanings, and new terms are constantly taking their place in the vocabulary of the people. But while this is true, it is quite clear that the goal toward which we move in definition is a truer insight into reality, and the preservation of this insight in terms that adequately express it.

The Relation of Purpose to Definition. We have said that the aim of every definition is to make our knowledge of things more precise, and to fix this knowledge in words. So far as this is true, we are emphasising the relation of definition to the objects of our knowledge. But what objects and what features of objects are of importance for knowledge, depends upon the purpose for which the definition of these objects is sought. A change in purpose is liable to produce a corresponding change in the definition. For example, if my purpose is to ascertain the structural character of a circle, my definition will call attention to the fact that a circle is a plane figure bounded in such a way that all straight lines drawn from a fixed point, called the centre, to this bounding line are of equal length. If, on the other hand, my purpose is to enable one to draw a circle, it will then be defined as the plane figure described by a point moving at a given distance around another fixed point. This is called, in contrast to the structural, a genetic definition. It should be ob-

served that the distinction between structural and genetic definition is not confined to mathematics; the distinction, that is, does not depend on the subject matter, but upon the purpose for which the definition is sought. But every case of definition whether structural or genetic is *per genus et differentiam*, a form of definition which we must take as typical of all definition whatsoever.

Tests of a Definition. In text-books of logic, it is customary to enumerate a number of other rules of definition than the one to which our attention has been exclusively directed. There results from this mode of presenting the subject an impression of complexity and difficulty that is really foreign to it, and the student is deprived of the incentive that comes from a simple and unencumbered statement to make a practical use of that rule. We, therefore, emphasise again the fact that all definition is to be sought *per genus et differentiam*, and that this rule, when properly applied, relieves of ambiguity any statement that is caused by indefiniteness in the meaning of words, and that it is the only one whereby this result can be accomplished.

The only question remaining on our hands is, then, how we are to know whether the rule has been properly applied. It is as an answer to this question that the other so-called rules have their value; and because they provide us with an answer to this question they are not rules of definition at all, but tests of the accuracy of definitions reached in some other way. We shall consider them in this light in

the following paragraphs, and thereby make the one rule of definition itself more definite.

In the first place, let us recall the fact that the object of definition is to limit the extent of a word's meaning, and in limiting it to make it more exact. *Omnis determinatio limitatio est*: we make our knowledge of objects definite by limiting the range of the object's nature of which our knowledge, at any time, takes account. Nowhere more than in the intellectual life does the proverb 'grasp all lose all' have a more pertinent application. Only if we are willing to let some things go, can we gain anything at all. But in order to gain something, we must systematically eliminate what does not have a bearing upon the purpose for which knowledge is being sought. And, as we have shown, definition is a means of systematically eliminating encroaching meanings which are threatening the definiteness of our assertions. It would seem, therefore, that one of the dangers to which our definitions may be exposed is to be found in their particular range: our definitions may be either too broad or too narrow. We shall examine each of these cases separately.

If the *genus* of a definition is not proximate our definition is likely to be too broad. For example, if we define a square as a quadrilateral figure (*genus*) whose angles are right-angles (*differentia*), it is obvious that we are in danger of confusing a square with an oblong which is also a quadrilateral rectangular figure. The definition is too wide. To

rectify the definition we shall have to compare the square with an oblong, and then we shall observe that while both are quadrilateral, this is not their nearest *genus*, that is, it does not contain all the qualities that they have in common. Besides being quadrilateral they are both rectangular. The class, therefore, to which they both belong is quadrilateral rectangle. Now, within this class, the *differentia* of the square will be equilateral, having all its sides equal. Our finished definition of a square will, therefore, be 'a quadrilateral rectangle having all its sides equal.'

If the *differentia* of a definition is not exact our definition is likely to be too narrow. By the *differentia* being exact we mean that it must be such that only the objects denoted by the term we are defining possess the attribute in question. Suppose we were to define man as a civilised animal, the *differentia* 'civilised' would not be exact, and the definition too narrow, because it excluded from the class, man, all individuals who had not reached a certain stage of culture. To test the accuracy of our *differentia* we may ask ourselves the question: Do all and only the objects denoted by the term we are defining possess the quality connoted by the *differentia* by means of which we are seeking to mark off the meaning of the term from that of other closely related terms? If not, the definition will be too narrow.

After we have done the best we can to secure the accuracy of our definition in the ways mentioned,

we may then examine the definition as a whole. We can do this if we remember that a true definition can always be simply converted. If it is true, for example, that 'a house-dog is a domestic animal that barks,' then, if our definition is accurate, it must be true that 'a domestic animal that barks is a house-dog.' That is to say, the object defined (the *definiendum*) and the definition must be commensurate. Or, to state the same fact another way, whatever can be relevantly predicated of the object defined must be predicable of the definition, and vice-versa. This rule guards against anything superfluous in definition.

Further, the terms of the definition must be of the same order as the term defined. If, for example, we defined the body as 'the visible garment of the soul,' it is obvious that the word soul belongs to the order of spiritual facts, and is transferred to the physical order, the order of the term defined. The uselessness of definitions of this kind may be overcome if we take our clue to the order of the terms in which our definition must be stated from the order of the term to be defined. Hence, to satisfy this requirement, we may ask ourselves the question: Are all the terms of the definition homogeneous with the term defined? In this way we avoid the indefiniteness that arises from the use of figurative language.

Another requirement of a good definition is that the terms of the definition should be more elementary than the term defined. The elementariness

of a term is determined with reference to the universe within which the term to be defined belongs. For example, the terms 'rational' and 'animal' are more elementary than the term 'man' of which they are predicated as the definition; and in the same way the terms 'mammal' and 'living in the ocean' are more elementary than the term 'whale.' By elementary, that is, is meant elementary in the field of knowledge to which the term to be defined refers, and not to the range of information that may at any time be current. Hence, for one who has no acquaintance with the field of knowledge with which the term is connected, we may remove the ambiguity that exists in the meaning of the term better by pointing out a specimen of the class or kind of thing that the term denotes than we can by a definition.

Again, a term must not be defined by the use of the term itself or by one of its synonyms. If this occurs we have what is called a circular definition. For example, if we define a power as 'a force that tends to produce motion,' we are obviously guilty of a breach of this requirement since the word force has the same meaning as the word to be defined.

Finally, the form of our definition must be positive if the term to be defined is positive, and negative if the term to be defined is negative. For example, the term 'gentleman' has a positive meaning, and, therefore, it would be a breach of this requirement if we were to define it as 'a man who

has no definite means of support.' But if we were to define an alien as 'a man who is not a citizen of the country with respect to which he is declared an alien,' we should satisfy the requirement, because the term alien carries with it a negative meaning.

The Problem and Classification of the Predicables. Whatever can be predicated of a subject is called a predicable. Now if we ask ourselves the question, What sort of things can be predicables, i. e., can be predicated of a subject? we have definitely placed before ourselves an inquiry that engaged the attention of Aristotle, and again of Porphyry six hundred years after Aristotle's time. The answers that Aristotle and Porphyry gave to this question may be placed in two separate columns thus:

ARISTOTLE'S LIST OF
PREDICABLES

Definition
Genus
Differentia
Proprium
Accident

PORPHYRY'S LIST OF
PREDICABLES

Genus
Species
Differentia
Proprium
Accident

The various kinds of things that can form the predicates of logical propositions are the kinds of things indicated by the names in these two lists.

An examination of these lists will show that they differ in the substitution by Porphyry of 'species' for Aristotle's 'definition,' the other four predicables being the same. But although apparently only a slight difference, this change indicates a dif-

ference in the doctrine of definition of sufficient significance to demand a word or two of explanation. The significance of this difference in the two lists can, perhaps, be seen if we remark that the question, What sorts of thing can be predicated? is not unconnected with the question, What kinds of thing can be defined? We have seen, for example, that in logical propositions we are concerned with both subject and predicate terms, and it has been made clear that our predicate terms must not only be predicable, but predicable of the objects denoted by our subject terms. If, for example, there should be anything indefinable, such a thing would correspond to what is called an individual; and of such individual it is obvious that we cannot affirm a *genus* and a *differentia*. Now, it is the difference between the views of Aristotle and Porphyry as to the kinds of thing that are capable of definition that accounts for the difference in their lists of predicables. Aristotle held that it is only as a universal or species that anything could be defined. Porphyry held that the subject of predication might be not only a species, but also an individual. It was, therefore, reasonable to include species in his list of predicables, for, doubtless, its species can be predicated of an individual; but it would have been a mere tautology to predicate species of itself as would have happened if this predicable had been included in Aristotle's list.

Our exposition of the doctrine of the predicables will follow in the main the Aristotelian view. We

shall assume that the subject terms of logical propositions denote objects, or groups of objects, that are thought of as possessing attributes which express the common nature of all the objects to which the subject term can be applied. Now if it is asked what can be predicated of objects so conceived, the answer is given in the list of Aristotelian predicables. Before, however, we consider in detail what each of these predicables means, we may point out that they are not all related to a subject in exactly the same way. Some of the predicables are commensurate with their subjects, and some are non-commensurate. To the former class belong definition and proprium; to the latter, genus, differentia, and accident.

The Commensurable Predicables. By a commensurate predicable is meant one that is found wherever the subject of which it is predicated is found. It is an attribute, or group of attributes, that is always present when the object is present, and absent when the object is absent. This class of predicables we shall explain briefly in this section.

The *definition* we have considered already. It is, as we have seen, an answer to the question, What is it? and this question may be asked of anything, of an attribute as well as of a substance. For example, if we ask what acceleration is, and the physicist were to answer, change of velocity per second, he is offering this as a definition, that is, a statement of what it is that makes anything the kind of thing called acceleration. Similarly, when an

organism is defined as a material body of which the parts are reciprocally related as ends and means, we should refuse to recognize as an organism any object which did not manifest these particular characteristics. The propositions, therefore, in which the predicate terms claim to be definitions are those in which the predicates also claim to be commensurate with their subject terms. This is only another way of saying, as we said above, that the statements which embody definitions must be capable of simple conversion. The definition states what is essential to the existence of the objects denoted by the term that is defined.

A *proprium* or property is an attribute peculiar to or characteristic of an object, and is commensurate with its existence, but it is not an essential part of its definition. We may state the relation between the *proprium* and definition by saying that by means of the *proprium* we develop the meaning of a term from the point at which its definition stopped. From this it will be obvious that whether we go beyond the meaning that is assigned to a term in its definition will depend upon a motive other than that which makes definition necessary. In other words, the *propria* of terms are those characteristic features of an object's nature which remain after definition has removed any existing ambiguity; and whether these are predicated as part of the term's meaning will depend upon whether special motives operate to make them explicit. For example, an equilateral triangle may be defined as

a three-sided plane figure having all its sides equal. This statement is sufficient for the purpose of distinguishing this kind of triangle from all other kinds. But if we were interested further in equilateral triangles, we should find that they are also equiangular, and this fact of being equiangular is called a property of the kind of triangles in question. Likewise, contractility, irritability, assimilating food, reproduction after its kind, are properties of an organism as this term was defined above.

The Non-Commensurable Predicables. By a non-commensurate predicable is meant any quality which alone does not exhaust the nature of an object, or which belongs to an object in some only of the object's relations. In neither case is the nature of the objects denoted by the term made entirely explicit. The following illustrations will make this clear.

We may, in the first place, predicate of any subject the *genus* to which it belongs. That is to say, we may affirm the area of clear meaning of a term, and this may be all that is necessary to convey our meaning. It is obvious that we are here dealing with the class of cases that were considered above under the head of translation. But, as we saw there, the *genus* alone does not tell us all that is necessary for a complete understanding of any object's nature. The *genus* is, therefore, an incommensurable predicable. We may, in the second place, predicate of a subject its *differentia*, that is, that part of the essential nature of the subject which distinguishes it

from other species of the same *genus*. If, therefore, we are in danger of ambiguity from a tendency to confuse species, we may, to make our meaning clear, resort to the method of calling attention to the specific differences of the objects that we have hitherto failed to distinguish. But the *differentia* alone, as we have seen, does not exhaust the nature of any object, and it is not, therefore, a commensurable predicable. In the third place, we may predicate some attribute not included in the definition of a subject, that is to say, an attribute that may or may not belong to the subject. For example, there is nothing in the nature of a pen that it should be made by a particular firm, or that it should belong to a particular person. These are accidental relations, and belong to the pen merely as an individual. All such attributes, not contained in the definition or *proprium*, are called accidents.

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QUESTIONS

1. From what two standpoints may the logical discussion of terms be carried on, and to what new problems does such a discussion give rise?
2. Define each of the problems enumerated in your answer to the previous question.
3. Define the nature and limits of the demand for non-ambiguous statements.
4. What is the practical importance of being able to make non-ambiguous statements?
5. State and criticise the traditional doctrine of ambiguity.
6. In what sense can ambiguity be said to be the fundamental problem of logic?
7. Explain the nature of translation as a method of overcoming ambiguity.
8. What conditions call for the employment of translation in the removal of ambiguity?
9. What conditions call for the employment of definition in the removal of ambiguity?
10. Explain and illustrate: The method of definition is comparison.
11. What is meant by: Logical definition is *per genus et differentiam*?

12. What do you understand by real and verbal definition? In what two senses may verbal definition be understood?
13. What is the logical significance of the distinction of real and verbal definition?
14. What is the relation of purpose to logical definition?
15. In what two ways may our definitions fall short of being strictly accurate?
16. Explain and illustrate how a definition may be too broad.
17. Explain and illustrate how a definition may be too narrow.
18. How does the simple conversion of a definition guard against introducing anything superfluous into the definition?
19. How may we guard against figurative language in definitions?
20. Explain: The terms of a definition should be more elementary than the term to be defined.
21. What is meant by a circular definition, and how may circular definitions be avoided?
22. What do you understand by a predicable?
23. How many predicables are there, and what is the difference between Aristotle's and Porphyry's lists of the predicables?
24. What is the relation of the problem of the predicables to that of definition?
25. Explain why Aristotle's and Porphyry's lists of the predicables differ?
26. What is meant by a commensurable predicable?
27. Which of the predicables are commensurable?
28. In what sense is a definition a predicable?
29. What is a proprium?
30. What is meant by a non-commensurable predicable?
31. Explain genus, differentia, and accident as non-commensurable predicables.

EXERCISES

1. Write three ambiguous statements whose ambiguity can be removed by translation.
2. Write three ambiguous statements whose ambiguity can be removed by definition.
3. Define, and state, in each case, the purpose that the definition is intended to serve: anger, river, house, emperor, metal, justice.
4. Criticise the following definitions, and show what rules, if any, are violated by them:
 - (1) Thunderbolts are the winged messengers of the gods.
 - (2) Round is a quality that belongs to any rotund object.
 - (3) Man is a vertebrate animal.
 - (4) An inflammation is some abnormal condition of one of the body tissues.
 - (5) An absentee is a person who does not live in his own country, or who does not look after his own property.
 - (6) Prudence is the ballast of the moral vessel.
5. Construct three definitions for the purpose of showing how definition depends upon comparison.
6. To which of the Predicables does each of the following predicates belong:
 - (1) The whale is a mammal.
 - (2) The whale is a vertebrate.
 - (3) The whale lives in the ocean.
 - (4) The whale was caught and brought to land.
 - (5) Logic is a good mental discipline.
 - (6) The tiger is a predatory animal.
 - (7) Englishmen are keen sportsmen.
 - (8) Lord Shaftesbury was a great philanthropist.
7. Give the genus, differentia, proprium, and accidens of
 - (a) peninsula, (b) triangle, (c) affirmative statement,

CHAPTER VII

DIVISION AND CLASSIFICATION

Relation of Definition and Division. Whenever ambiguity threatens any statement on account of the vagueness of the outlying range of a word's meaning, we may remove the ambiguity by defining the word's meaning. The method of definition, as we saw in the last chapter, is comparison. By this method we distinguish a meaning from one or more others with which we are in danger of confusing it, and our result is a division of the *genus* into two or more species. There is, however, nothing in the nature or conditions of definition that, in principle, assures us that all the distinguishable meanings of a *genus* have been discovered, and nothing, therefore, that assures us that we have examined all the species which belong to the *genus* from which our definition starts. If, therefore, instead of asking how one species differs from another, we wished to know how many species there are in a given *genus*, we should be confronted with a new problem which it is the business of logical division to consider; for by division we mean, in general, the method that must be employed to develop systematically all the species which belong to a genus. Now, if it is asked why any one should raise such a question as this, we can only say that the demand for non-ambiguous statements may be left unsatisfied after making the

meaning of our words as clear as possible. Ambiguity, in other words, may be occasioned not only by uncertainty in the connotation of terms, but also by uncertainty in their denotation; and whenever there is vagueness in either respect there is likely to be a corresponding vagueness in the other. From this point of view, logical definition and division are closely connected problems, and they differ merely as one stage differs from another in a single process. In logical division we carry further the process that begins in logical definition, and the object is to remove any remaining uncertainty in a word's meaning.

What is Meant by Logical Division. The word 'division' is not well chosen to designate the process that we have now to describe. If it should suggest, as it ordinarily must, that it is the method by which we split up a genus into its component parts, in much the same way that a house-wrecker separates the doors, windows, joists, etc., of the building he is dismantling, we should be misled entirely in our understanding of what the term means or the process implies. If this were the sense in which we were required to understand the word, its application would not give us a clearer apprehension of a term's meaning, but would result in the destruction of the meaning altogether. It will, perhaps, set the matter in a clearer light if we say that logical division, like logical definition, starts with species or concepts, and that in both cases we are concerned with the kinds in which the species is found realised.

This being the case, we cannot say that a genus is the sum of its species; we can only say that it is differentiated into its species.

In logical division, we are concerned with the relation of a genus and its species. But that relation, as we have just seen, cannot be one of a whole to its parts; it is rather a relation in which the species disjunctively enumerate the alternatives which, when taken as an enumeration, denote the range of a term's meaning. That is, by division we analyse the denotation of a term's meaning. For example, if we should divide the genus 'rectangle' into two species, one with its adjacent sides equal, and the other with its adjacent sides unequal, we cannot mean by rectangle, in such a case, the sum of these two species. 'Rectangle' does not mean the square *plus* the oblong; it means *either* the square *or* the oblong. That is to say, the genus is realised in each of its species; the species of any genus exhibit distributively, not collectively, a word's meaning.

If, now, we should seek to derive from these statements about the nature of logical division a rule which, when applied to any case, will serve as a test of the accuracy with which the division has been carried out, we may perhaps say that we must be able to predicate the genus, or whole that is divided, of each of the species of a logical division. Thus, in the previous example, we called the oblong a rectangle, and also the square a rectangle. Each is the whole of what a rectangle is. To take another

illustration. The genus 'human being' may be logically divided into 'man,' 'woman,' etc., and the genus — human being — of which they are the species, may be predicated of each one, thus: 'a man is a human being,' 'a woman is a human being,' etc.

In this discussion of the nature of logical division we have seen (1) that logical division is the process by which the range of a term's meaning can be exhaustively determined. It follows from this that the process of logical division can be applied only to what is in its nature general, in other words, to a species or concept. The genus, species, or concept to which the process of division is applied is called the *totum divisum*. We have seen also (2) that the sub-species of the whole that is divided enumerate disjunctively the kinds into which the whole is differentiated, that they are not the parts which when taken together constitute the whole. From this it follows (3) that we must be able to predicate of each kind the qualities connoted by the genus or whole that is divided. When this is not possible logical division has not taken place.

Other Kinds of Division. It will help to keep the meaning of logical division distinct, if we describe the characteristics of other operations that go by the same general name of division. Of these, there are in particular three: metaphysical, physical, and verbal division. These are, like logical division, mental operations, but each serves a distinct purpose. The purpose of logical division, as we have

seen, is to enumerate the kinds or species into which a genus is differentiated, or, what is the same thing, the kinds or species in which the qualities of the genus are realised. The purpose of *metaphysical* division is to distinguish the several attributes which are capable of being predicated of a genus. This kind of division obviously lies at the foundation of the problem of the predicables which was considered in the last chapter. For the problem of the predicables, as we saw, was concerned with the character of the attributes that could be predicated of any species. When, therefore, we divide a species into its genus, differentia, proprium and accidens, we are not employing what is technically known as logical division, but a method that is known as metaphysical division. The difference between these two methods may be brought out in the statement that in metaphysical division we must be able to predicate of the genus the species which our division has been the means of distinguishing; whereas in logical division the converse statement holds true, that is, it is the genus which must be predicable of the species. For example, if we divide 'organism' into 'animals,' 'birds,' 'fishes' and 'reptiles,' we have a logical division, — a division which makes it possible to predicate of each species the genus which has been divided. If, on the other hand, 'organism' is divided into 'living creature,' 'capable of locomotion,' 'quadruped,' etc., this is called a metaphysical division, — a division which makes it possible to predicate of the genus

each of the attributes which the division distinguishes. The purpose of *physical* division is to analyse a whole or aggregate into the parts of which it is composed. Thus, we may divide 'horse' into head, legs, ears, tail, etc. This process is sometimes called physical partition. It is distinguished from the other kinds of division by the fact that neither parts nor wholes can be predicated of each other. For example, it is obvious that 'horse' is neither head, nor legs, nor ears, etc.; nor can we say of any one of these that it is horse. *Verbal* division is sometimes mentioned as a separate form of division. It is used to denote the act of distinguishing the homonymous meanings of a word. For example, the word 'hand' may mean either 'a labourer' or 'an organ of the human body.'

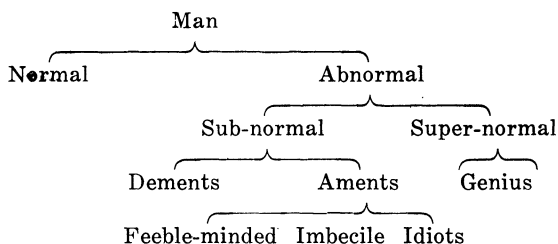
Fundamentum Divisionis. We have seen that the motive to logical division is found in the interest we have in determining the whole denotation of a term's meaning. Logical division is, therefore, the method by which we systematically develop the denotation of terms whose meanings have been fixed by definition. It follows from this that we cannot divide a term whose meaning is unknown to us. This fact is frequently overlooked by the student, and he is apt to make the effort to put the method into operation in a case in which his knowledge of the *totum divisum*, or term to be divided, makes his failure a foregone conclusion. Logical division presupposes that the *totum divisum* has been given a meaning by logical definition. If, consequently, we

are required to determine the denotation of a term with whose connotation we are unacquainted, we must first determine the meaning of that term by the method of logical definition. While, therefore, logical division and logical definition are, as we have seen, closely related processes, each has a distinctive rôle to play in the development of meaning.

With this caution in mind, we pass to a statement of certain features of the method of logical division which throw light upon the significance of the process itself. What, we may ask, is the basis of division? On what principle or principles are we to proceed in any given case? By what are we to be guided in working out a logical division? The answer to such questions as these is usually given in an exposition of what logicians call the *fundamentum divisionis*. By a *fundamentum divisionis* is meant some characteristic of the *totum divisum* which, when systematically applied, develops differences among its members. Or, as it is sometimes said, it is that aspect of the genus in respect of which its species are differentiated. If, now, the purpose of logical division is to develop differences in respect of which species may be differentiated, we shall have to seek in our definition of the *totum divisum* for some indeterminate attribute which can serve as the basis of our division. Now that there should be something indeterminate in a definition will become perfectly clear if we recollect that all that a definition aims to give is a meaning relevant to the situation and purpose which made the defini-

tion necessary. There is no such thing as a complete definition in any other sense than a relevant definition. The only other sense in which we can speak of definitions being complete is that we are acquainted with all the known meanings of a word. But in that case we should not have a single definition but a series of definitions; and, of course, all that we could say about that is that the series is complete, not that the definition is. If, consequently, our definition of a term enables us to make unambiguous statements, that is all that we can expect of it; but it does not follow that the definition will be sufficiently determinate for carrying out a logical division. For example, 'man' has been defined as a rational animal, and, as a definition, it serves its purpose well enough. But if we consider the denotation of the term's meaning thus defined, we shall find that there is something indeterminate in the word rational, for rationality, as we all know, admits of degrees. When, therefore, we assert of any individual that he is a man, that is, a rational animal, we cannot be understood to imply a definite degree of rationality, but only some degree. If, however, we asked what degrees of rationality exist, we should have to adopt some standard of rationality, or state more carefully what we mean by rationality in our definition of man. This standard or statement would be, for the purpose of logical division, our *fundamentum divisionis*. With respect to this *fundamentum divisionis*, some individuals will meet the requirements it lays down, and these

we call normal; others will fail to meet it, and these we call abnormal. In the latter class, we find some who fall below, and some who fall above the standard: the sub-normal and the super-normal respectively. The sub-normal, again, may be divided into dementes, those in whom mental deficiency appears subsequently to mental development, and aments, those who have never attained normal mental development. The aments may be divided, according to the degree in which they depart from the normal, into feeble-minded, imbecile, idiot. This division may be shown in tabular form as follows:



We may point out in this connection that every division reacts upon the definition from which it starts to make it more exact. For example, we found the definition of man as a rational animal defective because it neither stated what is meant by rationality, nor what degrees of rationality were to be recognised. . Both these deficiencies are made good in logical division. Thus, to make our *fundamentum divisionis* clear, we may say that by rationality we mean 'a degree of intellectual capacity suffi-

cient to enable its possessor to perform his duties as a member of society in that position of life to which he is born,' and the classes of those who fulfill these standard requirements or depart from them either by way of excess or defect are the normal, demented, feeble-minded, imbecile, idiot, genius. When, therefore, we look back upon the original definition from the standpoint of our completed division, it has taken on a more definite meaning. We now know not only what is specifically meant by calling any man rational, but we also know what are the different degrees in which rationality may in any case be realised. In other words, our division enables us to say that the concept 'man,' when determined with respect to rationality, may be realised in either one or the other of the forms specified in the division.

Questions of Terminology. We shall postpone further discussion of the problems of logical division until we have explained the technical terms in which the discussion must be carried on. For the sake of completeness, we may recall that the term *totum divisum* was used above to denote the whole that is divided; it is the genus with which any division starts. When viewed in relation to all the species to which logical division gives rise, the *totum divisum* is called the *summum genus*. The species with which a division ends are called *infirmae species*. If, as in the illustration used in the last section, our division is carried through a number of stages, the intermediate species, that

is, those that fall under the *sumum genus*, are spoken of as *subaltern genera*. When, however, we wish to point out the relation that any one of the *subaltern genera* has to the next below it in the series, we designate this subaltern genus the *proximum genus*. It will be observed that the *totum divisum* is a *proximate genus* with respect to the next below it in the series, but a *sumum genus* with respect to all, — whether one or more, — that lie below it in the series. The *constituent* or *co-ordinate species* are those which are differentiated at any single stage in the process of logical division. These distinctions, it should be noted, are purely relative, and the student will have no difficulty in mastering the terminology if he observes that they all hinge upon the meaning assigned to *genus* and *species*. A *genus* is anything considered as a subject of division, and a *species* is always the end-product of a division. The same thing, therefore, may be both a *genus* and a *species*, but whenever that is the case, the same thing is being looked at from different points of view.

Rules of Logical Division. There are other features of logical division than those that have been already considered with which it is important that the student should become acquainted. These we shall state and illustrate in connection with our enumeration and exposition of the rules of logical division.

The first rule is that a logical division must proceed in all its stages upon one *fundamentum divi-*

sionis. The reason for this, of course, is found in the nature of the problem that logical division is set to solve. That problem is to develop the meaning of a term so as to make determinate whatever may have been left indeterminate in its definition. If we did not keep within that aspect of a term's meaning with which our logical division starts, we should fail to develop this meaning from an indeterminate to a more determinate form. In the example used above, we found that a number of degrees of rationality exist, and that these must be included in our definition of 'man' if this concept is to be completely determined. If we had found, in our division of the term, species of another kind, — let us say red man, yellow man, — it would be evident that we had gone outside the *fundamentum divisionis*, rationality, and that we had consequently failed to determine that aspect of the term's meaning with which we were concerned. Whenever this happens, whenever, that is, we fail to carry the division through on a single principle, we are said to have a *cross division*. All the species to which a logical division gives rise must be the same in kind; and this is possible only if we retain the same principle of division throughout. Again, in carrying out a principle of division, we aim to make our species exclusive, to include, that is to say, in any one species only that which is definitely excluded by the other co-ordinate species. The constituent species of a genus must not overlap. This requirement is obviously related to that feature of a logical definition

which emphasises the difference between two or more conflicting meanings (*differentia*), as the former requirement connects logical division with the other aspect of definition which emphasises the common likeness of these conflicting meanings (*genus*).

It is not difficult to apply the first rule of division when the *fundamentum divisionis* is completed in a single step. For example, if 'man' is defined as 'a rational animal possessing a skin-color of some kind,' the latter part of the definition may become for us a principle of division. Dividing man then according to skin-colour we find the denotation of the term to be as follows:

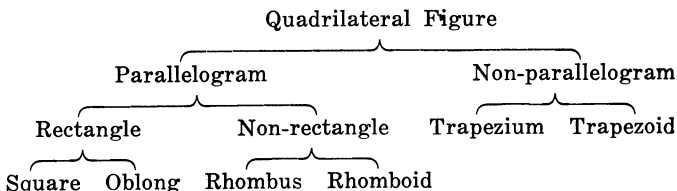
Man				
┌───────────────────────────────────┐				
White man	Black man	Yellow man	Brown man	Red man

When, however, the division is carried through several stages, as it frequently is, it may not be apparent how a single principle can control the whole series of divisions. If, for example, we were to divide 'human being' into 'male' and 'female,' and were then to divide 'female' into 'brunette' and 'blonde,' it is obvious that the *fundamentum divisionis* in the first division is sex difference, and in the second, complexion colour. In cases like this, we cannot look upon the division as a series which develops a single theme; we must regard it rather as a chain of discontinuous divisions which contains as many separate divisions as there are distinct *fundamenta*. The fact that they can be written

consecutively does not have any logical significance; the reason for giving this form to them is economical; it saves time, when the end product of one division is the starting-point of a new one, to continue in the serial order. If, however, our practice were guided by logical requirements, we should re-write our term whenever we intend to apply to it a new principle of division.

Let us now consider the case of a division which has the characteristics of a continuous series, that is, a series which develops a single idea through a number of stages. The significance of this case is that it emphasises the importance and illustrates the method of systematically developed knowledge, of knowledge, that is, which proceeds at each step by the smallest possible degrees of relevant difference. Thus, in carrying out a division which develops its meaning in a continuous series, we are counselled to make the steps of the series as small as possible. This is the purport of the Latin injunction, *Divisio ne fiat per saltum*: you must not make your division go by leaps. A division that is not systematically developed is useless for the purpose that put it in operation. But since, throughout all the stages that we carry our division, we need to be guided by a single principle, it is evident that at each stage of the division we must use a *fundamentum divisionis* that is some modification of the principle with which we began the division. That is to say, each subordinate *fundamentum* must fall within the meaning of the original *fundamentum*. An illustration, borrowed

from Gibson, will make this evident. Let us take as our *fundamentum divisionis*, the side-relations of 'quadrilateral figures,' understanding by quadrilateral figure 'a plane rectilinear quadrilateral figure.' Our purpose is to develop the number of types which belong to this class. According to their side-relations, 'quadrilateral figures' are either 'parallelograms,' that is, figures whose opposite sides are parallel and equal, or 'non-parallelograms.' Dividing these species further according to the inclination of their sides, a special form of the original *fundamentum*, we get, in the case of the parallelograms, rectangle and non-rectangle; and in the case of the non-parallelograms trapezium, a figure in which there are no parallel sides, and trapezoid, a figure in which two of the sides are parallel but not equal. Another specification of the original *fundamentum* would be relative side-length. If we apply this to rectangle we get square and oblong; and to non-rectangle, rhombus and rhomboid, oblique figures which correspond to the square and oblong respectively. All these relations expressed in the form of a genealogical tree are as follows:



The second rule is that a logical division must be exhaustive. The reason for this rule is connected obviously with the purpose that controls such division, namely, to denote in orderly relation whatever belongs to a certain genus. What the rule states is that a place must be found, when the division has been made, for everything that belongs to the genus in one or other of the constituent species. It is only another way of stating the purport of the rule to say that the constituent species together must be coextensive with the whole genus. The rule is intended to guard against too narrow and too broad division. When the division is too narrow something is omitted from, and when the division is too broad something is added to, the genus, and whichever fault we may fall into, the division is rendered useless. Instances of too narrow division are: 'men' into 'good' and 'bad;' and 'objects' into 'useful' and 'ornamental.' Too broad division is likely to occur whenever there is an indistinct apprehension of the connotation of the terms we employ in our divisions. Unless, therefore, a division is exhaustive, that is, neither too narrow nor too broad, we have not really divided at all; for when the division is too narrow, the constituent species are equivalent to only part of the genus, and when it is too broad, they are equivalent to the genus and something else. Whenever a genus has been properly divided, the denotation of genus and species must exactly coincide.

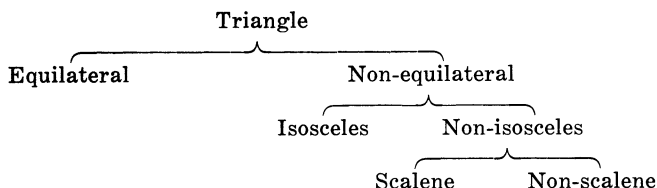
Dichotomy. Our discussion of logical division has emphasised two things: (1) that our ability to

use the method depends upon our knowledge of the subject to which it is applied, and (2) that to insure against omitting anything that really belongs to a genus we must make our division exhaustive, and that when this requires us to carry the division through several stages, the steps between one subdivision and another must be made as small as possible. Division is not, consequently, an easy method to apply, although it is not difficult to understand; but the difficulty is due primarily to the great complexity of the material that nature presents for our study.

Dichotomy is a traditional method of division which has been supposed to relieve us of the difficulties to which we have just referred, and to make it possible, in the absence of any considerable knowledge of the subject under investigation, to determine the species which may belong to it. If this were true, we might omit the consideration of it entirely on the ground that it is a merely formal process; if it is not true, we must consider its value in relation to our effort to make our knowledge of things systematic. That it is not true is the position of modern logicians as it was of Aristotle. We must, therefore, place this method in its relation to our foregoing discussions.

First, let us inquire how dichotomy differs from the form of division which we have just considered. The characteristic of a dichotomous division is, as the name implies, that it proceeds at each step of the division by dividing a genus into two alternative

species. Now, if each step of such a division exhausts the genus we are dividing, it is apparent that one of the terms must be an indefinite or so-called negative term. Thus, if we divide 'triangle' according to the relations of the sides, we shall have at the first division, if we take equality of all the sides as the *fundamentum*, two species, 'equilateral' and 'non-equilateral.' Dividing the 'non-equilateral' term with reference to the equality of two of the sides, we shall have once more two species, 'isosceles' and 'non-isosceles.' If we divide now the 'non-isosceles' term on the basis of the only remaining possibility, namely, the equality of one side with itself, we get also two species, 'scalene' and, if there are any, 'non-scalene.' The arrangement of these divisions in a tree will illustrate the typical form of division by dichotomy. Thus:



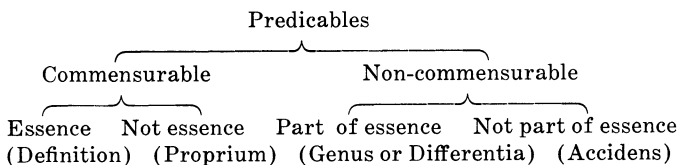
From the foregoing statements and illustration, it will be apparent that we cannot begin to divide a term by dichotomy without having before our minds some definition of the genus that we wish to divide. Unless we knew, for example, that a triangle was 'a plane figure bounded by three straight lines,' we should not be able to divide it to determine

its species. Dichotomy, as much as any form of division, presupposes definition. Now, the connection between dichotomy and definition *per genus et differentiam*, may be expressed by saying that every such division suggests a definition of the class we are dividing, and that every definition implies a division by dichotomy. Thus, to take the illustration already used, a 'triangle' may be defined as 'a three-sided plane figure whose sides have a certain relation to each other;' and if we wish to make the indeterminate part of this definition determinate, we divide by dichotomy in order to bring out the specific relationship of the sides to each other.

We may observe, further, that, applied to relatively simple cases, dichotomy is a rather cumbersome method of doing what can be accomplished more expeditiously at a single step. There is no reason, for example, why we should not divide triangle into its constituent species at once; for, in such a case, it is not by the method of division, but by reason of our acquaintance with the subject-matter, that we know the species that our division is a means of classifying. We may also observe that it is unavoidable in division by dichotomy that the end-product should be a hypothetical term, — 'non-scalene' in the case above — on the existence or non-existence of which the division itself can throw absolutely no light.

What now, it may be asked, is the value of a division by dichotomy? It is valuable in the first place as a test of the accuracy and exhaustiveness

of our analysis of a term's meaning. To take an illustration from Aristotle, we may classify the Predicables as follows:

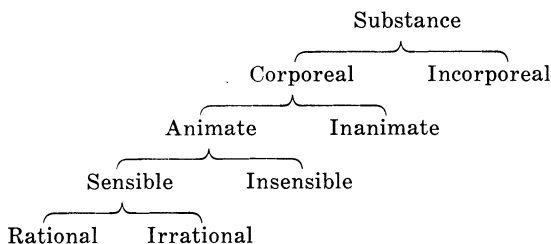


It will be noticed in this case that the *fundamentum divisionis*, or, as we may say, the *differentia*, is the relation of the species according to the degrees of the essential connection that they severally have to the genus, and that the division is throughout dichotomous. But, of course, the Predicables were already known before they could serve as an illustration of how dichotomy may verify a classification.

We may also point out that a division by dichotomy is also valuable if we wish to ascertain the position of a particular species in the class to which it belongs. Thus, to quote Welton, "in the Analytic Key prefixed to Bentham's *British Flora*, which is intended to enable anyone who has a specimen of a certain plant before him to discover its species and its technical name, the arrangement is nearly entirely dichotomous, and, for such a purpose, this form is the most useful. But to adopt dichotomy as a *final* arrangement would be absurd. A botanist, for example, starts at once with three classes of the *sum-mum genus* plant, viz., exogens, endogens, and acro-

gens, and each of these is sub-divided into varying numbers of orders, and these again into still further varying number of genera, and so on, with little or no regard to dichotomy, the object being to make the classification agree with the distinctions existing in the plants themselves."

The Tree of Porphyry. In a typical division by dichotomy, as we saw in the last section, it is the negative or indeterminate term that lends itself to further division. If, now, the division proceeds through all its stages by the dichotomous division of the positive or determinate term, we have a species of division which is illustrated by what is technically known as the 'Tree of Porphyry.' An illustration will show not only the possibility of such a division, but exhibit the form that is characteristic of it. Thus:



In any case of division by dichotomy whether of this type or not, the indeterminate or negative term and the determinate or positive term, in their relation to each other, are sometimes referred to as Contradictory Opposites, Contradictory Relatives, or Contradictories. This designation is apt to be

misleading unless it is pointed out that contradiction can be logically postulated of terms only when these have a common or underlying basis. There is no contradiction, for example, between virtue and beef-steak, because they do not belong to the same universe of discourse, that is, because they have no common basis. Animate and inanimate are contradictories because they are the alternative ways in which corporeal substance is capable of getting itself expressed. The alternative species, therefore, to which a dichotomous division gives rise may be spoken of as contradictory in the sense that they are the definite and exclusive forms in which the genus to which they belong is realised. It would, perhaps, avoid all ambiguity if we spoke of the relation between the determinate and indeterminate terms of a logical division as Complementary. If we did so, we should have to mean that the indeterminate term included within it all of the denotation of the genus that was not definitely included in the determinate term. Only on this interpretation can these terms be regarded as the names of alternate species.

It will be observed that in a division of the type illustrated by the Tree of Porphyry the indeterminate term is rejected at each of the subordinate steps. This fact is technically known as an *abscissio infiniti*.

Classification Defined. One does not need to go to text-books of logic to become acquainted with the process that is denoted by the term classification.

Before he has entered the university, nay before he has entered the common school, the normal child has learned to arrange the objects with which he is brought daily into contact into groups, and to do this on the basis of their observed likenesses and unlikenesses. Whenever anyone groups objects together on the ground of their likenesses, and distinguishes between groups on the ground of their differences, he is employing the method of classification. Classification, therefore, may be defined as the process of thinking together a number of objects on the ground of their common attributes, and of thinking apart groups of objects whose attributes are different.

Classification and Definition. It will be seen that classification, as thus defined, is closely related to definition. Definition completes the process that is begun in classification. Classification underlies the defining process. This relationship will become evident if we recall that definition aims to place the object whose meaning is not clear in a class with other objects which share with it a common set of attributes. When, therefore, we ask how we can find the genus of any term we wish to define, the answer will point us to the method of classification which we are now studying. This is only another way of saying, with a slightly different emphasis, what was said in the last chapter, namely, that the method of definition is based upon the comparison of the object we are defining with other objects that are in danger of being confused with it. Compar-

ison is common both to definition and to classification. But we compare objects with each other in classification for the purpose of ascertaining their common attributes; in definition, we compare the objects of a class for the purpose of discovering the differences which keep the sub-classes distinct. This statement has important practical and philosophical bearings. It enables us to see, for instance, how groups of objects can be considered as a single object, a fact that gives significance, as we have seen, to the general concrete term. Whenever we overlook the differences between the objects of which a class is composed, and emphasise the points of likeness by which each object retains its place in the class, we have constituted of the class a distinct kind of object. In such a class the individuals are no longer important, there is no discernible difference between one and another; they are, as Leibnitz would say, identical. But if, for any reason, the statements we make were dependent for their clearness upon the differences in the class, we should have to go beyond the method of classification by which our classes are constituted, and we should be forced, as we have pointed out, to adopt the method of definition. In classification, we are interested in the likenesses, in definition, in the differences of objects.

Classification and Division. Whenever we are in danger of overlooking the distinctions that exist within a genus, whenever, that is, we are in danger of making ambiguous statements through our fail-

ure to keep in mind the whole denotation of a term, there exists a situation that may be relieved by the employment of the method of division. The starting point of a logical division is the genus, the whole which is possessed of a number of common qualities; and the purpose of division is to mark out within the genus the species which realise, in different ways, these common qualities. The movement of thought is downward from the more general to the less general. If now we consider the characteristics of a classification, we shall find that they are the reverse of this. The situation in which we feel obliged to classify is one in which we are in danger of failing to observe the likenesses of objects on account of the prominence that their individual attributes have come to assume. Popularly expressed, we are exposed to the danger of "missing the wood for the trees." The starting point of all classification, therefore, is the particular, the objects which are kept distinct and apart on account of their differences; and the purpose that classification is intended to serve is to detect within these differences certain relatively permanent groups of qualities. Our most available method for doing this is comparison; but that in respect of which the comparison is made is determined by the interest that our classification is to promote. For instance, we may classify objects with respect to their use or with respect to their sense-attributes. It is the character of our interest in things that determines which of these principles of classification we choose;

but, in any case, the movement of thought in classification is upward from the particular to the general. Classification is the beginning of a process that we shall study more in detail when we come to the problem of induction.

Kinds of Classification. It has been customary to distinguish two kinds of classification, — the artificial and the natural. By an artificial classification is usually meant one in which the grouping of objects takes place on the ground of their possessing some attribute or group of attributes which does not serve as the basis of any further statement about these objects. We may, for example, classify certain objects of domestic furniture as chairs; but having done so, we are not able, on that basis, to predicate anything further of the objects to which the name chair is given. By a natural classification is generally meant one that groups objects on the ground of their possessing an attribute, or group of attributes, which serves as the basis of further predication. For example, if we classify a group of animals as mammals, we are able to say, by virtue of the fact that they are mammals, that they are also vertebrates. This view has received, among modern writers, a classic expression by Mill. It is based upon the supposition that there exist in nature classes of objects which are distinct from each other, and that if we make the differences of these classes the basis of our classification, we shall thereby mark off the 'natural kinds' from others which by way of contrast may be called artificial kinds.

Since the spread of the doctrine of evolution, it is no longer possible to maintain, in all fields of knowledge, the distinction of natural and artificial kinds. We no longer think of species of animals, for example, as marking fixed and distinct differences between groups of objects; we have to think, according to the doctrine of evolution, of the whole animal series, from amoeba to man, as having sprung from a common ancestor. Species are not, in the meaning of the distinction we are now considering, separate and distinct kinds.

But while we no longer find it possible to classify objects in the way just considered, the distinction between natural and artificial suggests a truth of which we may take advantage in our discussion of this topic. The fallacy that underlies the separation of classes into natural and artificial seems to be, from the logical standpoint, that it denies that relevancy to purpose is a factor in all classification. The classification of objects, for example, as natural is based on the supposition that the attributes of species are determined wholly by the nature of the objects themselves, and that they are in no way and to no extent determined by the interest which prompts us to classify; while the classification of objects as artificial is predominantly, if not wholly, determined by the interest that controls the classification. This can not be maintained. For, as we have seen, knowledge is always the product of both kinds of factors,—of both subjective purpose and objective fact. And it is this analysis of what we mean by knowledge

that gives us the clue to the distinction of natural and artificial kinds, and which enables us to understand the two main kinds of classification. Our classification does not depend upon the presence or absence of interest, but upon the particular kind of interest which we seek to satisfy. We may be interested, for example, in grouping objects according to their essential attributes, that is, according to the attributes which constitute them members of a class. We may also be interested in grouping objects according to their non-essential attributes, that is, according to the attributes which are either implied by their essential attributes, or which are only accidental ones. In view of our discussion of the Predicables, this statement means that we may make the qualities which characterise the genus the principle of classification; or the qualities which characterise the proprium, or the accidens, the basis of classification.

Classification according to the essential attributes of the objects compared gives a theoretical classification; and classification according to the non-essential attributes, a practical classification. In the former case, the theoretical interests which prompt us to study the nature of things are uppermost; and in the latter, the practical interests which are concerned with the uses of things are in control. To take an illustration usually found in text-books of logic, the Swedish botanist, Karl Linnaeus (1707-1778), based his classification of plants upon the comparison of the sexual organs of plants. The

distinct advantage of this method was that it made possible the indexing of plants in a convenient way into genera and species so that their relative position to each other in a series could be made out. This must be taken not as a theoretical, but as a practical classification, and, in principle, is not different from the arrangement of words in a dictionary in their alphabetical order. If, on the other hand, we make a classification according to the natural relationships of the species, we shall have an illustration of a theoretical classification. It will be obvious that many variations are possible within these two divisions; but for the details, the student is referred to the larger books on logic and the principles of science.

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QUESTIONS

1. In what way is logical division related to logical definition?
2. Name one way in which logical division may not be interpreted.
3. Explain: By division we analyse the denotation of a term's meaning.
4. What is meant by saying that the species enumerated in a logical division are to be read disjunctively?
5. What is the test of a logical division?
6. What is a metaphysical division, and how does it differ from a logical division?
7. What do you understand by a physical division? Illustrate.
8. What is a verbal division?
9. Define a *fundamentum divisionis*.
10. Explain how one discovers for a particular division an appropriate *fundamentum divisionis*.
11. In what respects can a logical definition be improved from the standpoint of logical division?
12. Explain the following terms: genus, species, totum divisum, summum genus, infirmæ species, subaltern genera, proximate genus, constituent species.
13. What is the first rule of logical division?
14. What is a cross division? Explain how it occurs.
15. What is meant by saying that the species to which a logical division gives rise must be exclusive?
16. What are the character and importance of a continuous series?
17. How can we keep within a single fundamentum when our division is carried through a number of stages?
18. State and explain the second rule of logical division.
19. What difficulties in the method of logical division are supposed to be met by dichotomy?
20. What are the characteristics of a dichotomous division?
21. What is the relation of dichotomy to definition?

22. What advantages are sometimes claimed for a dichotomous division?
23. What is the Tree of Porphyry?
24. What do you understand by contradictory opposites?
25. Explain *abscissio infiniti*.
26. What is classification?
27. How are classification and definition related?
26. What is the relation of classification and division?
29. State Mill's doctrine of classification.
30. Why has the distinction of natural and artificial kinds broken down?
31. What is the relation of purpose to classification, and what effect does this have upon the distinction of natural and artificial classification?
32. What bases of classification may be recognized as implicit in Mill's doctrine of classification?
33. Explain theoretical and practical classification.

EXERCISES

1. Divide logically the following terms, stating in each instance (a) the definition and (b) the fundamentum divisionis.

psychology
European

gun
student

culture
government

2. Examine the following divisions, and point out which rule of logical division, if any, is broken by them:
 - (1) Substances into material and spiritual.
 - (2) Books into scientific, philosophical, and literary.
 - (3) Religions into true and false.
 - (4) Students into those who are diligent, female, and athletic.
3. What is the metaphysical division of the following terms?

landscape
capital

philosophy
child

toy
amusement

4. Divide physically six terms of your own selection.
5. Give six illustrations of verbal division.
6. Illustrate a cross division, and explain how it is brought about.
7. Give two examples of dichotomy.
8. Illustrate by two examples the Tree of Porphyry.
9. Give examples which illustrate the characteristics of
(a) a theoretical and (b) a practical classification.

CHAPTER VIII

CATEGORICAL PROPOSITIONS

Types of Propositions. In a former chapter we saw that predication is the act of affirming or denying something (P) of something else (S). Any statement which rests upon predication, that is, which affirms or denies something of something else, is called in logic a proposition. Predication may be of various degrees of directness or explicitness, according to the degree of our knowledge of the subject of predication. For example, if my knowledge of natural history is only slight, I shall probably be able to say of a given animal merely that it is either a lion or a tiger or a leopard or something else. If a further study of animals makes me acquainted with the characteristics of the species lion, I shall then be able to say that if a given animal has these characteristics, it is a lion. If, again, the examination of a particular animal showed that it does or does not have these distinguishing qualities, I should then be able to say that it is or is not a lion. The forms of predication illustrated in these instances we have learned already to call alternative, hypothetical, and categorically respectively. The statements in which these forms of predication are expressed are called disjunctive, hypothetical and categorical propositions. The classification of propositions into disjunctive, hypothetical, and categorical

is said to be made from the standpoint of relation, of the relations, that is, that obtain between the elements that enter into the proposition. It is quite obvious, for instance, that the predicate and subject of each of the above propositions dealing with what, under given conditions, can be predicated of a particular animal, are related in quite distinct ways. The predicate term is asserted differently in each case, and it is this difference in our manner of asserting that lies at the foundation of the distinction we make between disjunctive, hypothetical, and categorical propositions. The form of assertion corresponds to a real element in the knowledge that an assertion claims. We have shown this in detail in the chapter on propositions. Our present task is to study the various kinds of propositions and the logical problems to which they give rise. In the present chapter, we shall consider a number of questions which relate to simple predication and which throw light upon the nature of categorical statements; in the next, we shall consider problems connected with the other forms of the proposition.

Classification of Categorical Propositions. The possibility of a classification of categorical propositions, that is, propositions which predicate P of S simply, rests upon the fact that unconditional statements (1) do not always, but sometimes do, apply to a determinate part of the subject-matter of such statements; or (2) do not always, but sometimes do, unite the predicate with the subject of such statements; or (3) do not always express the same de-

gree of certainty. We shall illustrate each of these cases in the order in which they have been named. As an illustration of the first case, where the statement applies to an indeterminate part of the subject-matter of the statement, let us take 'Not all men are honest who say they are.' This is a proposition in which honesty is predicated of some only of the men who declare their honesty. As an illustration of the second case, where the statement does not unite the predicate with the subject of the proposition, let us take the illustration just used and observe that honesty is denied of some of the individuals who assure us that they are honest. As an illustration of the third case, where our statements express different degrees of certainty, let us take the following: 'He may be honest though poor;' 'He is honest but poor;' 'He must be honest.'

The three characteristics of propositions just mentioned and illustrated are so constant, and they occur in such a wide range of our knowledge, that early in the history of logic it was found necessary to distinguish them by the use of technical terms, that is, by terms which mean, whenever they are used, just these characteristics of propositions. Thus, when we wish to call attention to the fact that our statements do or do not apply to a determinate part of the subject-matter of the proposition, we are said to be dealing with the proposition from the standpoint of its Quantity. When, on the other hand, we wish to emphasise the fact that the predicate does or does not belong to the subject, we are

said to be dealing with the proposition from the standpoint of its Quality. When, thirdly, we wish to distinguish the degree of certainty that attaches to our statements, we are said to be dealing with the proposition from the standpoint of its Modality.

If to this statement we now add what was said in the first paragraph of the present chapter about the degrees of explicitness in predication being made from the standpoint of Relation, we have before us the four ways in which propositions may be classified. We shall now write down, for reference, the classification of propositions when looked at from each of the above-mentioned points of view.

	RELATION	
	Categorical	
	Hypothetical	
	Disjunctive	
QUANTITY		QUALITY
Singular		Affirmative
Universal		Negative
Particular		
Indesignate		
	MODALITY	
	Assertoric	
	Necessary	
	Problematical	

Quantity of Propositions. The problem of the quantity of propositions is one which is concerned with the question whether we intend our statements to be true of a determinate or indeterminate part of the subject-matter of such statements. It is, for

example, quite clear that when any one says, 'I have seen Paris,' he is ordinarily understood to mean that he has been to Paris, and visited some of its important boulevards, churches, galleries, etc. No one would understand him to mean that he had become acquainted with the whole of Paris. The truth of the claim to have seen Paris would be that some unspecified parts of Paris had been visited. Again, no one who made the above statement would ordinarily wish to be understood that he had a thorough acquaintance with those parts of Paris which had come under his observation. The statement is intended to claim an acquaintance, such as it is, with certain parts of the French capital. From these considerations, it is quite evident that the statement, 'I have seen Paris,' leaves undetermined both the range and the thoroughness of the observation that is claimed by the statement. Now, although a great number of the statements we make about even the most familiar and ordinary events of life are, in the sense of these illustrations, quite undetermined, we sometimes make statements which we intend to be understood with no such limitation. To take a simple illustration, if one were asked, 'What is the most direct route from Chicago to London?' the answer, if one were given, would claim to be determinately true of the subject of the inquiry, either of a specified part of the inquiry or of the whole of it. For instance, the reply might take this form, 'Go by way of the New York Central lines to New York, and then inquire;' or it might take this form, 'Go to

New York, and then take passage with the Cunard Line to London *via* Liverpool.' Let us take another illustration. A mathematical problem is an inquiry of a definite kind, and we may work the problem up to a certain point quite sure that all the steps we have taken lead us toward the true solution of the whole problem. We may or we may not be able to proceed beyond that point. The fact that we cannot, does not invalidate the claim that we may make of having solved a definite part of the problem. That is to say, if the answers we make to questions, and the statements in which we express our knowledge contain in them a distinct reference to the extent of the subject covered by such answers and statements, they are called determinate whether the whole or some definite part of the whole subject has been considered.

We have seen that statements may be determinate or indeterminate in what they assert. Our thought, that is to say, may refer to a definitely specified area of a given subject, or it may refer to an indefinitely specified area. In the former case, the definitely specified area may be either the whole, or an indicated part of the whole. In the latter case, we cannot tell whether the knowledge expressed in the statement is true of the whole or of only a part of the whole. Now these distinctions in the ordinary use of language lie at the foundation of the logical consideration of propositions from the standpoint of quantity. Hence, the question of the quantity of propositions is whether what a statement connotes

is true of a specified or unspecified part of what its subject denotes. When, consequently, we speak about the quantity of a proposition, we must be understood to be interested in the meaning of a statement from the standpoint of the number of the objects to which the statement applies. It is obvious that the subject terms of all propositions which express quantity must be read in denotation.

If now we ask, To how many objects can we intend our statements to apply? we may answer to one, to more than one but less than all, and to all. These are all the possible cases, and they afford a clue to the classification of propositions from the standpoint of quantity.

The Singular Proposition. When a statement is made of one specified individual, that statement, in logic, is said to be a *singular proposition*. 'Hannibal was a great Carthaginian general' is, in this sense, a singular proposition. It is a proposition that expresses what an examination of a single instance of the genus 'general' has led us to affirm. This is not an exceptional case, for we are frequently led, in the course of our experience, to examine single objects and to attempt some classification of them. The child's questions, 'What is this?' 'Who made that?' and the like are illustrations of the importance of this class of judgments in the progress of intelligence and the advancement of knowledge. Singular statements are concrete, or keep us in close connection with what is concrete, and for this reason are of great help in making exact, particular, and

precise the knowledge that, in one way and another, we are daily acquiring. The examination of individual objects is, also, a large part of the work of any science, and much of the knowledge that science has consists of the singular judgments it is able to assert. We may say in passing that the student's danger is that he will get away from the observation of particular things, and that his education will force him to premature generalisation in which a form of words is substituted for a face to face acquaintance with the facts of his special subjects of study. If such should be the case, it is hardly possible that he will appreciate the importance of this type of proposition.

The Universal Proposition. Another class of propositions is met with when the subject terms of such propositions are general instead of individual. 'All democratic governments tend to issue in despotism' is a statement expressing a judgment that purports to be based upon the common character of the individuals that the general term signifies. Greek, French, British, American, etc., are the governments of which such a statement is held to be true, and yet not as Greek, French, etc., but as democratic. Statements of this kind are made because there is conceived to be a necessary connection between what the predicate and subject terms connote, and they apply to all individual or particular instances which possess the attributes of the subject term. In other words, whenever we affirm or deny of each of a total number of instances the meaning that attaches to

the predicate terms of propositions, we have what, in logic, is called a *universal proposition*.

Kinds of Universal Proposition. A question may arise as to whether there is any logical difference between a singular and a universal proposition, since in both cases, the subject term must be understood to be distributed, that is, to apply to the whole number of the instances that the connotation of the term implies. There can be no objection to taking these two cases as illustrating the same logical principle, namely, that from the standpoint of their quantity we intend our statements to apply to all the instances, whether to a specified one or to all, to which, in the nature of the proposition, the subject term can apply. The singular proposition may thus be taken as a particular form of the universal proposition. Another type of proposition which, like the singular proposition, has a concrete subject term, and which, on account of the definiteness of its denotation, has to be interpreted as a universal proposition may be considered in this connection. For example, in the proposition, 'A part is always less than the whole,' a part is a perfectly determined idea of quantity from the logical point of view, and the proposition in which it serves as a subject term must, therefore, be treated as a universal proposition.

The Particular Proposition. Whenever the number of instances denoted by the subject term of a proposition is, from the logical standpoint, undetermined, we are said to have a *particular proposition*. The question for us to bear in mind in decid-

ing the quantity of a proposition is simply whether the number of instances of the kind connoted by the subject term is a determined number or not. For example, if we say that 'Five per cent of all college students fail in their work,' it is clearly not meant that we know the exact number of college students, and therefore that we know the exact number who fail; the statement, if it is true, must be based upon the examination of large numbers of students working under a great variety of conditions, and, when so based, we feel warranted in saying, even with respect to those students who have not had an opportunity to pass or to fail in any subject, that in the period covered by their college course five per cent of these will fail. Now what we wish to illustrate by this example is that the statement is a universal one because, although arithmetically we have no idea of the number of failing students, we do have in the subject term of our proposition, 'five per cent of all college students,' an idea logically determined from the standpoint of its quantity. 'Five per cent' is just as definite as 'this one' of a given number, and both are as definite as 'all' of the given number. But if, on the contrary, in the absence of all knowledge of any such statistical study as we have supposed, we were required to express our judgment as to the number of failures likely to be incurred at any given time, we could not logically go beyond the statement that 'Some students will fail in their work.' This statement is true of an undetermined number of students, and 'some,' as a sign of quantity

in propositions, means this. The propositions that express judgments that have not determined the quantity of their subject-matter are called particular propositions.

The Indesignate Proposition. Whenever we express judgments of quantity, our statements, as a rule, bear upon their face some definite indication of the extent of their subject terms. From the standpoint of quantity, all our statements are offered as true of a number of determined or undetermined individuals, and when a statement is made without specifying which of these two possibilities is meant, we are left in doubt as to how the statement is to be taken. Now sometimes this occurs. The popular wisdom of an age which gets itself hardened into proverbs is frequently defective in this respect. 'Dead men tell no tales,' is an illustration in point. Are we to understand this of all dead men or only of some dead men? Is there, that is, a necessary connection between being dead and being unable to appear as a witness in a trial, for example; or are we to suppose that there is only an accidental connection between them, and that in some cases the testimony of dead men is admissible? 'Virtue is to be rewarded,' is a proposition that is defective in the same respect. Are we to understand this of each and all the virtues, of the ordinary virtues of our daily life as well as of the more heroic virtues that are called out in exceptional circumstances; or are we to limit the statement to the latter class of cases? If the latter, then the same set of questions recurs,

and we should wish to know whether a reward must be attached to all of them or to only some of them. These illustrations are sufficient to indicate a kind of statement we sometimes meet with even in common life, statements that are quantitative by implication but are not explicitly so. How are we to deal with them logically? Our first inclination is to receive all such statements, according to the character of their subject-matter, as universal or particular propositions. But if it is remembered that all propositions are expressions of the judgments of the person who states the proposition, and that they are, from the standpoint of the person who hears them, proposals for his acceptance or rejection in the sense in which they are made, it will be obvious that we are not allowed to interpret these statements in any sense we wish; our business is to understand them in the sense they have for the person who makes them. But, as we saw, these propositions create a doubt that is based upon the possibility of understanding the statements in either of two ways. Whenever a statement raises a doubt of this kind, a doubt that may be resolved by accepting either the one or the other of alternative possibilities, without indicating which alternative is to be adopted, it is called an *indesignate proposition*. An *indesignate*, or as it is sometimes called, an *indefinite* proposition is one which implies the existence of a determined or undetermined number of individuals of which the statement is offered as true, but which fails to state explicitly whether it is made of the

number of the determined or of the undetermined instances.

Quality of Propositions. Our study in a previous chapter has shown that terms always intend more than they express, that they are aspects of a whole meaning which as terms they do not fully bring out. We have also learned that in order to get at the full meaning of any term we must find it functioning in some way in a proposition. It is only in a proposition that we discover the full meaning of a term; the proposition is the express statement of the whole meaning within which the meanings of terms may properly be sought. In other words, it is the way that terms function in propositions that gives the key to the meaning of the proposition and of the terms themselves. That is to say, we have to interpret terms from the standpoint of their propositions and proposition from the standpoint of their terms. The reason is that each implies the other, and neither can exist alone.

Coming now to the particular problem of this section, we may see, in the light of what has just been said, that terms may be related to one another in propositions in one or the other of two ways, in the way of inclusion or of exclusion. The fact that terms are elements of the meaning expressed by the proposition, is indicated, as we have already seen, by the copula. The copula, we said, is the sign of predication, the sign that S and P are to be taken in the way required by the whole meaning expressed in the proposition. One of the ways in which prop-

ositions require us to take their terms is, as we said, with respect to whether they are inclusive or exclusive of each other. And this requirement is indicated by the copula, which according as it has or has not the negative particle 'not,' distinguishes for us the two kinds of Quality of propositional statements. When the terms are inclusive, this is indicated by the copula 'is' or 'are;' when they are exclusive, by the copula 'is not' or 'are not.' The propositions that have respectively these two kinds of copula are called affirmative and negative propositions. Affirmative and negative propositions, therefore, are statements that give expression to the inclusive and exclusive relationship of the terms through which our meanings are expressed.

The Affirmative Proposition. Affirmative propositions, formally considered, are those in which the copula is either 'is' or 'are.' From the standpoint of the character of the thought expressed in them, affirmative propositions may be defined as the expression of our positive meanings. By a positive meaning we understand one which combines in a whole content of thought two or more features of the object thought about. Stated differently, a positive meaning is one that may be analysed into elements each of which expresses some feature of the meaning which the proposition as a whole expresses. For example, the meaning of the proposition, 'All mammals are vertebrates' is, that there is a genus which, when looked at from the standpoint of anatomy, must be described as a vertebrate, but which

if looked at from the standpoint of physiology must be described as a mammal. Let us call the genus, or the whole meaning of any proposition, SP. Then our positive meaning, SP, may be analysed into S and P, and these terms may be used to express the intended meaning in the proposition 'S is P.' To develop the significance of affirmative predication further, we may say that in any statement of the form 'S is P,' S and P are not isolated or independent terms; S is not merely S, nor is P merely P. What the proposition affirms is that the S that is P is an SP, and only because it is SP can we say that it is P. But P can be predicated of S only if it also is SP. SP underlies both terms, and gives to the meaning that the proposition 'S is P' expresses its positive character.

The Negative Proposition. Negative propositions, formally considered, are those in which the copula is either 'is not' or 'are not.' Materially, that is, from the standpoint of the character of the thought expressed in them, negative propositions may be defined as the expression of our exclusive meanings. By a negative meaning, therefore, we understand one which excludes from a whole content of thought one or more attributes of the object thought about, either because it does not at all, or because it does not now, form part of the connotation of the whole content of thought. Stated otherwise, a negative meaning is one which may be analysed into elements one or more of which is incompatible with some fact or truth which we have already accepted.

In the chapter on terms we discussed negative meaning, and the conditions under which such meanings arise were explained. These conditions can be reviewed profitably in the present connection, and they will be found to throw light upon the general statement in the present paragraph of what we understand a negative meaning to be.

We shall pass on to other aspects of the same problem. We may call attention then to the fact that the predicate terms of negative propositions are to be read in connotation. The reason for this is that logical negation expresses incompatibility in the meanings of terms. For example, 'Apples are not pears,' does not express merely the perceptual differences of the things we call apples and the things we call pears; what the proposition means is that if we know what attributes pears have, they will be found not to be the attributes which apples have. Now, as we see by this illustration, the difficulty about negative statements is that they seem to break the connection that, as we said, must bind S and P together in logical propositions. For example, we have shown that, in order to make any statement of the kind S-P, we have to suppose that P is predicated of S on the ground of the common likeness that underlies the differences of S and P. It is, in other words, because S and P make a reference to the same subject, although to different aspects of it, that predication can take place at all. Can we say that in negative propositions there is the same identity in difference that is the ground of every logical state-

ment? It would seem that we can; only the differences, in this case, have come in our thought to be a special motive for the judgment we assert. In other words, in affirmative propositions we express likenesses amid differences, while in negative propositions we express differences amid likenesses. The negative proposition has this characteristic because, by making difference the motive of our assertions, we can sometimes best conserve the unity of idea that every propositional statement requires.

The Relation of Affirmative and Negative Propositions. A study of the relation of affirmative and negative statements will throw still further light upon the meaning of negation. In affirmative statements, thought moves between two extremes, the extreme of tautology on the one hand, and the extreme of nonsense on the other. If we take affirmation in one extreme way, we can only express identical meanings, 'A boy is a boy,' and this is tautology. If we take affirmation in another extreme way, we can only express incompatible meanings, 'A stove is a gold mine,' and this is nonsense. Now it is the function of negation to save us from meaningless — tautological or nonsensical — assertions. For instance, the case of tautology, when taken absolutely, is a denial of the minimum of difference which must exist if we are to make our assertions significant. Merely to predicate an object's existence of itself, or to affirm a meaning of itself, is nothing but trifling. If we should mean by the

predicate only what we mean by the subject of a proposition, we should state in the whole proposition no more than we stated in either of its terms. Thus, 'A is A' has exactly the meaning of A, whether A is read denotatively in both terms, or connotatively in both terms. If A is A is a significant statement, we must at least read A denotatively in one term, and connotatively in the other. In other words, the identity which unites the subject and predicate of logical propositions must not go to the extreme of eliminating all differences from the meaning which logical propositions state. The affirmative proposition, because its chief interest is in identity, is particularly open to the danger of overlooking the differences that make the subject and predicate of such propositions distinct terms. Now, when this happens, when identity usurps the whole functions of assertion, we can be saved from the fatuity of tautology by resorting to negation as the only means at our disposal for opening the way to some significant statement. Thus, if we oppose any tautology with a denial, the burden of the proof of what we assert is thrust upon us, and this consists in showing the grounds on which our negation rests. For example, if we deny that 'A boy is a boy,' we may do so because, as we may point out, he is also a son, and that this relationship is compatible with the fact of boyhood from which, in the tautological statement, we were unable to get away. Negation serves, in such a case, to prevent us from

standing intellectually stock-still; and, as we often find, the denial of truism may sometimes be the open way to the truth.

The other extreme to which affirmative judgments may lead us consists in the development of differences to such an extent that the unity of the judgment is threatened or completely destroyed. This may occur, for example, when the material of the judgment is forced to render so many predicates that the universe of discourse, which in all logical judgment sets the limit to which differences can be carried, is in danger of being superseded, and the qualities which the material may have in other universes of discourse are brought in to satisfy the demand for differences which has, so to say, got out of hand. When, for example, the student of psychology speaks of auditory sensations being brought in over the auditory nerve, and when the text-books of physiology speak about the striated muscles being under the control of the will, there is a confusion of universes of discourse, and we can only force the statements within their respective bounds again by a refusal to accept them as significant assertions. The statements are incompatible with the body of knowledge of their respective sciences, and the negation is in each case in the interests of this body of accepted knowledge.

Exceptive and Exclusive Propositions. Whenever a subject of discourse is qualified by such words as *except*, *only*, *but*, *unless*, *alone*, *none but*, etc., it immediately becomes uncertain to what, in such

cases, the predicate applies. For example, if we say, 'All free persons know how to obey,' it is quite obvious that we are predicating 'the knowledge of how to obey' of 'the persons who are free.' If, however, we were to say 'None but the free know how to obey,' we should be saying something quite different, although most persons would understand that we were still predicating 'the knowledge of how to obey' of 'the persons who are free.' If, again, we said, 'Only the free know how to obey,' it might be asked whether we had said something identical with one of the other statements, or something different from both. And, further, if we said, 'No one unless he is free knows how to obey,' the problem would become still more complicated.

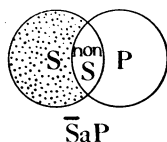
In view of the difficulties of interpretation occasioned by such statements as those to which attention has just been called, it is customary for logic to point out that all propositions whose subject terms are qualified by such words as *except*, *only*, *but*, are either exceptive or exclusive propositions. In the exceptive and the exclusive proposition the subject term is an infinite or non-S term, because all the qualifying words, *except*, *but*, *only*, etc., are understood as carrying a negative signification. Examples are: 'All except honor' = 'All that is not honour;' 'None but the free' = 'No one who is not free;' 'Only natives' = 'No one who is not a native.' It should be observed, moreover, that exceptive and exclusive propositions are always universal, there are no particular propositions of either kind. That

is to say, the subject terms of these propositions denote always a determinate number of individuals. But while the exceptive and exclusive propositions distribute their non-S terms, they differ from each other in quality. That is to say, all propositions which qualify their subjects in the manner indicated are either affirmative or negative. In the light of these statements we may consider these classes of propositions separately.

The exceptive proposition is affirmative. It has the general form, $\bar{S}aP$. Examples are: 'All planets except Venus and Mercury are beyond the earth's orbit' = 'Any planet that is not Venus or Mercury is beyond the earth's orbit;' 'All members but the Irish voted for the measure' = 'All the members who do not belong to the Irish party voted for the measure.'

In an exceptive proposition the individuals denoted by the subject term are determined by taking out of a class a certain number of individuals to which the predicate term is meant not to apply. It will be seen that if the part of the class to which the predicate term does apply is to be distributed, the part taken out must be distributed also. What an exceptive proposition says, therefore, is that a statement is true of that part of a class which remains when stated exceptions to it are definitely removed. The truth of such a statement is maintained of everything that cannot be brought under the exception, that is, of everything that is not the specified exception. Consequently, what we are talking

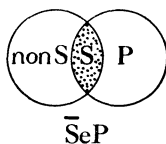
about, in an exceptive proposition, is anything, within the universe of discourse, that is definitely non-S. Thus, if a circle represents a universe of discourse, and any shaded portion of it the excepted part, which may be called S, the unshaded part will then represent the denotation of the subject of predication, that is, non-S. The exceptive proposition may in these terms be represented thus:



The exclusive proposition is negative. It has the general form, $\bar{S}eP$. Examples are: 'None but the brave deserve the fair' = 'No one who is not brave deserves the fair'; 'Only graduates are eligible' = 'No one who is not a graduate is eligible.'

The exclusive proposition determines its subject term in the same way as the exceptive proposition, that is, by taking out of a class some determinate number of individuals, and making all other individuals of the class the subject of predication. Thus, if S be the excluded individuals, non-S will be the subject of which predication takes place. But since exclusive propositions are negative, the individuals denoted by non-S will necessarily have qualities which are incompatible with their possessing those connoted by the predicate term; incompatibility of qualities is what we mean by negation. Hence, the

exclusive proposition definitely denies P of every individual which does not have the qualities also of S; nothing that is not S is P. If, now, we represent any excluded part of a class by the shaded portion of a circle, the exclusive proposition may be shown as follows:



Compound Propositions. We have said that exceptive and exclusive propositions are always universal. This is not because particular propositions do not qualify their subject terms by such words as only, except, etc., but because when they do, they must be treated as compound propositions. By a compound proposition, or, as the older logicians called them, exponible propositions, is understood one whose meaning requires a number of distinct propositions for its complete expression. 'Milton and Dante wrote religious poetry of the highest merit,' is a simple example. The meaning of the statement obviously is, (a) 'Milton wrote religious poetry of the highest merit,' and (b) 'Dante wrote religious poetry of the highest merit.' A more complex example is, 'Men who are honest though poor will be respected and helped.' The whole sense of this can be expressed in the two statements (a) 'Poor men who are honest will be respected,' and (b) 'Poor men who are honest will be helped.' From

these examples we can see that a separate expression must be given to each significant idea of any complex of ideas.

Bearing in mind what a compound proposition is, let us turn to the particular propositions which qualify their subject terms by the use of exclusive words or phrases. These propositions, we have said, are compound. An example is, 'Some only who enlist will find their way to the firing line.' This obviously cannot mean, 'Some who do not enlist will find their way to the firing line;' that is, it cannot be treated as a particular exclusive proposition. What the statement means is (a) 'Some who enlist will find their way to the firing line,' and (b) 'Some who enlist will not find their way to the firing line.' The following must also be treated as a compound proposition, 'Excepting a few, all these answers are wrong.' What the proposition states is (a) 'Some of these answers are wrong,' and (b) 'Some of these answers are not wrong.' From these examples it seems obvious that whenever we qualify, in the way indicated, the subject terms of particular propositions, that is, whenever 'some' is used in its exclusive sense, the proposition must be resolved into two independent particular propositions, the one affirmative and the other negative. In the light of these cases, we may also understand why the particular proposition which has no such exclusive significance must be interpreted as either affirmative or negative, and not as both. For instance, 'Not all foreigners are to be mistrusted' means 'Some foreigners

are not to be mistrusted;' and if any one were to add to this interpretation, 'Some foreigners are to be mistrusted,' he would be going beyond what is said to a secondary implication of what is said. The latter statement is doubtless implied in the original statement, but it is not what that statement asserts.

Modality of Propositions. The knowledge we have of different subjects, as we are all aware, varies not only with respect to the nature of these subjects themselves, so that one kind of knowledge we call history, and another science, and another philosophy, but also with respect to the degrees of our attainments in any of these departments of knowledge, so that we speak about these several subjects with differing degrees of confidence. If, for example, the student has diligently pursued the study of bacteriology for a number of years he will not only know a great deal about the nature and conditions of the changes that microscopic organisms produce in their hosts, but he will have developed, with his increasing knowledge of the subject, a certain degree of confidence in his ability to describe these changes, to prepare cultures, and to do many things besides. If the same student has not given a good deal of time to the study of history, he will not feel the same degree of confidence that we observed in the former case, if he were asked to name, for instance, the causes that produced the French Revolution. There are, then, different degrees of knowledge, and these get reflected in the degrees of confidence with which we express what we know. It

is this fact that logic refers to under the term Modality. The meaning of this statement may become clearer if it is pointed out that the term modality does not refer to any uncertainty that may exist as to the nature and relations of the object of judgment, but only to the degrees of certainty of our knowledge of the object. Modality, that is to say, is a logical distinction; it characterises our thoughts about things, and not the things themselves. Whether there is any uncertainty in things themselves cannot be discussed in an elementary logic, but that varying degrees of certainty and uncertainty attach to our knowledge of things is a fact well known to all.

It is usual, in logic, to distinguish three degrees of certainty, and to throw these into typical propositional forms when we wish to express the modality of our thought. The propositions which set forth the modality of our judgments are called assertoric, problematic and apodeictic. We may state what is to be understood by each of these in order.

The Assertoric Proposition. An assertoric proposition has the same form as the affirmative categorical proposition, and yet is not to be understood in the same way. The categorical proposition expresses the way that S and P are related in the universe of discourse that defines the limits of the judging activity, and, therefore, relates to the matter of the judgment. The assertoric judgment, on the other hand, expresses one of the degrees of certainty of our knowledge about the subject-matter. For

example, if we were to say, 'All men are biped,' we should most likely be understood to express a judgment founded upon common observation, and that is what the categorical proposition of the form, 'S is P,' usually does. If, however, with respect to this judgment, we were asked, 'What makes you think so?' we should possibly hunt around the various corners of our knowledge for a reason for thinking that walking on two feet is characteristic of all men. If from such an excursion and search we were to return without anything that threw light upon the question, we should probably reply, 'I don't know, but he *is* a biped.' We should, that is, assert with emphasis our original statement, and thereby express our confidence in the correctness of the original statement. What, therefore, an assertoric proposition does is to assert that S is actually P, or, stated another way, that 'S is P' is actual, whether any reason, other than perception or memory supplies, can be given or not.

The Problematic Proposition. A problematic proposition states that 'S is possibly P,' or the fact expressed by 'S is P' is possible. The possibility does not imply any uncertainty in the fact expressed, but only an uncertainty in the state of our knowledge about the fact, an uncertainty that makes it impossible for us to state with any higher degree of confidence the correctness of what we say. For example, the student who is reading this section of logic may say, 'My understanding of the subject is possibly correct.' What he means by this is not

that his understanding may be described as possibly correct, for there is no such thing as 'possibly correct,' but that the statement, 'My understanding of this subject is correct' is a possibility in his own case. If we take a simpler case this will become clear. 'The rose is possibly white' does not mean that 'possibly white' is the colour that the rose has, for there is no such colour. We are to understand the adverb to qualify not any part of the statement, but the whole assertion — 'The rose is white,' and then, as in the other case, this statement expresses a possibility.

If this is the view that we must adopt of problematic propositions, what, we may ask, is the ground of the implied uncertainty? The uncertainty is not due, as in assertoric propositions, to the fact that there are no discoverable reasons for the statement, but that the discovered grounds of the statement are of such a nature that we can never be sure that we have all the elements necessary to give our statement any higher degree of certainty. To illustrate: the student's understanding of this paragraph must, as we know, be correct or incorrect, just as the rose must have any one or other of the colours that belong to the species rose. If we have not seen the rose in question, we know only that it is either red or pink or white or yellow, and because it may be something else than what we assert, we merely say that it is possibly white. So in the case of the student: his understanding is either correct or incorrect, but because the alternatives have not been de-

terminated with respect to the case in hand, he can only say that his understanding of the paragraph is possibly correct. A possibility, therefore, means the presence of alternatives between which no ground of decision has as yet been found.

Judgments which involve human volition as one of their factors, even when these judgments involve no alternatives, are said to be problematic. For example, 'I may go tomorrow,' means that 'I will go tomorrow' is the statement of a fact that is possible. Here we have not a series of alternatives as the ground of the uncertainty of the judgment, but the absence of some of the factors that are necessary for a more positive statement. All actions that depend on human volition are said to be contingent, and therefore a judgment that involves volition as one of its factors must always be problematic.

The Apodeictic Proposition. Apodeictic judgments are necessary: 'S must be P.' There exists, in this case, necessary grounds for the matter of fact judged. 'S is P' is necessary. Necessity is a word that must be used carefully. This is obvious if we call attention to the different kinds of necessity upon which our apodeictic statements depend. Mathematical statements, although not expressed in the form of apodeictic propositions, are necessary. For example, 'A straight line is the shortest distance between two points.' What the mathematician understands by this statement is that the shortest distance between two points must be what we mean by a straight line. The idea of a straight line

carries that meaning necessarily. Here, then, we have necessity resting upon the clearness with which an idea itself is conceived or apprehended, and not on anything else. Another class of necessity is met with when our judgments rest upon other judgments which may or may not be necessary. For example, the equality of triangles on equal bases and between the same parallels is a case of necessity resting upon grounds that are themselves necessary. In all truly apodeictic judgments there is presumed to be an inter-connection of fact that makes the fact asserted by the judgment necessary.

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QUESTIONS

1. What is the ground of the difference between the three types of propositions?
2. Describe the four main classes of categorical propositions.
3. Explain and illustrate what you understand by the quantity of propositions.
4. What is a singular proposition? Illustrate and state its importance.
5. What is a universal proposition?
6. What is the relation of the singular and universal proposition?
7. What is a particular proposition, and with what form of universal proposition is it most likely to be confused?
8. Describe and illustrate the indesignate proposition.
9. What do you understand by the quality of propositions?
10. What are the characteristics of affirmative propositions?
11. Describe and illustrate a negative proposition.
12. Why must the predicate of negative propositions be read connotatively?
13. What is the relation of tautology to affirmative statements?
14. In what way can tautological statements be overcome?
15. What is the logical status of nonsense?
16. In what respects are exceptive and exclusive propositions alike?
17. State and illustrate the characteristics of exceptive propositions.
18. What is an exclusive proposition? Illustrate.
19. Why are there no particular exceptive or exclusive propositions?
20. What is a compound proposition? By what other name is it known? Illustrate.
21. Explain what is meant by the modality of propositions.

22. What is an assertoric proposition, and how does it differ from the categorical proposition?
23. Explain and illustrate the problematic proposition.
24. How will you classify judgments which involve human volition as one of its factors?
25. Describe and illustrate the apodeictic proposition.

EXERCISES.

1. Write in parallel columns a list of six singular and six general propositions, and be ready to defend the assigned character of these propositions.
2. What kind of proposition is, 'Water is a liquid composed of one part oxygen and two parts hydrogen.' Give reasons for your answer.
3. What is the distribution of the predicate term of the proposition in the preceding question?
4. Give three illustrations of indesignate propositions.
5. What kind of proposition is, 'Oxygen is necessary for the maintenance of life'?
6. Without using the sign of quantity 'some,' give three illustrations of particular propositions.
7. Classify the following propositions from the standpoint of quantity, and in each case state the reason for your classification:
 - (1) Each of the hunters shot a bird.
 - (2) Rivers generally run into the sea.
 - (3) Blood is thicker than water.
 - (4) Afflictions are often salutary.
 - (5) Can the leopard change his spots?
 - (6) Veteran soldiers are the steadiest in battle.
 - (7) Suspicion ever haunts the guilty mind.
 - (8) Knowledge is power.
 - (9) The longest road has an end.
 - (10) Almost any Turk hates a Greek.
 - (11) Philosophy and Psychology are necessary to a well-rounded education.
 - (12) Only those who know are in the end to be trusted.

8. Illustrate how affirmative statements can be made either tautological or nonsensical.
9. Write a complete account of how one negative statement of your own selection may conceivably have come to be made.
10. Write a list of three assertoric propositions, and show how they differ from affirmative categorical propositions.
11. Explain the problematic nature of six propositions of your own selection.
12. Write a list of three apodeictic propositions.

CHAPTER IX

IMPLICATIONS OF CATEGORICAL PROPOSITIONS

Relation of Quantity and Quality in Categorical Propositions. When the subject term of a proposition is read in denotation, it refers to a determined or undetermined number of individuals, and when the predicate term of a proposition is read in connotation, it either affirms or denies the meaning of the predicate term of the individuals denoted by the subject term. Now, since every proposition undertakes to set forth both these relations of its subject-matter, there are no propositions which do not possess both quantity and quality.

From the standpoint of their quantity, categorical propositions, as we have seen, are either universal or particular; that is, logical statements are always about a determinate or indeterminate part of the subject-matter of such statements. From the standpoint of their quality, categorical propositions are either affirmative or negative; that is, logical statements either affirm or deny an attribute of some determinate or indeterminate part of their subject. Combined in a single statement, these facts and relations may be expressed thus: Propositions with distributed, as also those with undistributed, subject terms are either affirmative or negative in quality. There are thus four propositions, — the universal affirmative, the universal neg-

ative, the particular affirmative, the particular negative, — whose relations may be shown in a table as follows:

		QUANTITY	QUALITY
Propositions	{	Universal	a. Affirmative
			b. Negative
	{	Particular	a. Affirmative
			b. Negative

This table may be read: Propositions may be universal in quantity and either affirmative or negative in quality; or they may be particular in quantity and either affirmative or negative in quality.

Structural Formulae indicating the Quantity and Quality of Propositions. The problems that we are to study in this chapter arise out of the fact that propositions involve the relations both of quantity and quality. Whatever else the student can or cannot do with a proposition, it is of fundamental importance that he should be able to determine it both with respect to its quantity, and also with respect to its quality. The student will find it entirely possible to acquire a considerable degree of facility in thus determining the propositions that come before him for study; and, moreover, a little additional industry in this matter will carry him a long way toward overcoming some of the difficulties that he will meet farther along the road. When he has become practically familiar with these distinctions, it will be a

great convenience if, for the propositions that embody these distinctions, he write formulae which indicate their quantity and quality. The value of such formulae is that they enable one to deal only with the essential aspects of assertions, and they shorten the method of referring to these essential aspects. Like any other formulae, those used in logic are more or less arbitrary devices, and are to be justified by their usefulness. They are in no sense a substitute for the logical facts and relations which they are merely the means of expressing.

Our first aim then will be to distinguish by a convenient symbol the four typical propositions mentioned above. This is usually done by taking the first vowel of the Latin word for affirmation (*affirmo*), and the first vowel of the Latin word for negation (*nego*) to denote the universal proposition of the quality indicated by the word, and the second vowel of each word in like manner to indicate the respective particular propositions. Thus, A and E, the first vowels in the words *affirmo* and *nego*, are used to indicate the universal affirmative and universal negative propositions respectively. The second vowels in the same Latin words are I and O, and these are used to indicate respectively the particular affirmative and particular negative propositions. If, now, we add these facts to the table printed above, we have the following result:

	QUANTITY	QUALITY	SYMBOL
Propositions	1. Universal	a. Affirmative	A
		b. Negative	E
	2. Particular	a. Affirmative	I
		b. Negative	O

Whenever, in the foregoing pages, we have had occasion to refer to the structure of a proposition without respect to its quantity, we have written 'S is P,' or 'S is-not P.' Whenever we have wished to indicate the quantity as well as the quality of propositions, we have written 'All S is P,' 'No S is P,' 'Some S is P,' 'Some S is-not P.' This, however, is too cumbersome a method when a simpler one is possible. And, therefore, logicians have adopted the device of substituting for the copula in propositions of the general form S-P, one or other of the four vowels by which the four kinds of categorical propositions are distinguished. In this way we get the following structural formulae: SaP, SeP, SiP, SoP, which are read 'All S is P,' 'No S is P,' 'Some S is P,' 'Some S is-not P.'

The following table summarises the important facts:

	QUANTITY AND QUALITY	STRUCTURAL FORMULA	READ
Propositions	1. Universal Affirmative	SaP	All S is P
	2. Universal Negative	SeP	No S is P
	3. Particulare Affirmative	SoP	Some S is P
	4. Particular Affirmative	SiP	Some S is P

This table should be read: Propositions as to their quantity and quality may be either universal affirmative, with the structural formula SaP which is read 'All S is P;' or universal negative, with the structural formula SeP which is read 'No S is P;' or particular affirmative, with the structural formula SiP which is read 'Some S is P;' or particular negative with the structural formula SoP which is read 'Some S is-not P.'

Distribution of the Terms of the Four Categorical Propositions. The logical meaning of distribution has been discussed in the chapter on terms, and we met the same question again when considering the quantity of propositions. We need, therefore, only repeat what was shown more at length above, that by distribution is meant the width or extent of the meaning expressed in a proposition. We have seen, for example, that from the standpoint of quantity the subject terms of universal propositions are distributed; they denote a determinate number of individuals, each of which possesses, or does not possess, in addition to the qualities connoted by the term itself, those qualities also that are connoted by the predicate term of such propositions. From the same standpoint, the subject terms of particular propositions, on the other hand, are undistributed; they refer to an indeterminate number of denoted individuals.

We have now to ask what influence the quality of propositions has upon the distribution of predicate terms. And first let us consider affirmative

propositions. In affirmative propositions we predicate the attribute connoted by P of the object or objects denoted by S. So far as they are affirmative, propositions do not enable us to say whether there are any objects, other than those denoted by the subject term, to which the attributes of the predicate term apply. Sometimes there are such objects; as, for example, 'The cat is a domesticated animal,' for, as we know, there are other domesticated animals than cats. Sometimes there are not, as, for example, 'Perception is the consciousness of particular material objects present to sense,' for there are no other objects to which such a description can apply except perception. In view of these facts, it is obvious that, from the form of the proposition, it is quite impossible to determine whether universal affirmative statements are to be understood in the one or the other of the ways just indicated; whether, that is to say, they may be said to have distributed or undistributed predicate terms. On account of this uncertainty, it is customary, in the absence of any reason connected with the subject matter to the contrary, to regard the predicate terms of such propositions as undistributed.

The predicate terms of negative propositions, on the other hand, are distributed. The reason for this is that it is only when every object which can be included in the denotation of the predicate term is excluded from the subject that the proposition can have a negative force at all. For if all P's were not definitely separated from the S, some P's might

be included in it, and this would not express the meaning of negation. Negation, as we have seen, has always a definitely exclusive force. And this is true of negative propositions irrespective of their quantity; it is true of the particular as well as of the universal negative proposition. Of the universal proposition, SeP, this is easily seen to hold. For if 'No S's are P's' has any meaning at all, it must surely mean that not any of the S's are any of the P's. That is to say, of none of the objects that have the qualities connoted by S can we affirm that they have also the qualities connoted by P. If that is true, no P's are any of the S's. But the same thing can be shown also of the particular proposition, SoP. 'Some S's are-not P's' must mean that none of the objects denoted by S has the qualities connoted by P; and that is equivalent to saying that none of these S's are at the same time P's. The objects denoted by P, if there are any, are entirely excluded from the number of objects referred to by S. If, in this case, any difficulty arises on account of the undistributed nature of the subject term, we can only say that the meaning of the proposition is that there are at least some unspecified instances of S's that do not have the characteristics of P; and that if, as may or may not happen, you discover any S's that do have these characteristics, then they are not the S's to which the proposition refers. It follows, from these considerations, that the predicate terms of all negative propositions are distributed.

We may summarise the foregoing discussion in the four following statements:

1. The subject terms of propositions that are universal in quantity are distributed.

2. The subject terms of propositions that are particular in quantity are undistributed.

3. The predicate terms of propositions that are affirmative in quality are undistributed.

4. The predicate terms of propositions that are negative in quality are distributed.

Thorough familiarity with the distribution of the terms of categorical propositions is so essential to the student's understanding of what follows that he is urged to master the subject at this point of his studies. Because of the importance of the subject, and as a further aid to its mastery, we add the following tables each of which represents the facts contained in the foregoing summary. The letters D and U stand for distributed and undistributed respectively:

		PROPOSITION		
STANDPOINT		SUBJECT	PREDICATE	STANDPOINT
Quantity	{	D	a U	Quality
		D	e D	
		U	i U	
		U	o D	

This table may be read as follows: From the standpoint of quantity, the subject term of an A or uni-

versal affirmative proposition is distributed; from the standpoint of quality, the predicate term of an A or universal affirmative proposition is undistributed, etc.

Another way of stating the same set of facts is to say that the subject terms of propositions that are universal in quantity are distributed, and those that are particular in quantity are undistributed; that the predicate terms of propositions that are affirmative in quality are undistributed, and those that are negative in quality are distributed. The following table represents these facts:

	QUANTITY	DISTRIBUTION
Subject terms	{ Universal	D
	{ Particular	U
	QUALITY	DISTRIBUTION
Predicate terms	{ Affirmative	U
	{ Negative	D

Meaning and Classes of Implication. The group of topics that are now to be discussed, and to which the preceding pages are an introduction, is by some writers brought together under the title of 'immediate inferences,' and by others under the title 'interpretation of propositions.' The objection that is usually urged against 'immediate inferences' is that an inference always involves the use of a middle term, and is therefore mediate; and, consequently,

that it is a contradiction to speak of 'immediate' inferences. The objection that may be brought against 'interpretation of propositions' is that it tends to confuse two different things, namely, the import or meaning of propositions, and the question of what other statements we are committed to in making any statement whatever. We shall speak of the 'implications of propositions,' since the questions that are to be considered in this chapter relate to the various methods of determining the relations that exist between our formally expressed propositions, and other unexpressed propositions with which they are immediately connected. Implication etymologically means 'folded-in,' so that the idea the topic suggests is that folded within any statement we make there are other statements that may be brought to light by the use of appropriate methods. The practical value of this chapter consists in the opportunity it affords of finding out that our statements ordinarily mean much more than we actually express in them.

We have to consider two main classes of implications:

(1) The Opposition of Propositions. By opposition of propositions is meant the relation that the truth or falsity of one proposition has to the truth or falsity of other propositions dealing with the same subject-matter, that is, having the same subject and predicate.

(2) Eductions. By eductions is meant the relation that one proposition accepted as true has

to other propositions dealing with the same subject-matter from other points of view.

The Opposition of Propositions. The term opposition applied to propositions would strictly mean that propositions could stand in this relation only when the truth of the one involved the falsity of the other, and vice versa. This was the original meaning and use of the term in logic. But it was found convenient to include under the same title a consideration of propositions that are not incompatible with one another. Opposition, as it is now used, embraces, therefore, a study of the relations of any two propositions referring to the same subject-matter whether the truth expressed by one is or is not compatible with that expressed by the other. Thus, propositions relating to the same subject, but differing in quantity, or quality, or both, are said to be opposed to each other.

Since there are only four categorical propositions, it is evident that each stands in relation to three others. Thus:

SaP is related to SeP, SiP, SoP.

SeP is related to SaP, SiP, SoP.

SiP is related to SaP, SeP, SoP.

SoP is related to SaP, SeP, SiP.

If we examine this list of possible oppositions, we shall see that each proposition is related to (1) a universal proposition, (2) a particular proposition, (3) an affirmative proposition, (4) a negative proposition. In other words, all propositions are related to others (1) of the same and (2) different

quantity, and (3) of the same and (4) different quality. There are, consequently, four kinds of opposition in which any proposition may stand. Thus, the relation between

1. Universal propositions of different quality is said to be one of contrariety: SaP, SeP.

2. Universal and particular propositions of the same quality is said to be one of subalternation: SaP, SiP; SeP, SoP.

3. Universal and particular propositions of different quality is said to be one of contradiction: SaP, SoP; SeP, SiP.

4. Particular propositions is said to be one of subcontrariety: SiP, SoP.

We shall consider each of these kinds of opposition in the order in which they have been named; and since any statement we make may be either true or false, we shall have to examine the propositions that stand in opposition on the supposition both of their truth and falsity.

Contrariety. Contrary opposition exists between universal propositions of a different quality; that is, between SaP and SeP. Thus, contrary propositions differ in quality only.

If what any universal proposition states is true, what its contrary states must be false. For example, if it is true that 'All men die,' it is false that 'No men die.' If it is true that 'No men are angels,' it is false that 'All men are angels.' If, therefore, we accept the truth of what either SaP or SeP states, the falsity of what its contrary states follows as a

matter of course, for the reason that the same predicates or qualities cannot belong, and not belong, to the same subject. On the other hand, if what any universal proposition states is false, what its contrary states is left in doubt. If, for example, it is false that 'All men are six feet tall,' we are left in doubt as to whether 'No men are six feet tall,' because some men may be.

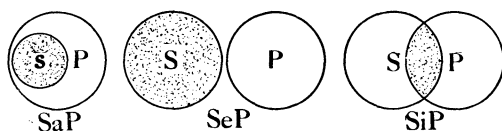
Let us connect these facts with what we have already learned. We saw, in the chapter on terms, that if two species of a genus differ from one another more than they do from any of the other co-ordinate species, the relation in which they stand is one of contrariety. If, for example, within the genus *G* the species *S* have the characteristic *P*, then however great and numerous the increasing differences between one species and another, they cannot be so great and numerous that, within the genus, *P* can be denied of any of the species. Based upon the degree of their variation from the genus, all the species, as we have seen, have a recognised place within the genus. But whatever the variations that occur, the species are held together by their common likeness, and so long as predication is concerned with this common likeness, so long can we not entertain any proposition which denies it of any or of all the species of the genus. If *SaP* is true, the proposition that denies that *S* is *P*, *SeP*, must be false, and vice versa.

Let us look at the other case where something is falsely predicated of all the species. Now what is

thus predicated cannot be the genus, for the genus can only be predicated truly. If anything is falsely predicated, it must be something that is either another genus, when the statement would be absolutely false, or something that is or may be a differentia, when the statement will be partially false. If, for example, 'All flesh is grass' were to be interpreted literally, the statement would be false, because it predicates a genus that is wholly misleading; and if one were to say that 'All vertebrates live in the water,' that would be false, because it predicates what is or may be a differentia, something that belongs to one of the species though not to all of them. In order to detect the falsity in these instances, we must know that what is predicated is either a wrong genus or a differentia, and this is not possible without going beyond what is given in a false statement. Hence, if all we know is that P is wrongly predicated of S, we cannot say whether P belongs in part or not at all to S, and, therefore, cannot infer the truth of the contrary of the original proposition. The contrary of a false universal, that is to say, is involved in doubt.

Again, the relations of contrary propositions may be studied from the side of the denotation of their terms. Thus, it cannot be true that 'All' and 'None' of the objects denoted by the subject term of a proposition have the qualities connoted by the predicate term; the truth of the one excludes the truth of the other. That the falsity of one proposition involves its contrary in doubt may be shown

as follows: If some of the objects denoted by the predicate term of an affirmative proposition, and none of the objects denoted by the same term in a negative proposition, belong to the group of objects denoted alike by the subject terms of these propositions, then it is obvious that in the one case P falls partly within, and in the other wholly without, the group of objects denoted by the subject terms of the two propositions. If, now, we make the statement falsely that 'All S falls within P without exhausting it,' we cannot affirm truly the contrary of this statement, 'No S falls within P,' because there is another alternative to the one offered by the contrary, namely, the case where S falls partly within and partly without the class of objects denoted by P; and which of these two cases is true cannot be determined without examining the cases themselves. Representing these facts diagrammatically thus:

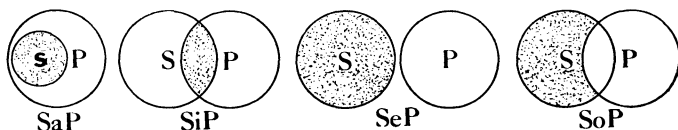


we must say that if SaP is false, SeP cannot be true without disposing of the alternative case SiP. But this cannot be done formally, but only by an examination of the subject-matter of these propositions.

Subalternation. Subaltern opposition exists between a universal and a particular proposition of the same quality; that is, between SaP and SiP;

SeP and SoP. Thus, subaltern propositions differ in quantity only.

If what the universal proposition states is true, what its subaltern states is also true. This follows from the fact that universal propositions distribute their subject terms. Thus, if something is true or false, as the universal states, of each of a class of objects, it is true or false of any number of them, since these must be identical with some of the former. The truth, therefore, of the particular is involved in the truth of the universal proposition. Hence, as may be seen by studying the accompanying diagram, if any part of the S that falls wholly



within P, (SaP), or wholly without P, (SeP), were to fall partly without or partly within P, some part of S would, in the one case, fall partly within (SiP), and, in the other, partly without (SoP) P, and it is of these parts that the statements of the respective subaltern propositions is made.

On the other hand, if what the universal proposition states is false, what its subaltern states is left in doubt. The truth of the particular is not involved in the falsity of the universal, and cannot, therefore, be the ground of the latter's falsity. But, as we shall see, it may be. We have studied already one ground of falsity of universal statements. We

have seen, for example, that when the contrary of any proposition is true, that proposition is absolutely false. But we have also seen that if we start with the falsity of a universal statement, nothing can be affirmed as to the truth or falsity of its contrary. Now, in contrast to the falsity involved in the contrary relation, that involved in the subaltern relation is only relative, that is, the falsity may be due not to the fact that S-P asserts something contrary to fact, but that it is asserted without regard to existing limitations. In other words, since there are degrees of error, as well as of truth, SaP and SeP may owe their falsity to the fact that only SiP and SoP are true respectively. But, here again, if we start with the falsity of the universal, we cannot say that its subaltern is true or false; truth or falsity, in this case, simply remain possible alternatives between which we can decide only by going outside what is given in the original proposition. And this is equivalent to saying that the subaltern is always involved in doubt when all that we know is that the universal or subalternans is false.

What we have seen is that the truth of a universal involves the truth of its subaltern; but that the falsity of a universal involves its subaltern in doubt.

Let us now consider how the universal is affected by the truth or falsity of its particular, subaltern, proposition. And, first, let us take the case where the subaltern is true. If the only thing we know about a group of objects is that some of them are in a particular place, it is obvious that we cannot

say that they are all in that place; and the reason is not that some are in another place, for that exceeds the knowledge contained in the original proposition, but that some may be. For a similar reason, we cannot affirm that 'No trees are in leaf,' if all that we know is that 'Some trees are-not in leaf.' From the truth of a particular proposition, therefore, we can only conclude that its subalternans—the universal of the same quality—is doubtful. Let us take, secondly, the case where the subaltern is false. Here the falsity of the subaltern involves the falsity of the subalternans. Thus, if 'Some apples are blue' is false, it is likewise false that 'All apples are blue,' and if 'Some apples are-not red' is false, it is likewise false that 'No apples are red.' The reason for this case is found in the fact that particular propositions affirm or deny a predicate of certain unspecified objects of a class, and not of the class as a whole. If, therefore, we know that a statement is false in some case, we know that it cannot be true of all cases, since all cases must include the case in which it is known to be false. *Falsus in uno, falsus in omnibus.* We are dealing here with the exception under which a rule breaks down.

Contradiction. Contradictory opposition exists between a universal and a particular proposition that differ from each other in quality; that is, between SaP and SoP, SeP and SiP. Thus, contradictory propositions differ both in quantity and quality.

If what any universal proposition states is true,

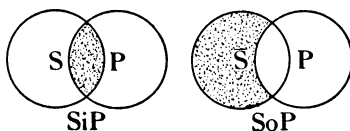
what its contradictory states must be false. If it be true that 'All men are willing to give their lives for a cause,' it is false that 'Some men are-not willing to give their lives for a cause.' If it is true that 'No men are utterly selfish,' it is false that 'Some men are utterly selfish.' If, on the other hand, what a universal proposition states is false, what the contradictory opposite states must be true. For example, if it is false that 'Every man has his price,' it is true that 'Some men cannot be bought,' and if it is false that 'None of the candidates were successful,' it is true that some were. Further, the truth or falsity of particular propositions involves the falsity or truth of their contradictories. If 'Some one is going to get hurt,' is true, it is false that 'No one is going to get hurt;' but if the statement is false, then 'No one is going to get hurt' is true. If 'Some people are short sighted' is true, it is false that 'No people are short sighted;' but if it is false that 'Some kinds of ignorance promote virtue,' it is true that 'No kinds of ignorance promote virtue.'

We may express these results in the single statement that the truth of one of the propositions in contradictory opposition involves the falsity of the other; and the falsity of one, the truth of the other. Of any pair of contradictory propositions, one must be false, and the other true. Contradiction is, therefore, the most complete form of logical opposition.

Subcontrariety. Subcontrary opposition exists between particular propositions of different quality;

that is, between SiP and SoP. Thus, subcontrary propositions differ in quality only.

If we make the two statements that 'Some men are blue-eyed' and 'Some men are-not blue-eyed,' each of these statements is true, and they are in subcontrary relation to each other. But the truth of either one of them is not sufficient to establish the truth of the other. If, for example, we represent our two propositions thus:



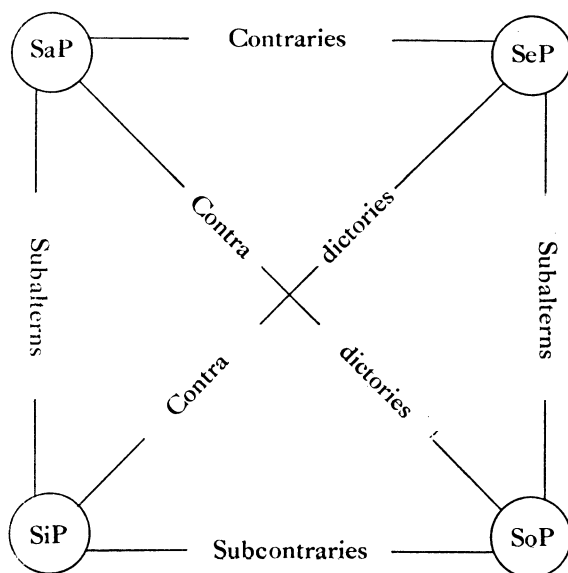
it will be evident that in each case P, in part or whole, is predicated of a different part of S. If, consequently, we know that 'Some indeterminate number of men are blue-eyed,' (SiP), this is not a sufficient ground for saying that the rest are, or are not, blue-eyed. The fact that we are dealing with particular propositions which have undistributed subject terms, that is, terms that do not take into account a determinate number of instances of the class, but only more or less of them, prevents us from saying anything definite of the rest of the class with respect to their being or not being blue-eyed. Of the propositions, therefore, that exist in subcontrary opposition, we may say that the truth of the one leaves the truth of the other in doubt.

Propositions in the subcontrary relation cannot

both be false. In other words, the falsity of one involves the truth of the other. For example, if it is false that 'Some men are four-footed,' it is true that 'Some men are-not four-footed.' That is, it is only on condition that some men are not four-footed that we can deny the truth of the statement that 'Some men are four-footed.' This view of subcontrary opposition is required by the fact that if both particular propositions were false, their contradictories would both be true; but, as we have seen, SaP and SeP cannot both be true, for if one is true the other is proved to be false. But if, as we have seen, the falsity of the universal — 'All men are four-footed,' for example — involves its contrary in doubt, we can at least say that its contradictory — 'Some men are-not four-footed' — is true.

The Square of Opposition. The accompanying diagram, called the Square of Opposition, is a mechanical device which has long been traditional in logic, and is intended as an aid to the better understanding of the doctrine of opposition. The universal propositions are placed at the top, the particular propositions at the bottom of the diagram; the affirmative propositions are placed on the left, and the negative ones on the right. The lines connecting any two propositions indicate the kind of opposition that exists between them. The top and bottom lines which are horizontal and parallel to each other connect propositions of the same quantity; the side lines which are perpendicular and parallel connect propositions of the same quality. The long diagonal lines

connect propositions that are different in both quantity and quality.



Let us now see how it may be used to throw light on the doctrine of opposition. It will be observed at once that every proposition has its contradictory, and in contradiction, it will be remembered, we have the most complete form of logical opposition. To work the relations of opposition out under the control of the contradictory relation seems to simplify the whole matter for most students. We summarise the doctrine of opposition below, and the

student is required to follow the statements out on the Square of Opposition. Thus, if

SaP is true, its contradictory and its contrary are false, and its subaltern is true.

SaP is false, its contradictory is true, and its contrary and its subaltern are doubtful.

SeP is true, its contradictory and its contrary are false, and its subaltern is true.

SeP is false, its contradictory is true, and its contrary and its subaltern are doubtful.

SiP is true, its contradictory is false, and its subcontrary and subalternans are doubtful.

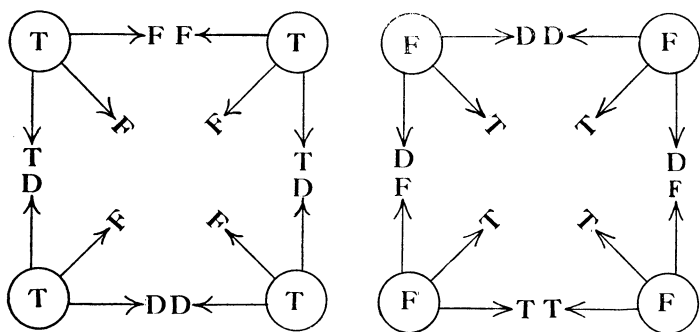
SiP is false, its contradictory and sub-contrary are true, and its subalternans is false.

SoP is true, its contradictory is false, and its subcontrary and subalternans are doubtful.

SoP is false, its contradictory and sub-contrary are true, and its subalternans is false.

It may be helpful to represent these facts and relations diagrammatically. This we may do by drawing only so much of the Square of Opposition as embodies the relations in which a given proposition stands, utilising the arrow head to indicate the direction in which given relations are to be read. For example, the lines $\searrow \swarrow$ diverge from a common point and indicate, according to their direction, the propositions we are considering and the relations in which they stand. It is obvious that the example chosen is the O proposition. Since, then, the direction of the lines tells us what propositions we are

dealing with, and in what relations of opposition they stand, we may indicate at the ends of the lines whether, in those relations, the proposition is true (T), false (F), or doubtful (D). Further, we may indicate the original proposition by writing its given truth or falsity in a circle. Following these directions, we have the accompanying result:



A summary of the doctrine of opposition is given in the table on page 279.

Nature and Methods of Education. In the course of our study, we have seen that all our judgments are related to other judgments referring to the same subject-matter, and, consequently, that every judgment implies more than it asserts. Put more directly, every judgment commits us not only to what we assert, but also to the acceptance of all the other judgments which are implied in what we assert. Now when these implied judgments involve a change in the point of view from which the sub-

ject-matter of the original judgment is viewed, they are called eductions.

We may classify the methods of eduction as obversion, conversion, contraposition, and inversion. But only two of these methods, obversion and conversion, are fundamental. In the following sections, we shall study the fundamental methods first, and follow this with a consideration of the derived methods, contraposition and inversion.

Given		SaP	SeP	SiP	SoP
SaP	T		F	T	F
	F		D	D	T
SeP	T	F		F	T
	F	D		T	D
SiP	T	D	F		D
	F	F	T		T
SoP	T	F	D	D	
	F	T	F	T	

TABLE OF OPPOSITIONS

Obversion. Other terms by which this method is sometimes known are Permutation, Aequipollence, Immediate inference by Private Conception. Obversion is the logical form of the double negative with which we have all become acquainted in the study

of grammar. Like the double negative, it has the force of changing an affirmative statement into a negative one, and a negative statement into an affirmative one. Obversion effects no change in the meaning of what is asserted, but only in its emphasis. For example, 'Barkis is willin'' by obversion becomes 'Barkis is-not unwillin';' 'To do this is-not unnecessary' becomes by obversion 'To do this is necessary.'

In obversion only the quality of the original proposition, called the obvertend, undergoes change; the quantity and the meaning remain the same. But if the quality changes, the predicate term of the obvertend must also undergo modification if the meaning is to remain unaffected. What exactly is this modification? It cannot be enough to say that the characteristic of the obverse of a proposition is that it has for its predicate term the negative of the predicate of the proposition from which it is derived, although, of course, this is, from the formal point of view, true. Nor can we say that the connotation of the obvertend is affected, for that is the question we are raising. If we are to understand the particular significance of obversion, we shall have to interpret it as a method that affects the meaning of a proposition by a change in what the proposition denotes. That is to say, the particular range of the objects signified by the obvertend is different from that of its obverse. For example, in the illustrations used above, it is obvious that Barkis' attitude must be one of willingness or un-

willingness; and of any action we may say that it is necessary or unnecessary. Now, since obversion is, in part, a matter of negation, we carry over into that process all the characteristics that make negation significant. We have seen that negation is very largely a matter of the emphasis that we give to our knowledge, and of the definiteness or indefiniteness of our acquaintance with what we assert. Thus, if we are not sure of the degree of Barkis' willingness, we may express the actual state of our information or opinion by saying that he is not-unwilling. If we wish to suggest, but not to state explicitly, the degree of necessity of the things we do, we may say that they are not-unnecessary. A guarded statement of the latter kind, it will be observed, becomes more emphatic by obversion, for we are thereby led to assert without qualification the necessity of our actions.

There are two steps in the process of obversion; the first consists in changing the quality of the original proposition without changing its quantity; the second consists in negating the predicate term of the original proposition. To consider the first step we should observe, with respect to quantity, that if the original proposition is universal the derived proposition is universal, and if the original proposition is particular the derived proposition is particular; and, with respect to quality, that if the original proposition is affirmative the derived proposition is negative, and that if the original proposition is negative the derived proposition is affirm-

ative. Combining these two considerations, we find that, in obversion, an A proposition always changes to an E, an E to an A, an I to an O, and an O to an I. With regard to negating the predicate term of the original proposition, the second step in obversion, we may observe that if we use a short horizontal stroke over the symbol of an affirmative predicate term to indicate that that term has been negated, it will be obvious that obverted propositions are characterised generally by having for their predicate terms the so-called infinite or non-P term. It follows that if the predicate term is already a negative or infinite term, the effect of negating it will be to remove the sign of negation. The method of indicating negation by the short horizontal stroke over the symbol of an affirmative term is used throughout this book, and this fact should be noted; a horizontal stroke over any term, whether predicate or subject term, will have the meaning of negation. Thus, \bar{S} will mean non-S, \bar{M} will mean non-M, just as \bar{P} means non-P. With these explanations the method of obversion should be sufficiently clear. In the following table we illustrate the method of obversion in each of the four categorical propositions:

Obvertend	SaP	SeP	SiP	SoP
First Step	SeP	SaP	SoP	SiP
Second Step Obverse	Se \bar{P}	Sa \bar{P}	So \bar{P}	Si \bar{P}

TABLE OF OBVERSIONS

Conversion. This is a process by which we affirm or deny of the predicate term of an original proposition, called in this relation the convertend, what was asserted by the subject term of the same proposition. If, for example, we know that 'Pilocarpin when taken internally is followed by a slowing of the heart beat,' we are able, by conversion, to place pilocarpin among the agents which have the effect of slowing the heart beat in any investigation we may make about the conditions under which the heart beat is slowed. In other words, the terms by which in one proposition we qualify a subject, may themselves become of direct interest to us, and in looking about for predicates which shall qualify them, we find in the original proposition something that is serviceable for this purpose. To state generally the fact that conversion denotes, we may say that if we make any statement in the S-P form, conversion is the answer to the question whether we are able, because we have made this statement, to make another which will have the form P-S. Having said something about S, can we, therefore, say something about P?

That this is a very reasonable inquiry will appear if we recall what was said about the possibility of predicating of any term S another P. We saw that this possibility rests upon the fact that S is not merely S, and that P is not merely P. In any judgment we are dealing with a complex phenomenon, and our statements reflect this fact. We, therefore, said that the S that in any proposition is P is an

SP, and, as we also saw, the P that is predicated of S has its roots running down into, and derives its character from, the same ground that makes of S a significant term. In other words, the P of any proposition is also an SP. It is this fact that underlies the process and problem of conversion. If S is P at all, then the conditions of its being P are those that will enable us to reverse the relation, and to declare that P is S. This is the general statement of the fact of conversion, but what the limitations are under which conversion may legitimately take place must be ascertained from the study of concrete cases. In other words, conversion is not a wholly formal process, it is not a mere manipulation of symbols; but a study of the formal relations of the terms of the proposition brings to light some of the limits of this type of thinking.

Formally considered, conversion is the method by which the predicate term of one proposition, called the convertend, becomes the subject term of another, called the converse, and the subject term of the former proposition becomes the predicate term of the latter. It will be seen, then, that the terms of the converse must be the same as the terms of the convertend. Stated simply, the terms of the converse are derived from the convertend by the transposition of its terms: S-P becomes P-S. Again, we should observe that in conversion there is no change in the quality of the proposition: an affirmative proposition remains affirmative, and a negative proposition remains negative. And, further,

the quantity of propositions sometimes is, and sometimes is not affected by their conversion. This will be shown in our examination of the four categorical propositions, and the reason for it will then become clear. But before we examine each kind of proposition to ascertain whether and in what form it is capable of conversion, we wish to call attention to the rule which governs all cases of conversion, namely: If a term is distributed in the converse it must have been distributed in the convertend. Practical direction for applying this rule may be laid down thus: Quantify the predicate, and transpose terms.

The simplest cases of conversion are met with in the universal negative and the particular affirmative propositions: SeP and SiP. The universal negative proposition has two distributed terms, and the particular affirmative proposition has two undistributed terms. It follows from these facts that if we transpose the terms of each proposition, without changing the quality, no term in the converse is distributed that was not distributed in the convertend. If, for example, the whole of P, in the universal negative proposition, lies outside the whole of S, then the whole of S falls outside the whole of P. In the case of the particular affirmative proposition, some of S is identical with some of P, and, therefore, it follows that the same part of P is identical with the same part of S. Under these conditions, it is possible not only to say that 'S is P,' but also that 'P is S.' When, as in these two

cases, we convert a proposition without changing its quantity, we are said to convert simply, or, as the Latin word has it, *simpliciter*.

Conversion by limitation, or *per accidens*, that is, conversion that involves a change in the quantity of a proposition, is necessary in the case of the universal affirmative proposition, SaP. The reason for this is that, being an affirmative statement, the predicate term is undistributed, and when this becomes the subject term of a new proposition, it does not present a determinate number of objects for the predicate to qualify. For example, 'All the books are on that shelf' cannot imply that 'All the things on that shelf are books;' there may be other things on the shelf than books. All, therefore, that we are warranted in saying is that 'Some of the things on that shelf are books,' a statement that differs in quantity from the original proposition. The student may find examples of universal affirmative propositions which, on account of the nature of their subject-matter, admit of simple conversion; but the form of the proposition does not show this. 'A straight line is the shortest distance between two points,' that is to say, definitions, may serve to show what is meant. So long, however, as we confine attention to the form of the proposition, SaP converts by limitation to PiS.

A particular negative proposition cannot be converted at all. In such a proposition the subject term is undistributed, and the predicate term is distributed. If, therefore, we transpose the terms, we

have a proposition with a distributed subject and an undistributed predicate, that is a proposition of the form 'All S—some P.' Now the only proposition that we have of this form is the universal affirmative, and we should be required to take this as such a proposition. But this we cannot do, because the converse of a proposition must be of the same quality as the convertend, and, moreover, no negative proposition has an undistributed predicate term. But if, in order to overcome this difficulty, we were to distribute this term, we infringe the rule of conversion which states that if a term is distributed in the converse it must have been distributed in the convertend. In no way, therefore, can we convert a particular negative proposition.

It may sometimes be true that SoP and PoS are both true, but we must go outside the formal relations of S and P in the propositions to see this. For example, 'Some anarchists are-not Italians,' and 'Some Italians are-not anarchists' are both true. But we do not always get a like satisfactory result merely by transposing the terms of particular negative statements. For example, 'Some men are-not professors' is true, but 'Some professors are-not men' is false, so long, that is, as we only mean by men membership in the human race. These cases depend upon a knowledge of more than the original proposition states, and there is no possibility of ascertaining the precise nature of that knowledge from a study of the formal relations of S and P in a particular negative proposition.

We add below a table of all possible conversions :

Convertend	SaP	SeP	SiP	SoP
Converse	PiS	PeS	PiS	

TABLE OF CONVERSIONS

Contraposition. Sometimes called Conversion by Negation, and Contrapositive Conversion. The contrapositive of a given proposition may be described, according to the method by which we arrive at it, as the converted obverse. The rule, consequently, that we should follow to reach the contrapositive of any proposition is: First obvert the original proposition, and then convert the proposition thus obtained. If we follow this rule, the contrapositive will be seen to be a proposition that has for its subject term the negative of the predicate of the original proposition, called in this case the obvertend. S-P becomes \bar{P} -S. Now, since obversion is involved in the process of reaching a contrapositive, and conversion does not change it back again, the contrapositive differs in quality from the original proposition. And since the quantity of a proposition, as we have seen, is sometimes affected by conversion, namely, when the proposition is a universal affirmative proposition, the quantity of the contrapositive will, in that case, be changed, but in all other cases it will be unaffected.

The contrapositive of any proposition may be obverted, and the proposition thus obtained will have the characteristics that we have enumerated

in the section on obversion. Such a proposition is called the obverted contrapositive. In this case $\overline{P}\text{-}S$, the contrapositive of $S\text{-}P$, becomes by obversion $\overline{P}\text{-}\overline{S}$.

The results of the above statements, expressed symbolically, are given in the following table of contrapositives:

Original Proposition	SaP	SeP	SiP	SoP
Obverse	Se \overline{P}	Sa \overline{P}	So \overline{P}	Si \overline{P}
Contrapositive	\overline{P} eS	\overline{P} iS		\overline{P} iS
Obverted Contrapositive	\overline{P} a \overline{S}	\overline{P} o \overline{S}		\overline{P} o \overline{S}

TABLE OF CONTRAPOSITION

Not many examples of contraposition are necessary if the student has mastered the fundamental processes of obversion and conversion. We confine ourselves to one:

Original Proposition	SaP	All acids are substances that turn blue litmus paper red
Obverse	Se \overline{P}	No acids are substances that do not turn blue litmus paper red
Contrapositive	\overline{P} eS	No substances that do not turn blue litmus paper red are acids
Obverted Contrapositive	\overline{P} a \overline{S}	All substances that do not turn blue litmus paper red are non-acid

Inversion. The inverse of a proposition has for its subject the contradictory of the subject of the original proposition (invertend). $S-P$ becomes $\bar{S}-P$. Like contraposition, inversion is arrived at indirectly through the use of the two fundamental forms of eduction. And since we can obtain by obversion the contradictory of a term only when that term forms the predicate of a proposition, S must be made the predicate of a proposition before its contradictory can be ascertained, that is, conversion must have taken place at some stage in the process of inversion. In other words, S must have been made the predicate of a proposition, and then that proposition must have been obverted if we are to get a non- S term. Inversion can apply only to the two universal propositions, SaP and SeP , and in each case we get a particular proposition of the general form $\bar{S}-P$. We must follow a separate rule in each of these cases. To invert a universal affirmative proposition, the rule reads: Obvert and convert alternately until a proposition of the required form is found. The rule for the universal negative proposition reads: Convert and obvert alternately until a proposition of the required form is found. We show the necessary steps in each of these cases in the following table:

Original Proposition	SaP
Obverse	Se \bar{P}
Converse	\bar{P} eS
Obverse	\bar{P} a \bar{S}
Converse	\bar{S} i \bar{P}
Inverse	\bar{S} oP

Original Proposition	SeP
Converse	PeS
Obverse	Pa \bar{S}
Inverse	\bar{S} iP

TABLES OF INVERSION

We may obvert these inverses, and thus obtain the obverted inverses. Thus, \bar{S} oP will give \bar{S} i \bar{P} , and \bar{S} iP will give \bar{S} o \bar{P} . It will be observed that from the universal affirmative proposition we get, by inversion, a particular negative proposition, and from the universal negative proposition a particular affirmative proposition, and that by obversion these inverses are given the same quality as their respective original propositions.

Summary of Eduction. In this section we give first an outline schema of the characteristics of the propositions arrived at by eduction, that is, we show the character and relations of the terms of all the eductive propositions. Thus:

Original Proposition	$S - P$
Obverse	$S - \bar{P}$
Converse	$P - S$
Contrapositive	$\bar{P} - S$
Obverted Contrapositive	$\bar{P} - \bar{S}$
Inverse	$\bar{S} - P$
Obverted Inverse	$\bar{S} - \bar{P}$

SCHEMA OF EDUCTIVE PROPOSITIONS

In the following table we show, secondly, all the eductions of each of the four categorical propositions in a form which indicates throughout the quantity and quality of the propositions:

Original Proposition	SaP	SeP	SiP	SoP
Obverse	Se \bar{P}	Sa \bar{P}	So \bar{P}	Si \bar{P}
Converse	PiS	PeS	PiS	
Obverted Converse	Po \bar{S}	Pa \bar{S}	Po \bar{S}	
Contrapositive	\bar{P} eS	\bar{P} iS		\bar{P} iS
Obverted Contrapositive	\bar{P} a \bar{S}	\bar{P} o \bar{S}		\bar{P} o \bar{S}
Inverse	\bar{S} oP	\bar{S} iP		
Obverted Inverse	\bar{S} i \bar{P}	\bar{S} o \bar{P}		

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QUESTIONS.

1. Classify categorical propositions from the standpoint (1) of their quantity and (2) quality.
2. What are the quantitative and qualitative characteristics of logical statements.
3. What symbols are used to distinguish the four categorical propositions, how are they derived, and what exactly do they represent?
4. Explain what is meant by a structural formula. What is the general structural formula of a categorical proposition?

5. Write the structural formulæ of all the categorical propositions, and explain how they are derived.
6. What do you understand by the distribution of terms?
7. What determines the distribution of the subject terms of propositions?
8. From what standpoint is the distribution of the predicate terms of propositions determined?
9. What is the distribution of each of the terms in each of the four categorical propositions?
10. Explain why the predicate terms of affirmative propositions are undistributed.
11. Explain why the predicate terms in SeP and SoP are distributed.
12. Classify in any suitable way the distribution of all the terms of all the categorical propositions.
13. What is meant by the implication of propositions?
14. What other terms have been used to describe the fact of implication, and why are these not suitable?
15. What are the main classes of implication?
16. What do you understand by the Opposition of Propositions?
17. Name all the relations of opposition of categorical propositions.
18. Between what propositions can the relation of contrariety exist?
19. How do contrary propositions differ from, and in what respects do they agree with, each other?
20. If one of the propositions existing in a contrary relation is true, what can you say of the other?
21. Explain why a proposition is doubtful if its contrary is false.
22. Between what propositions can the subaltern relation exist?
23. How do subaltern propositions differ from, and in what respects do they agree with, each other?
24. Why does the truth of the subalternans involve the truth of the subaltern?

25. Explain why a proposition is doubtful if its subaltern is true.
26. Why is a proposition false if its subaltern is false?
27. Between what propositions can the contradictory relation exist?
28. How do contradictory propositions differ from, and in what respects do they agree with, each other?
29. Explain why the truth or falsity of a proposition involves the falsity or truth of its contradictory.
30. Between what propositions can the subcontrary relation exist?
31. How do subcontrary propositions differ from, and in what respects do they agree with, each other?
32. Why is a proposition doubtful if its subcontrary is true?
33. Explain why a proposition is true if its subcontrary is false.
34. What is the Square of Opposition?
35. What are Eductions?
36. Classify the Eductions.
37. Explain and illustrate the method of obversion.
38. Upon what characteristics of propositions does the method of conversion rest?
39. Explain and illustrate the method of conversion.
40. What is the rule of conversion?
41. Name and illustrate the classes of conversion.
42. Why cannot SoP be converted?
43. Are there any particular negative propositions that can be converted? Illustrate.
44. If your answer to the previous question is in the affirmative, explain why logic says that SoP cannot be converted.
45. What is contraposition?
46. What steps must be taken to secure the contrapositive of a proposition?
47. Why is the contrapositive of a proposition sometimes called the converted obverse?
48. What is inversion?

49. What is the method of finding the inverse of SaP ?
50. How do you proceed to find the inverse of a universal negative proposition?

EXERCISES

NOTE — State propositions in logical form when answering a question, and write the formula for each proposition involved in your answers.

1. Write all the relations of Opposition of:
 - (1) We are not all college professors.
 - (2) Few books in logic are easy reading.
 - (3) The old paths are best.
 - (4) Improbable events happen almost every day.
 - (5) The longest road comes to an end.
 - (6) Unasked advice is seldom acceptable.
 - (7) Only ignorant persons hold such opinions.
 - (8) Infallibility is a myth.
2. Give the converse of:
 - (1) Mercy but murders, pardoning (= if it pardons) those that kill.
 - (2) We cannot all command success.
3. Give the obverse of:
 - (1) Not every advice is a safe one.
 - (2) Rivers generally run into the sea.
4. Give the contrapositive of:
 - (1) No one is altogether reliable.
 - (2) Every mistake is not a proof of ignorance.
5. Give the inverse of:
 - (1) He that bridled his tongue is greater than he that taketh a city.
 - (2) Natives alone can stand the climates of Africa.

CHAPTER X

CATEGORICAL ARGUMENTS.

The Problem of the Chapter. In the previous chapters we have studied mainly the three following problems: (1) the nature of simple assertion or the judgment, (2) the logical expression of these judgments or the proposition, and (3) the problem of logical implication. In the next two chapters, we shall still be occupied with judgments and their propositions; and, although the new problems that are to come before us are more complex than those we have studied, they will appear simpler, because everyone may be presumed to have already some acquaintance with them. The fact of common experience that underlies the topics of these chapters is, that we connect statements of various kinds with one another in numerous ways, and that we do this with a view to making other assertions that, it seems to us, could not be made as surely in any other way. Often the actual statements we make are not debatable, they are readily accepted by the person to whom we are speaking, and may, therefore, be regarded merely as reminding him of certain points of common agreement; and if, beyond this simple function, they have any use, this must be found in what they suggest, and not in what they actually state. In other words, we are accustomed to use certain significant statements for the purpose of

making them the bases of certain other significant statements which we wish the person to whom we are speaking also to accept. Now it is quite obvious that about such a general fact as this we may ask whether there are any rules of procedure that may be followed, if the suggested truth is to be accepted. We may ask, that is, whether there are any serviceable ways of connecting statements when these are used as the basis of others the truthfulness of which we wish particularly to recommend. This is the question we are to consider in this and the following chapters, a question that is technically known as the problem of the syllogism. To state the problem somewhat technically, we are now to enter upon a study of the combination of propositions, and of the laws of their inter-connection.

Two Preliminary Questions. Before taking up the specific topics to which the present chapter is devoted, there are two questions to which some attention must in the first place be given, — a question about the syllogism, and a question about the particular kind of syllogism that we are immediately concerned with. About the syllogism we may ask whether it is a problem that is set by the judgment, by the nature and limitations of human thought, and one, therefore, that reveals the structure of the thinking or knowing mind; or, if this is not the case, whether it is a problem that grows out of our effort to communicate what we know. In the latter case, the syllogism is not a problem that is set directly by the judgment, but one that arises out of the

nature of the proposition. Now it is not possible, nor perhaps is it necessary or desirable, to enter into a full discussion of the question thus suggested; but, because of its bearing upon our understanding of the nature and function of the syllogism, we may point out more specifically that the question really asks whether the syllogism is a method by which the human mind thinks when it is seeking to gain knowledge; or whether it is a method by which we lay bare, for ourselves and others, the grounds on which our knowledge is supposed to rest. That is, we are asking whether the syllogism is, as in the one case, an instrument for acquiring knowledge; or, as in the other, an instrument for the exposition of knowledge. In making this statement disjunctive, we are not to be understood to mean that the syllogism may not serve both functions, nor to deny that the two functions may be inter-dependent. Indeed, what we have seen to be true of the relation of the proposition to the judgment, namely, that the former, being the expression in words of the latter, is controlled by motives that grow out of our desire to communicate what we know, may be expected to be true also of the syllogism — a series of interconnected propositions — in its relation to the more complex process of reasoning. The view that the syllogism is a method by which we expound and communicate the knowledge that is acquired by reasoning seems to have a high degree of probability, and it may be allowed, in the main, to determine our discussions in the following sections. From this

point of view, the syllogism will come before us as a series of problems connected with the nature and connection of propositions, and we must consider the present chapter as a continuation of those immediately preceding it in which we were developing the character of propositional statements.

Turning now to the second inquiry, we may raise the question whether we are to study in this chapter the connection between any kinds of statement, and any kinds of connection between statements. The answer to both parts of the question must be, No. There are forms of statement that, so long as these forms are retained, do not belong to logic at all. We pointed this out above when we were studying imperatives, interrogatives, optatives, and exclamations. There are also, as we have seen, forms of logical statement that are not categorical, and which, as we shall see later, present problems quite distinct from those that are. Such, for example, are the hypothetical and disjunctive propositions. We exclude from the scope of this chapter both classes of statement, and consider only the relationships that hold between categorical propositions. There are also kinds of connection between categorical statements that are not logical in the restricted sense in which we are using the word here. Such, for instance, are the connections between ideas that the psychologist discusses under the head of association. Such, also, are the connections between ideas based upon the mathematical principle of proportion: A is greater than B, B is

greater than C, therefore, A is greater than C. Whatever the logical character of these cases may be, they do not, on any strict interpretation, belong to the present chapter which is confined to a study of the connection of categorical propositions which is illustrated in the Aristotelian syllogism.

Nature of the Syllogism. We may describe a syllogism as a series of three propositions which are related to each other in such a way that the truth expressed in one of them, called the conclusion, depends upon the truth expressed in the other two, called the premises, taken together. Let us illustrate this statement in the following simple case:

All insects are arthropods.

All bees are insects.

All bees are arthropods.

In this syllogism, there are three inter-related propositions, each one of which has been written separately. It is obvious to first inspection that the truth of the last written statement, 'All bees are arthropods,' avowedly rests upon the truth of the other two taken together. That is to say, the truth of every conclusion is a truth that rests upon a connection of facts that it requires two propositions, at least, completely to express.

Our understanding of this type of argument will be facilitated if we learn to think about the syllogism from the standpoint of the conclusion it states, and not from the standpoint of the premises it expresses. The reason for doing so is found in the

nature of the thought processes themselves. In these processes, as we saw when studying the judgment, emphasis is laid primarily upon the answer to the cognitive question, What is it? and only subordinate importance is attached to the method by which the answer is reached. Whether the process of thought is relatively simple or complex, substantive emphasis is laid upon the solutions that are found to the problematic situations that stimulate the judging activity, and it is the function of the conclusions of syllogisms to express such solutions. From this point of view, the premises must be looked upon as expressing the grounds of the fact or truth stated in the conclusion of the syllogism. Indeed, it is only as stating the grounds of what is expressed in the conclusion that the premises are premises at all. In like manner, a conclusion can be a conclusion only if it is regarded as the statement of a grounded assertion. The grounds of assertion may be relatively simple as in our judgments of perception; they may be relatively complex, as in the syllogism; but every judgment is a grounded assertion, and when such assertions are referred to their grounds, when, that is, the reason for holding the assertions is in any way appealed to, such assertions are properly called conclusions. Premises and conclusions are thus correlative terms, and can be understood only from the standpoint of their mutual implication.

Further light is thrown upon the nature of the syllogism if we recall that the mental process under-

lying the syllogism is a process of reasoning. The mental process underlying the proposition, as we have seen, is called a judgment. When, for example, we discern the aspects in which objects differ from, and are like, each other, directly, that is, by having the objects before us and comparing the one with the other, this mental process, involving discrimination, abstraction, and synthesis, is called judging, and the mental attitude in which the results of such a process is asserted is called a judgment. Now the mental process underlying the syllogism is not essentially different from that underlying the judgment. Like the judgment, reasoning rests upon comparison, but unlike it, there is no means of direct comparison of one object with another. In this case, we compare each of the objects we are trying to relate with a third object in order to detect whether they have anything in common with it. That is, we compare objects, through the observed likeness or difference that each has to a third. This is called indirect comparison, and the process of reasoning in which this is carried out is called mediate reasoning. In contrast to the so-called immediate reasoning or judgment, mediate reasoning is a process underlying the Aristotelian syllogism, and we may call the stated results of such a process an argument. It is such an argument that gets expressed in the syllogism.

Structural View of the Syllogism. We may, now, point out certain characteristics of the syllogism that are conditioned by the fact that it is composed

of a series of interrelated propositions. Propositions, for instance, may be said to bear the same relation to the syllogism that terms do to the proposition, that is, they may be regarded as its constituent elements. But as terms do not form propositions by being brought together by the copula, so, we may say, the syllogism is not built up by the union of propositions. The propositions of a syllogism express distinguishable features of a complex process of thinking, and it is the purpose of the syllogism to make clear the inter-connection between the several aspects of our thoughts-about-things. In other words, the movement of thought in syllogistic reasoning turns upon a common element represented in the syllogism by the middle-term, and it is only through this common element that there exist grounds for the truth of any mediated judgment. From this point of view, the middle term is as characteristic a feature of the syllogism as the copula is of the proposition.

Certain other features of the syllogism come into prominence when we consider it from the standpoint of its structure. For instance, every syllogism has three and only three propositions, — the conclusion, and two premises. The conclusion is a statement of the relation between two terms which are named, from the place they occupy in the syllogism, the major and minor terms. The subject (S) of the conclusion is the minor term of the syllogism, and the predicate (P) of the conclusion is the major term of the syllogism. Since now the

premises of a syllogism are propositions, they also state a relationship between terms, but not a relationship between any terms. Their function is to state the relationship that the major and minor terms have respectively to a common or middle term (M). The proposition that states the relationship between the major and middle terms is known as the major premise. The proposition that states the relationship between the minor and the middle terms is known as the minor premise. It should be observed that the nature and validity of categorical arguments do not depend upon the order in which the propositions are named, but upon the connection between the propositions of which they are comprised. It is usual, however, to write the major premise first, the minor premise second, and the conclusion last.

With these facts in mind, it is possible to give a definition of the syllogism that emphasises the connection of the elements that are always present in these kinds of arguments. From this point of view, the syllogism is a method by which we relate two terms (S and P) through the relation that each has to a common or middle term (M).

Rules of the Syllogism. In the previous section, we pointed out that the syllogism has a definite structure which is determined by the fact that its three constituent propositions are related to each other through a middle term. The rules of the syllogism may be classified with respect to these facts,

— with respect to the terms, and with respect to the propositions of the syllogism. With respect to its terms, the rules are definitely rules of distribution. With respect to its propositions, the rules refer to the quantity and quality of the propositions.

RULES OF DISTRIBUTION

I. The middle term must be distributed in one, at least, of the premises.

II. If a term is distributed in the conclusion, it must have been distributed in one of the premises.

The reason for the first rule is, that the validity of the syllogism depends upon the reference of the major and minor terms unambiguously to the same common term, and this is done with certainty only if the middle term is distributed, at least, once. If the middle term is undistributed in each premise, the major term might be related to one part in the major premise, and the minor term to another part in the minor premise. Whenever this is the case, we have four and not three terms in the two propositions, and thus the syllogistic character of the argument is destroyed. For example, from the statements that 'All rash men are confident,' and 'All brave men are confident,' we cannot conclude that 'All brave men are rash,' for the reason that the confidence that each has is or might be a different kind of confidence; there is, in other words, no middle term or common factor to serve as a basis of comparison. If, however, the middle term is distributed in one, at least, of the premises, the part

of the term denoted by the undistributed middle in the other premise is included in this distribution, and there is thus a common basis of comparison for the major and minor terms. Thus, we can affirm that 'No Americans are Englishmen' on the ground that 'All Englishmen are Europeans,' and 'No Americans are.' The fallacy involved in breaking the rule for the distribution of the middle term is called the fallacy of Undistributed Middle.

The reason for the second rule is, that we must not make a statement, in the conclusion, about a determinate number of the objects denoted by a term when, in the premises, our statement has been about an indeterminate number of these objects. In other words, we must not go beyond our accepted facts. Stated conversely, to support a conclusion which denotes, in either of its terms, a determinate number of the objects to which the term applies requires that all these objects shall have been denoted in one of the premises. When this rule is not observed by the major term, the fallacy that results is an Illicit Process of the Major, and when it is not observed by the minor term an Illicit Process of the Minor Term.

RULES OF QUALITY

I. If the conclusion is affirmative, both premises are affirmative; and conversely.

II. If the conclusion is negative, one premise, and only one, is negative; and conversely.

In discussing these rules, which refer to the relation of identity, or of non-identity, of two classes, on the ground of their identity or non-

identity with a third, it is important to observe that we can argue a relation of identity only on the basis of an identity of relation, and a relation of non-identity only on the basis of a non-identity of relation. In other words, the relation of S and P to M must be either so far identical as to warrant the statement of the complete or partial identity of S and P; or so far non-identical as to warrant the statement of the complete or partial non-identity of S and P. If the conclusion is a statement of identity, the grounds for such a statement cannot be found in a relation of non-identity; nor can we find grounds for a statement of non-identity in a relation of identity. Hence, if the conclusion is affirmative both the premises must be of the same quality; and if the conclusion is negative, one of the premises must be of that quality.

Why can we not ground any statement upon two negative propositions? Because, in that case, there is no identity of relation between S and M and P and M, on which to base such a statement. From the exclusion of both S and P from M, we cannot argue to the identity of S and P, nor can we argue to the non-identity of S and P. Two negative premises cannot, therefore, support any conclusion, nor can any conclusion be inferred from them.

RULES OF QUANTITY

I. A particular conclusion cannot be supported by two particular premises.

II. If a conclusion is particular, one, and only one, of the premises supporting it can be particular.

With regard to the first rule, let us suppose that the particular affirmative, and the particular negative, conclusions, SiP and SoP, can be supported by two particular propositions. In the former case, the premises must be both particular affirmative, and then the middle term is undistributed whichever place it occupied in the premises. This would give the fallacy of the undistributed middle. In the second case, one premise must be particular affirmative, and the other particular negative, and then there can be only one distributed term, namely, the predicate of the negative premise. If this is the middle term, the major term will be of wider extent in the conclusion, which is a negative proposition, than in the major premise, — the fallacy of illicit process of the major. If, on the other hand, the middle term is not the predicate term of the negative premise, the argument fails on account of the fallacy of undistributed middle.

With regard to the second rule, the particular conclusion must be either affirmative or negative. If it is affirmative, the premises will both be particular affirmative, II., that is, propositions whose terms are all undistributed. A syllogism whose premises were both particular affirmative would be fallacious on account of undistributed middle. If, on the other hand, the conclusion were particular negative, one of the premises would have to be particular negative, OI, IO, that is, propositions only one of whose terms is distributed. If this were the middle term, then, all the other terms being undistributed, the conclusion being negative would involve

the syllogism in an illicit process of the major; and if that term were not the middle term, the syllogism would involve an undistributed middle. Only one premise, therefore, of a particular conclusion can be particular.

Figures of the Syllogism. The propositions that together constitute the premises or grounds of a syllogism are characterised, as we have seen, by the fact that they have a common or middle term. Now since each premise has two terms, a subject term and a predicate term, and one of these is the middle-term of the syllogism, it is obvious that the middle term may be either the subject term or the predicate term of the major premise, and either the subject term or the predicate term of the minor premise. That is to say, the middle term of a syllogism may occupy four different positions in the propositions which, when taken together, are the premises of the argument which the syllogism is the means of stating. Now the fact that the middle term of a syllogism may occupy different positions in the premises of different arguments is referred to by the term *Figure*. The *Figure* of a syllogism, in brief, is determined by the position of the middle term in its major and minor premises.

The number of *Figures* depends upon the number of different positions that the middle term may occupy alternately in the premises of a syllogism. Theoretically, there are four. Thus:

1. The middle term may be the subject term of the major, and the predicate term of the minor premise.

2. The middle term may be the predicate term of each premise.

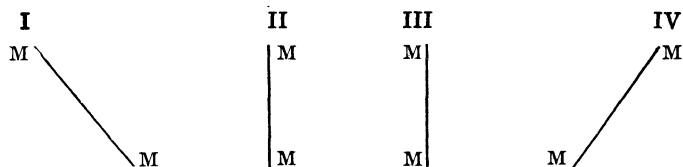
3. The middle term may be the subject term of each premise.

4. The middle term may be the predicate term of the major, and the subject term of the minor premise.

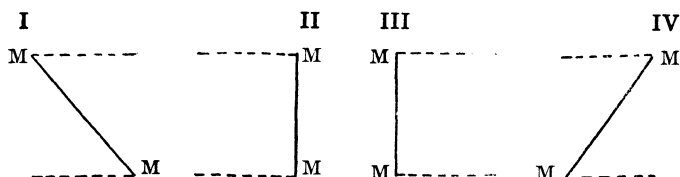
In the order named, the syllogisms whose form corresponds to the alternative possibilities just named are said to be in the First Figure, the Second Figure, the Third Figure, and the Fourth Figure respectively. Written in outline, syllogisms in these Figures will present the following appearance:

First Figure	Second Figure	Third Figure	Fourth Figure
M—P	P—M	M—P	P—M
S—M	S—M	M—S	M—S
<hr/> S—P	<hr/> S—P	<hr/> S—P	<hr/> S—P

As an aid to the retention of the positions of the middle terms in each of the Four Figures, we may add the following suggestion. If we connect by a line the middle terms of the premises in the Four Figures as just shown in outline, we shall get this result:



The upper point of these lines is the position occupied by the middle term of the major premise, and the lower point of the lines is the position occupied by the middle term of the minor premise in the Four Figures respectively. If now to these lines we add dotted lines to represent the major and minor premises, our diagram will assume this aspect:



A diagram like this, of course, may mean something or nothing according as the student can bring to it an interpreting idea. A student informed me that, for a number of a class in logic, the two end figures were thought of as snakes fighting or hissing at each other. The two middle figures were thought of as people sitting back to back because they were cross with each other. As this information came to me two years after the course was taken, it may be presumed to have, for one at least, some mnemonic value.

Special Rules of the Four Figures. Each of the Four Figures has its own special rules, which are applications of the general rules of the syllogism to the specific form that categorical arguments assume in these Figures. We shall state and prove the rules for each Figure in order.

FIRST FIGURE

RULE 1. The minor premise must be affirmative.

Let us suppose the minor is negative. Then,
 M—P the major must be affirmative, and the
 S—M conclusion negative. Therefore, the major
 ——— term will be undistributed in the major
 S—P premise, and distributed in the conclusion;
 illicit process of the major. Hence, the
 minor premise cannot be negative, and must be
 affirmative.

RULE 2. The major premise must be universal.

If the minor premise is affirmative, the
 middle term is undistributed, and must,
 therefore, be distributed in the major
 premise. And since the middle term is
 subject of this premise, the proposition
 must be universal.

SECOND FIGURE

RULE 1. One premise must be negative.

Since the middle term is the predicate of
 P—M each premise, one of the premises must be
 S—M a negative proposition to avoid the fallacy
 ——— of undistributed middle.
 S—P

RULE 2. The major premise must be universal.

The major term being the predicate of a
 negative proposition in the conclusion, is
 distributed. It must, therefore, be dis-
 tributed in the major premise. But since

there it is the subject of the proposition, the major premise must be universal.

THIRD FIGURE

RULE 1. The minor premise must be affirmative.
 Let us suppose that the minor is negative.
 M—P Then, the major premise must be affirmative, and the conclusion negative. Therefore, the major term will be undistributed
 M—S in the major premise, being the predicate
 ——— of an affirmative proposition, and distributed in the conclusion, being the predicate of a negative proposition. This involves illicit major. Hence, the minor premise must be affirmative.

RULE 2. The conclusion must be particular.
 Since the minor premise is affirmative, the minor term, being predicate, is undistributed, and must, therefore, be undistributed in the conclusion also. But the minor term is the subject of the conclusion which, therefore, must be particular.

FOURTH FIGURE

RULE 1. If either premise is negative, the major is universal.
 P—M If either premise is negative, the conclusion is negative, and the major term of the
 M—S conclusion must be distributed. It must,
 ——— therefore, be distributed in the major
 S—P premise. But since there it is the subject, the premise must be universal.

RULE 2. If the major premise is affirmative, the minor must be universal.

If the major premise is affirmative, its middle term is undistributed. It must, therefore, be distributed in the minor premise. And since there the middle term is the subject, the premise must be universal.

RULE 3. If the minor premise is affirmative, the conclusion is particular.

If the minor premise is affirmative, the minor term is undistributed. In the conclusion, therefore, this term must be undistributed. The conclusion, therefore, is particular.

RULE 4. If the minor premise is negative, both premises are universal.

If the minor premise is negative, the minor term is distributed. It must, therefore, be distributed in the conclusion which will then be universal. But a universal conclusion requires two universal premises.

The Moods of the Syllogism. If now we consider the structure of the syllogism from the point of view of the quantity and quality of the propositions which enter into it, we meet with the facts which are denoted by the term — the Moods of the syllogism. By the Moods of the syllogism is meant the character of a syllogism as determined by the quantity and quality of its constituent propositions. There are, as we have seen, four categorical prop-

ositions, SaP, SeP, SiP, SoP, and any combination of three of these, theoretically, may form the constituent propositions of a syllogism. For example, a syllogism may have in its premises and conclusion only universal affirmative propositions. But it may have in its major premise a universal affirmative, in its minor premise a universal negative, and in its conclusion a universal negative proposition. Every syllogism must express itself by the use of the four categorical propositions three at a time, and, therefore, no syllogism can fail to have a Mood. That is to say, each of the constituent propositions of a syllogism has both a quantity and a quality, and when we have named the quantity and quality of each proposition, in the order of major premise, minor premise, and conclusion, we have named the Mood of the syllogism. Thus, in the examples just used, the mood of the first is said to be AAA, that of the second, AEE.

The Valid Moods of the Syllogism. The Mood of a syllogism, we have said, is determined by the quantity and quality of its constituent propositions. The question that now comes before us is, How many of the possible combinations of three out of the four categorical propositions, taken as major and minor premise and conclusion, will give us valid moods, that is, syllogisms whose conclusions logically depend upon the premises on which they are said to rest? In answering this question, it will simplify our task if, leaving the conclusions out of consideration, we ascertain the combinations of propositions

which are possible as premises. Since, as we have seen, there are four categorical propositions, each of these may serve successively as the major premise of a syllogism, and each may serve successively as the minor premise of the same syllogism. Thus, writing the major premise first and the minor second, we shall get the following sixteen combinations:

AA	EA	IA	OA
AE	EE	IE	OE
AI	EI	II	OI
AO	EO	IO	OO

If, now, we examine this list of possible premises, it is apparent that some must be rejected by the rules of the syllogism that we have stated in another section. Certain combinations are disqualified because they violate the rule against negative premises, for example, EE, EO, OE, and OO; certain others must be eliminated because they offend against the rule about particular premises, for example, II, IO, and OI. There remain, after the disqualified members have been eliminated, the following combinations:

AA	EA	IA	OA
AE		IE	
AI	EI		
AO			

These we must consider as the only premises that can be appealed to in support of any conclusion which we may be concerned to maintain.

Relation of Moods to Figure. There is, however, a further question that requires an answer, namely, whether we can throw the defence of a conclusion upon any of the above-mentioned premises in any of the Figures without affecting the validity of the argument. Or, to state the same question conversely, Is each of the above pairs of premises capable of rendering a valid conclusion in each of the Four Figures? We are asking, what effect, if any, the statement of a syllogism in either of the Four Figures has upon the validity of the argument.

There are two methods that may be adopted in seeking an answer to this question. And first we shall examine each of the above combinations of premises in the light of the special rules of each Figure, accepting those that conform, and rejecting those that do not conform to the requirements of the particular Figure.

In the First Figure, the major premise must be universal, and the minor affirmative. The only combinations of premises that stand these tests are, AA, AI, EA, and EI. If we draw the proper conclusion in each case, we may write the four valid moods for this figure as follows:

AAA	AII	EAE	EIO
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It will be observed that in this figure we are able to support by premises a conclusion stated in any one of the four propositions A, E, I, and O.

In the Second Figure, the major premise must be universal, and one premise negative. The only

combinations of premises that satisfy these requirements are AE, AO, EA, and EI. With their respective conclusions, these premises give the following Moods:

AEE AOO EAE EIO

The conclusions in the Second Figure, it will be observed, are negative propositions, either E or O.

In the Third Figure, the minor premise must be affirmative, and the conclusion particular. The following are the combinations of premises in which the minor is affirmative: AA, AI, EA, EI, IA, and OA. From each of these we may draw, as required, particular conclusions, and the valid Moods will be as follows:

AAI AII EAO EIO IAI OAO

In the Fourth Figure, if either premise is negative, the major is universal; if the major premise is affirmative, the minor is universal; if the minor premise is affirmative, the conclusion is particular, and if the minor is negative, both premises are universal. The only combinations of propositions which satisfy these conditions are: AA, AE, EA, EI, IA. To draw conclusions from these premises, we must remember that when, in this figure, the minor premise is affirmative, the conclusion must be particular. The valid moods are:

AAI AEE EAO EIO IAI

In this figure, we may support conclusions stated in each of the four propositions, excepting the universal affirmative.

The second method of determining the valid Moods is to examine each of the combinations of premises in each Figure, and observe whether it satisfies the requirements of a valid argument. Since we have eliminated those combinations of premises which do not meet the rules of quantity and quality as stated above, the validity of the argument will depend upon its satisfying the rules of distribution. We shall give examples below for all the Figures, and the student is urged to work out the entire list. This will be a review exercise, and it will help him to fix definitely the essentials connected with the Figure and Mood of syllogisms, and the distribution of terms.

To begin, we may write the structural formulæ for the Figures in which we are to examine the given combination of premises. Thus:

I	II	III	IV
M—P	P—M	M—P	P—M
S—M	S—M	M—S	M—S
<u>S—P</u>	<u>S—P</u>	<u>S—P</u>	<u>S—P</u>

In the vacant spaces of each premise may be written the combination of premises we are wishing to examine, and in the vacant space of the conclusion, the conclusion that these premises support.

When this has been done, our first question will be: Is the middle term distributed in one, at least,

of the premises? The second question will be: Is each distributed term of the conclusion, when there is one, distributed also in one of the premises?

We give the following examples chiefly as a guide to the student in working out the entire list. AE in all the Figures.

I	II	III	IV
MaP	PaM	MaP	PaM
SeM	SeM	MeS	MeS
<hr/>	<hr/>	<hr/>	<hr/>
SeP	SeP	SeP	SeP

In the First Figure, this argument is invalid because the major term is distributed in the conclusion and is not distributed in the major premise: Illicit process of the major.

In the Second Figure the argument is valid.

In the Third Figure the argument is invalid because the major term is distributed in the conclusion and is not distributed in the major premise: Illicit process of the major.

In the Fourth Figure, the argument is valid.

AI in all the Figures.

I	II	III	IV
MaP	PaM	MaP	PaM
SiM	SiM	MiS	MiS
<hr/>	<hr/>	<hr/>	<hr/>
SiP	SiP	SiP	SiP

In the First Figure, the argument is valid.

In the Second Figure, the argument is invalid because the middle term is not distributed in either premise: Undistributed middle.

In the Third Figure, the argument is valid.

In the Fourth Figure, the argument is invalid because the middle term is not distributed in either premise: Undistributed middle.

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QUESTIONS

1. In what respects are the problems of this and the immediately following chapters related to those of the preceding chapters?
2. What general view of the syllogism is assumed in the discussions of this chapter?
3. What particular forms of statement are and what are not studied in this chapter?
4. Discuss the advantages and disadvantages of viewing the syllogism from the standpoint (1) of its premises and (2) of its conclusion?
5. What mental process underlies the syllogism, and how does it differ from that which underlies the judgment?
6. What do you understand by the major, minor and middle terms of the syllogism?
7. Define the syllogism from the standpoint of its contained terms.
8. Define and illustrate the following: Premise, Conclusion, Major premise, Minor premise, Undistributed middle, Illicit process.
9. With respect to what aspects of its contained propositions are the rules of the syllogism determined?
10. Explain why the middle term of a syllogism must be distributed in at least one of the premises.
11. Why is there no separate rule relating to a term in the conclusion that is undistributed?
12. Why must the premises of an affirmative conclusion be affirmative?
13. Explain why a negative conclusion can be supported only if but one of its premises is negative.
14. Why cannot a particular conclusion be supported by two particular premises?
15. What is meant by the Figure of a syllogism?
16. Describe the formal characteristics of each of the Figures.
17. What do you understand by the Special Rules of the Four Figures?

18. What is meant by the Moods of the syllogism?
19. How do you determine all the theoretically valid Moods of the syllogisms?
20. What is the relation of the Moods to the Figures of the syllogism?
21. What two methods may be adopted to determine whether a particular argument is valid in a given Figure?

EXERCISES

NOTE — Write the formula for each proposition and syllogism in your answers to the following questions.

1. Point out (1) the conclusion, (2) the middle term, (3) the major term, (4) the minor term, (5) the major premise, (6) the minor premise in the following arguments:
 - (1) The receiver of stolen property should be punished; you have received stolen property, and, therefore, should be punished.
 - (2) Theft is crime; theft was encouraged by the laws of Sparta, therefore, the laws of Sparta encouraged crime.
 - (3) Only those messages which have been prepaid will be delivered; this message has been prepaid, and, therefore, will be delivered.
2. Name all the rules of the syllogism which are broken by each of the following Moods:

AIA, EEI, IEA, IOI, IIA, AEI.
3. Prove from the rules alone, irrespective of Figure, that IE can yield no valid conclusion.
4. In what Figures do the following premises yield a valid conclusion:

AA, AI, EA, OA.
5. Why cannot a particular negative proposition stand (1) as a premise in the First, (2) as a major in the Second, (3) as a minor in the Third, or (4) as a premise in the Fourth Figure?

6. Arrange the following arguments in the order of major and minor premise, and conclusion, and determine their Figure, Mood, and validity:
 - (1) All colours are physical phenomena, but no sounds are colours, and therefore no sound is a physical phenomenon.
 - (2) Whatever is given on the evidence of sense may be taken as a fact; the existence of God, therefore, is not a fact, for it is not evident to sense.
 - (3) Every candid man acknowledges merit in a rival; every learned man does not do so; therefore, every learned man is not candid.
7. What can be determined respecting a syllogism under each of the following conditions:
 - (1) That only one term is distributed, and that only once.
 - (2) That only one term is distributed, and that twice.
 - (3) That two terms only are distributed, each only once.
 - (4) That two terms are distributed, each twice.

CHAPTER XI

OTHER FORMS OF CATEGORICAL ARGUMENT

Their Relation to the Syllogism. The syllogism may be considered from either of two points of view, from the point of view of its premises, or from the point of view of its conclusion. If we attend particularly to the premises, we shall think of the syllogism as a method by which from given data we arrive at a conclusion; if we attend particularly to the conclusion, we shall think of the syllogism as a method by which we state the grounds on which a position already arrived at is held to rest. These two ways of regarding the syllogism are not contradictory, nor is our view of the nature of the syllogism affected by the possibility of this two-fold approach to its problems. There results only a slightly different emphasis in one's exposition; in the one case, the conclusion, in the other, the premises, come to be the focus of our logical interest.

The reason why we may alter, in the way just indicated, our exposition of the syllogism is that the syllogism does not express a series of consecutive steps by which the judgment actually proceeds forward from data to conclusion, or backward from conclusion to grounds; but it represents, in a conventional form and order, the simplest inter-connections of a complex process of reasoning. It is sometimes brought forward as an objection to syllogistic

reasoning that it is not an instrument of the discovery of truth; but the objection rests upon a misunderstanding. The truth about any subject-matter is discovered by thinking, and the syllogism is not, and does not affect to be, a substitute for thinking. We may even say that unless truth already exists, unless some kind and degree of thinking has taken place, the syllogism has no function to perform. The syllogism presupposes the existence of truth, and is not the condition of its existence. It represents the results of our reflection upon the truth that is already in our possession, — a reflection that is directed towards finding out the reasons why truth assumes a demonstrative character. Now this is not a small nor an easy thing. It is, however, an eminently practical thing. For when thought takes itself to task, and, by reflection upon its own processes, demands that the conditions of truth be erected into principles of demonstration, we may be led to see how far from being demonstrative are many of the beliefs that we hold most confidently.

These remarks have a practical bearing upon the subjects of this chapter, and may prepare us to understand why our thoughts-about-things can be thrown into a variety of forms which conform in principle only to the Aristotelian syllogism. The discussion of these arguments should lead us to see that even in logic we are not bound in the statement of our thought to the formal primness of the three-term syllogism.

Enthymeme. This is the name of a categorical argument which omits, in statement, one of the constituent factors of the syllogism. Now since there are three propositions, two premises and a conclusion, in a categorical syllogism, the omission of either one of these from the syllogism results in an enthymeme. But it must not be supposed that the omission of one member of a syllogism destroys the syllogistic character of the reasoning, for, as the term itself implies, the omitted member still functions *in the mind* of the person who states his thought in this particular form. The enthymeme, consequently, is a particular way of stating a categorical argument, it is not a new form of categorical syllogism.

According as the omitted member of the syllogism is the major premise, the minor premise, or the conclusion, the resulting enthymeme is said to be of the First Order, the Second Order, and the Third Order respectively. An enthymeme of the First Order, consequently, is a syllogism which is abbreviated by the omission of the major premise. For example, 'Those who have no occupation have nothing in which to interest themselves, and are, therefore, unhappy.' An enthymeme of the Second Order is a syllogism which is abbreviated by the omission of the minor premise. For example, 'Those who have no occupation are unhappy, because they have nothing in which to interest themselves.' An enthymeme of the Third Order is a syllogism which is abbreviated by the omission of the conclusion.

For example, 'Those who have nothing in which to interest themselves are unhappy, and those who have no occupation have nothing in which to interest themselves.'

In the enthymeme, as in the fully expressed syllogism, the order in which the propositions are stated is wholly a matter of convenience, and does not affect the nature or validity of the argument. In case of the enthymeme, however, it must be made obvious by the use of such words as 'and,' 'because,' and 'therefore,' which members of the syllogism we are undertaking to express.

It may be questioned whether the enthymeme has both Mood and Figure. Perhaps the best answer to this inquiry would be that a syllogism in any Figure and Mood may be stated as an enthymeme by the omission of one of its constituent propositions. But since Mood and Figure are characteristics of arguments only when fully expressed in the three-term syllogism, the most that we can say is that an enthymeme implies a Figure and Mood, and not that it has either.

Polysyllogism. In the enthymeme, we have an example of the elliptical character of thought. Thinking, indeed, is ordinarily carried on by means of the smallest number of factors necessary to its success definitely present to consciousness. When the thought is a simple one, and the inter-connection of its parts consists of a relation of three terms, any one of the relations may fall into the background without invalidating or making doubtful its truth.

This is, as we have seen, the case of the enthymeme. When, however, thinking becomes complex, and the inter-connection of its parts involves relations of many terms, it is much more necessary to simplify our methods, if thinking is not to break down under the weight of the terms into which the subject-matter of thought is analysed. In this case, what we do is to build up a series of enthymemes or abbreviated syllogisms, and such a series is called a polysyllogism. A polysyllogism is, therefore, a series of inter-connected enthymemes representing a continuous chain of reasoning.

It is customary to denote the syllogisms into which a polysyllogism may be analysed by the terms *prosyllogism* and *episylogism*. The application of these terms depends entirely upon whether we are thinking of the constituent syllogisms in the forward direction, that is, of the first syllogism as being connected with the second, and the second with the third, etc.; or whether we are thinking of them in the reverse order, that is, of the second syllogism as being connected with the first, the third with the second, etc. When we think of the constituent syllogisms in the former way, each syllogism is called a *prosyllogism*; and when we think of them in the latter way, each syllogism is called an *episylogism*. The terms *prosyllogism* and *episylogism*, consequently, are used relatively, and any syllogism in a train of reasoning may be both a *pro*- and an *epi*syllogism. It is obvious that the first

syllogism in such a series can be only a prosyllogism, and the last only an episyllogism.

Types of Polysyllogism Trains of reasoning of the type just described fall into two main classes, progressive and regressive. We shall describe each of these briefly, and merely for the purpose of showing their logical structure.

A progressive, synthetic, or episyllogistic train of reasoning proceeds from the premises of a prosyllogism to the conclusion of an episyllogism. A polysyllogism of this character develops progressively the consequences of the premises which form its starting point. Stated otherwise, the progressive polysyllogism advances from the conditions to what is conditioned, from causes to effects. Probably the most familiar illustrations of this class of polysyllogism for the student are to be found in the direct proofs of geometrical propositions. A regressive, analytic, or prosyllogistic train of reasoning proceeds from the conclusion of an episyllogism to the premises of a prosyllogism. In such cases we go back from a conclusion to show the grounds upon which it rests. Stated otherwise, we are said to reason from the conditioned to its necessary conditions, from effects to causes.

Illustrations of these two classes of polysyllogisms will be found in the following sections in which we consider the various forms that such trains of reasoning may assume. In this and the preceding section we have been considering the general characteristics of a group of syllogisms; in the sections

that follow we are to study the cases that fall under one or other of the two main classes of the group.

Sorites. This is a polysyllogism whose constituent syllogisms are all in the First Figure, and from which all the conclusions, except the last, are omitted. Or, since a syllogism which omits one of its propositions is an enthymeme, we may regard a sorites as a series of enthymemes from each of which, except the first, one premise is omitted, and from each of which, except the last, the conclusion is omitted.

There are two forms of sorites, progressive and regressive, which are usually distinguished as the Aristotelian and the Goclenian respectively. An examination of each of these will serve to set the nature of this class of arguments more clearly in the light.

An Aristotelian sorites is a progressive series of enthymemes in which the omitted conclusion of each prosyllogism forms the minor premise of the succeeding episyllogism. A familiar example may be taken from Rom. VIII., 29, 30: 'For whom he foreknew, he also foreordained to be conformed to the image of his Son, . . . and whom he foreordained, them he also called; and whom he called, them he also justified: and whom he justified, them he also glorified.' A Goclenian sorites—so called from Professor Goclenius of Marburg (1547-1628) its discoverer—is a regressive series of enthymemes in which the omitted conclusion of each prosyllogism forms the major premise of the succeeding episyl-

logism. For example: 'One must take pains if he is to be a successful man; industry is necessary to taking pains; health is necessary to industry; a regulated diet and sufficient recreation and rest are necessary to health; and, therefore, a regulated diet and sufficient recreation and rest are necessary if one is to become a successful man.'

The symbolic expression and analysis of each of these forms of sorites may be given thus:

ARISTOTELIAN SORITES

S is A
A is B
B is C
C is P

S is P

GOCCLENIAN SORITES

Z is P
Y is Z
X is Y
S is X

S is P

Analysis:

$$\left\{ \begin{array}{l} S \text{ is } A \\ A \text{ is } B \\ (S \text{ is } B) \\ B \text{ is } C \\ (S \text{ is } C) \\ C \text{ is } P \\ S \text{ is } P \end{array} \right\}$$

$$\left\{ \begin{array}{l} Z \text{ is } P \\ Y \text{ is } Z \\ (Y \text{ is } P) \\ Z \text{ is } Y \\ (X \text{ is } P) \\ S \text{ is } X \\ S \text{ is } P \end{array} \right\}$$

Each of these forms is subject to special rules which we shall proceed to state and prove.

THE SPECIAL RULES OF THE ARISTOTELIAN SORITES

RULE 1. Only one premise, and that the last, can be negative.

Let us suppose that two of the premises are negative. Then the conclusion of the syllogism in

which the first negative premise occurs will be negative. But since this negative conclusion is necessarily a premise of the next succeeding syllogism, its conclusion will be negative, and so on, until a negative conclusion enters as a premise into a syllogism which has the second negative premise. But as we have seen, from two negative premises no conclusion can be inferred. Only one premise, therefore, can be negative.

This negative premise must be the last. Let us suppose that some other premise is negative. In that case, the conclusion of the syllogism into which it entered would be negative. This conclusion, serving as the minor premise of the following episyllogism, would give a conclusion that is negative, that is, one in which the major term is distributed. But since the major term in the affirmative major premise is undistributed, the argument would fail on account of an illicit process of the major. No premise, therefore, except the last can be negative.

RULE 2. Only one premise, and that the first, can be particular.

Since every premise, except the last, must be affirmative, it is evident that the conclusions of the constituent syllogisms, except the last, must be affirmative. If now the first premise is particular, the conclusion of the first prosyllogism will be particular. And if any other premise, except the first, were particular, there would occur somewhere in the series two particular affirmative premises, and this,

in the First Figure, involves the fallacy of undistributed middle.

THE SPECIAL RULES OF THE GOCLENIAN SORITES

RULE 1. Only one premise, and that the first, can be negative.

If more than one premise were negative, one of the constituent syllogisms would be involved in the fallacy of negative premises. And since, if any premise is negative, the conclusion must be negative, that premise must be the one which contains the predicate term of the conclusion, that is, the first premise in this particular case.

RULE 2. Only one premise, and that the last, can be particular.

If any premise is particular, the conclusion of the syllogism in which it occurs would be particular; and if this conclusion became a premise of another syllogism whose premise was already particular, there would be involved, in the First Figure, the fallacy of undistributed middle. Only the last premise, therefore, can be particular.

Epicheirema. This is a regressive chain of reasoning in which one, or both, of the premises is expanded by the addition of a reason. In other words, it is an argument in which one of the premises is an enthymeme, but in which both may be. If only one of the premises is supported by a reason, the epicheirema is called single, and when both are so supported, double. A single epicheirema, there-

fore, is a regressive chain of reasoning in which one of the premises is supported by a reason. For example:

All rational beings are to be treated with respect, inas-
much as they are made in the image of God.

Slaves are rational beings.

Therefore, slaves are to be treated with respect.

—*Father Clarke.*

A double epicheirema is a regressive chain of reasoning in which both premises are supported by a reason. For example:

All Malays are cruel, because all savages are.

All the aboriginal inhabitants of Singapore are Malays,
because all the natives of that part of Asia are.

Therefore, all natives of Singapore are cruel.

—*Welton.*

The formulæ for these two kinds of epicheirema may be written as follows:

SINGLE	DOUBLE
Every M is P, because it is X	Every M is P, because it is X
Every S is M	Every S is M, because it is Y
Every S is P	Every S is P

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QUESTIONS

1. What characteristic of human thinking underlies the forms of argument studied in this chapter?
2. What is an Enthymeme?
3. Name and illustrate the distinct types of Enthymeme.
4. How do we know whether any expressed proposition of an Enthymeme is major or minor premise or conclusion?
5. Do Enthymemes have Figure and Mood? Explain.
6. What is a Polysyllogism?
7. Under what conditions does human thinking assume a polysyllogistic form?
8. Explain and illustrate the meaning of the terms Prosyllogism and Episyllogism.
9. Describe and illustrate the general characteristics of a Progressive Polysyllogism. By what other names is it known?
10. Describe and illustrate the general characteristics of a Regressive Polysyllogism. By what other names is it known?
11. What is a Sorites?
12. Describe and illustrate the character of an Aristotelian Sorites. Write also its structural formula.
13. Describe and illustrate the character of a Goelenian Sorites. Write its structural formula.

14. What is an Epicheirema?
15. Explain and illustrate each type of Epicheirema, and write the formula.

EXERCISES

NOTE — Write the formula for each of the propositions and syllogisms contained in your answers to the following questions.

1. What is the logical character of the following arguments:
 - (1) He must have committed the murder, for he was the only person present with the deceased at the time.
 - (2) Nations are justified in revolting when badly governed, for every people has a right to good government.
 - (3) Death cannot be an evil, for it is universal.
 - (4) The planet Mars resembles the earth in possessing atmosphere, water, and moderate temperatures, and these conditions on the earth are necessary to its being inhabited.
 - (5) He must be mad to do such a thing.
2. What is the logical character of the following arguments:
 - (1) Misfortunes sometimes are circumstances tending to improve character, circumstances tending to improve character are promoters of happiness, what tends to promote happiness is for the general well-being, therefore, misfortunes sometimes promote general well-being.
 - (2) A wise man is one who seeks to gain knowledge, the man who seeks to gain knowledge must be industrious, the industrious man has to make sacrifices, to make sacrifices is always a hardship, and, therefore, a wise man always lives a life of hardship.
3. Prove that in an Aristotelian Sorites only the last premise can be negative, and only the first particular.

4. Construct an Aristotelian Sorites with a particular negative conclusion.
5. Prove that in a Goelenian Sorites only the last premise can be particular and only the first negative.
6. Construct a Goelenian Sorites with a particular negative conclusion.
7. Give one example of each class of Epicheirema.

CHAPTER XII

DISJUNCTIVE AND HYPOTHETICAL PROPOSITIONS AND ARGUMENTS

The Disjunctive Proposition. We have already distinguished the various types of logical proposition, and discussed their relation to each other. We have seen that in the categorical proposition P is affirmed or denied of S simply. The disjunctive proposition, on the other hand, is the expression of an alternative predication. In the disjunctive proposition, the character of S seems to waver between a number of alternative possibilities, P or Q or M, and we are not able to determine which of these really, or for our purpose, belongs to S. 'S is either P, or Q, or M,' is a typical disjunctive proposition.

Before proceeding to the discussion of the disjunctive syllogism, there are several questions about the disjunctive proposition which require attention. And, first, we may note that the alternatives which form the predicates of such a proposition, are not any alternatives, but are usually alternatives that have something in common. We have met the converse of this statement in the chapter on division, where we saw that the species of a genus were to be understood as a series of alternatives which had a common nature. What we mean to express in our disjunctive propositions, is, therefore, not unfamiliar; and the formal statement 'S is either P or Q

or M' may be interpreted to mean that P and Q and M are predicates which are, or may be, contained under the wider predicate R which can be asserted of S. For example, 'He is either a lawyer, a doctor, a teacher, or a clergyman,' may be expressed in the simpler statement, 'He is a member of a learned profession.' The alternatives of disjunctive propositions are usually of this class, and fall within a single domain or universe of discourse. They conform, as do all other statements, to the limitations of relevant assertion. But whether every instance of alternative predication is actually thought of in its relation to a limiting universe is what we cannot affirm. All we hold is that logical alternatives imply a common basis, and that it is this common basis that gives to disjunction its distinctive meaning.

Second, the question whether the alternatives of a disjunction should be treated as mutually exclusive or not has aroused a good deal of discussion, and there seems to be no simple answer to it. The meaning of the question may be made plain if we give it a formal expression. When it is said that 'S is either P or Q or M,' is it necessarily implied that S cannot be both P and Q and M? There can be no doubt that in a great many instances the alternative predicates do, as a matter of fact, exclude each other. For example, we may say that 'Plato was born in either 429 or 427 B. C.' About the insurrection in Mexico we may say, 'It will either succeed or be crushed.' Some logicians maintain that this is the only sense in which disjunctive predica-

tion can be understood in logic, and that the instances in which this does not occur are due, as Bradley says, to "our slovenly habits of expression and thought," and are "no real evidence against the exclusive character of disjunction." On the other hand, attention may be called to the fact that we do express our meanings in disjunctive forms which cannot or need not be interpreted in this exclusive manner. For example, 'He is either a fool or a knave' may mean that he is the one or the other, but it is quite possible for a knave to be a fool, and a fool, a knave. In such cases as this, where the truth-import and the statement-import are not necessarily the same, we are following the line of least error if we choose the less determinate meaning, in the absence of any indication that the more determinate meaning is the one intended. What the statement 'S is either P or Q or M,' therefore, means is that S cannot be neither P nor Q nor M; it must be one, and, unless otherwise indicated, it may be more than one. We must be guided in our interpretation, not by the form of the proposition, but by the nature of its subject-matter.

Third, the series of alternatives that constitute the predicates of disjunctive propositions must be exhaustive. That is, they must determine completely the alternative possibilities in the given instance. If we say 'The water is either hot or cold,' the statement is misleading, if we assume that all the alternatives of the disjunction are explicitly stated. But if we say 'The water is either

hot or cold or of some mean temperature,' the statement is true, and, in principle, exhaustive. It would seem, therefore, that to secure an exhaustive enumeration of alternatives were more fundamentally important than to make them exclusive; and in any case we can say that the latter is sometimes impossible where the former presents no particular difficulty.

The Nature and Classes of the Disjunctive Syllogism. The syllogism whose major premise is a disjunctive proposition is called a disjunctive syllogism. The significance of such a syllogism consists in the fact that conclusions may be inferred from premises that are indeterminate in content; or, that the truth of any proposition may rest upon a number of conditions, one or more of which may be undetermined in character.

We may distinguish two classes of disjunctive argument, according as the nature of the subject of the disjunctive major premise requires two, or more than two, terms to state exhaustively the alternative possibilities. If the major premise of a disjunctive argument has two alternatives in its predicate, the argument is called a simple disjunctive syllogism; if the predicate term of such an argument presents more than two alternatives, the argument is called mixed. The reason for this distinction is that the character of the other two propositions of the syllogism is affected by the considerations to which we have adverted. For in such arguments as we are at present concerned with, it

is evident that we can only affirm or deny one or more of the alternatives presented by the major premise, such affirmation or denial giving us the minor premise; and deny or affirm the remaining alternative, or alternatives, such denial or affirmation giving us the conclusion of the syllogism. Thus, the general rule to which disjunctive arguments must conform is that to affirm or deny one or more of the members of a series of alternatives is to deny or affirm the remaining member or members. Now if the disjunctive major premise presents only a pair of alternatives, the minor premise and conclusion of the syllogism are both categorical propositions; there is, in other words, only one disjunctive premise, and that the major. Such an argument is called a simple disjunctive syllogism. If, on the other hand, there are more than two alternatives in the disjunctive major premise, one, or both, of the other propositions of the syllogism must be disjunctive also. In such a case, we call the argument a mixed disjunctive syllogism.

The Moods of the Disjunctive Syllogism. Each class of disjunctive argument has two moods, depending upon whether the minor premise affirms or denies one or more of the alternatives presented by the major premise. The names of these moods, like those that we shall meet with in our study of the hypothetical syllogism, are derived from the Latin words which denote the quality of the propositions which constitute the minor premise and conclusion respectively. For example, if the minor premise is

negative, and the conclusion is affirmative, the mood of the syllogism is called the *Modus Tollendo Ponens*, that is, the mood which by denying affirms. If, on the other hand, the minor premise is affirmative, and the conclusion is negative, the mood is called the *Modus Ponendo Tollens*, that is, the mood which by affirming denies. These statements will be understood if read in connection with the formulæ for the two classes of disjunctive syllogism in each of these moods which follow:

MOOD	DISJUNCTIVE SYLLOGISM	
	SIMPLE	MIXED
Modus Ponendo Tollens	A is either B or C.	A is either B or C or D.
	A is B.	A is B.
	A is not C.	A is not either C or D.
Modus Tollendo Ponens	A is either B or C.	A is either B or C or D.
	A is not B.	A is not B.
	A is C.	A is either C or D.

The Validity of Disjunctive Arguments. A further question relates to the validity of these moods. We have seen that disjunctive propositions are required to state their alternatives exhaustively, that is, to enumerate alternately all the possibilities presented by the subject-matter under consideration. We have also seen that in some cases the alternatives are at the same time exclusive, that is, incompatible. Thus, 'A is either B or C' may mean (1) 'A is either B or C, but not statedly both;' or it may mean (2) 'A is either B or C, but statedly not both,'

that is, the equivalent of 'A is either B or else C.' Now in whichever way we understand the disjunctive proposition, an argument based upon it is valid in Modus Tollendo Ponens, because all disjunctive arguments imply the presence in the subject of one or more of the alternatives specified in the predicate of the major premise. That is to say, the subject cannot be neither of the predicated alternatives. The meaning of the subject professedly is to be found within the number of enumerated predicates. Hence, whether the predicates are exclusive or not, to deny one or more of a subject is to bring the subject nearer to one or more of the remaining predicates. That is, the Modus Tollendo Ponens emphasises the exhaustive character of the disjunctive major. On the other hand, the Modus Ponendo Tollens is valid only when an exclusive reading of the major is understood. For example, when we say of any one that 'He is a fool or a knave,' we cannot infer that because 'He is a fool,' that 'He is not a knave,' unless it is distinctly understood that the individual case precludes the possibility of him being both. Thus, if we wish to state this meaning unambiguously, we should have to say 'He is either a fool or else a knave,' thereby implying that he is not both together. When, therefore, the disjunctive major takes the form of 'statedly not both or all the predicates,' the Modus Ponendo Tollens is a valid mood for disjunctive arguments. This mood emphasises the exclusive character of the disjunctive major.

The Hypothetical Proposition. Any proposition which states a connection between two possibilities is hypothetical. For example, 'If water is heated at standard pressure, it will boil at 100° C.' There are two characteristics of a hypothetical proposition to which attention should be given. The first is that it is the form of statement in which we express our judgments when the material of these judgments is possible, not actual. This does not mean that the material dealt with may not be actual, it means only that we are not concerned with it as existing, but with the situations in which it is possible for it to exist. Thus, to take the judgment which is expressed in the proposition 'If eggs are scarce, they will be dear.' In this proposition we are not asserting that 'eggs are scarce,' nor are we affirming that 'they are dear;' we are merely stating that 'when-ever eggs are scarce, they are at the same time dear.' In other words, we are stating a connection between 'it is possible for eggs to be scarce,' and 'it is possible for them to be dear.'

The second characteristic of hypothetical propositions relates to the kind of connection that they affirm. This may be brought out if we regard the judgments underlying hypothetical statements to be concerned with the development of the consequences that follow from a supposition. What leads to the erection of a supposition is, doubtless, some feature of an actually existing situation which cannot be brought under direct observation; but the supposition itself is not part of the situation in the same

sense that the feature itself is. Any supposition is a suggestion which is based upon a wider range of experience than the circumstances which calls it forth, and we fall back upon this broader experience to guide us in dealing with a situation that is problematical. Now problematic situations can be developed only in a series of possibilities, and possibilities are expressed in hypothetical propositions: 'If A is B, it is C.' The part of the hypothetical proposition which states the supposition or condition is called the antecedent, the part which states the result of the supposition is called the consequent. The connection which is thus affirmed by hypothetical propositions is a relation of dependence between antecedent and consequent, such that the truth of the consequent follows necessarily upon the establishment of the antecedent. The hypothetical proposition thus states a rule under which we may bring for testing any fact or situation about which we are at all doubtful. It should be observed that it is the nature of the thought expressed, and not the form of the expression that constitutes any proposition hypothetical. While usually we introduce hypothetical statements with 'if,' they may be prefaced by 'granted that,' 'suppose that,' 'allowing that,' 'whenever,' 'wherever,' and sometimes there is no verbal sign to indicate that our thought is hypothetical, as, for example, 'Trespassers will be prosecuted,' 'Deserters will be shot on sight,' etc.

The Hypothetical Syllogism. Any syllogism in which the major premise is a hypothetical proposi-

tion is called a hypothetical syllogism. In hypothetical arguments the minor premise and the conclusion are both categorical propositions. Now since the major premise states a connection between two possibilities, the minor premise must state something about one or other of these possibilities categorically if there is to be an inference. The only things that can be stated are that either of the possibilities has or has not been verified. Theoretically, this would give four kinds of hypothetical syllogism, the minor premises of which would affirm and deny separately the antecedent and consequent of the major premise. Two of these, however, fail to give the proper basis for an inference, namely, when the minor premise denies the antecedent, and when it affirms the consequent. The reason for the failure is that we are in each case required to go beyond what is contained in the major premise. For example, when we say 'If A is B, it is C,' we are not to be understood to mean that the only condition of 'A being C' is that 'A is B.' We said above, for instance, that 'If eggs are scarce, they are dear;' but, of course, they may be dear for other reasons as well, — a corner on the market by the cold storage interests, for example. Hence, if we were to deny the antecedent of this proposition, 'Eggs are not scarce,' and use this as a minor premise, we could not infer that 'they are not dear.' The antecedent of a hypothetical proposition states a condition, but does not offer itself as the sole condition of the truth of the consequent. If, on the other hand,

we were to affirm the consequent, 'Eggs are dear,' we cannot infer that 'they are scarce,' and the reason is the same. The forms of the syllogism are reduced, therefore, to two, in one of which the minor premise affirms the antecedent of the major, and in the other the minor premise denies the consequent of the major. In reference to these forms of the syllogism, we may state the general rule of hypothetical arguments as follows: When two possibilities are related as antecedent and consequent, the truth of the consequent follows from the truth of the antecedent, and the falsity of the antecedent follows from the falsity of the consequent.

The Moods of the Hypothetical Syllogism. Corresponding to these two forms of hypothetical argument we may distinguish two moods of the syllogism. There is first the argument in which the minor premise affirms the antecedent of the major premise, — the *Modus Ponens*; and, second, the argument in which the minor premise denies the consequent of the major premise, — the *Modus Tollens*.

The antecedent and consequent of the major premise of any hypothetical syllogism may be either affirmative or negative. In each of the two moods, therefore, we may distinguish four forms of hypothetical argument as follows: (1) the antecedent and consequent of the major premise may both be affirmative; (2) the antecedent may be affirmative, and the consequent negative; (3) the antecedent may be negative, and the consequent affirm-

ative; (4) both antecedent and consequent may be negative. It will be evident that arguments in these moods will have sometimes an affirmative and sometimes a negative proposition in the minor premise, and the same will be true of the conclusion. The names of the several forms of the moods of hypothetical syllogisms, like those of the disjunctive syllogism, are derived from the Latin words which describe the quality of the propositions which constitute the minor premise and the conclusion. We shall briefly characterise each mood and give examples and formulæ for the different forms of the two moods.

1. *Modus Ponens*,—the mood in which the minor premise affirms the antecedent of the major premise, and the conclusion affirms the consequent. Arguments in this mood are said to be *Constructive*. The validity of arguments in this mood is based upon the fact that we are bound to accept whatever follows from any data we have accepted. If, for example, we accept as a possibility a fall in temperature, we are bound to accept whatever is conditioned upon this possibility, say, colder weather. We may, therefore, construct a hypothetical argument in this mood as follows:

A fall in temperature means colder weather.
The temperature is falling.
Therefore, the weather will be colder.

The different forms of the Modus Ponens are given here with illustrations of each form.

(1) Modus Ponendo Ponens, — the mood which by affirming affirms.

If A is B, it is C. If any county increases in wealth,
it increases in power;

A is B, The United States is increasing in
wealth,

A is C, Therefore, the United States is in-
creasing in power.

(2) Modus Ponendo Tollens, — the mood which by affirming denies.

If A is B, it is-not C. If any import duty is imposed
simply for revenue purposes, that
duty is not protective;

A is B. The English import duties are im-
posed simply for revenue purposes;

A is-not C. Therefore, they are not protective.

(3) Modus Tollendo Ponens, — the mood which by denying affirms.

If A is not B, it is C. If any swan is not white, it is
black;

A is-not B. Australian swans are not white,

A is C. Therefore, they are black.

(4) Modus Tollendo Tollens, — the mood which by denying denies.

If A is-not B, it is-not C. If any war is not defensive,
it is not just;

- A is-not B. The wars waged by Napoleon the Great were not defensive,
 A is-not C. Therefore, they were not just.

2. **Modus Tollens**,—the mood in which the minor premise denies the consequent of the major premise, and the conclusion denies the antecedent. Arguments in this mood are said to be Destructive. The validity of arguments in this mood follows from the relations expressed by the major premise. For if B follows of necessity from the existence of A, — ‘If A then B,’—the failure of B to appear must mean the absence or non-existence of A. An illustration of an argument in this mood is:

If no men were mad, asylums would be useless,
 Asylums are-not useless,
 Therefore, some men are mad.

The different forms of the **Modus Tollens** are given here with illustrations of each form.

(1) **Modus Tollendo Tollens**, — the mood which by denying denies.

If A is B, it is C. If any country is civilised, it has a population amongst whom education is general;

A is-not C. The people of Russia are not generally educated;

A is-not B. Therefore, Russia is not a civilised country.

(2) **Modus Ponendo Tollens**, — The mood which by affirming denies.

If A is B, it is not C. If any social institution is justifiable, it oppresses no class of the community;

A is C. Slavery does oppress a class of the community,

A is-not B. Therefore, it is not justifiable.

(3) **Modus Tollendo Ponens**, — the mood which by denying affirms.

If A is-not B, it is C. If any railroad is not required in the district through which it runs, it is a financial failure;

A is-not C. The great American lines are not financial failures,

A is B. Therefore, they are required in the districts through which they run.

(4) **Modus Ponendo Ponens**, — the mood which by affirming affirms.

If A is-not B, it is-not C. If any country has no capital invested abroad, its imports will not exceed its exports;

A is C. England's imports do exceed her exports,

A is B. Therefore, England has capital invested abroad.

Dilemmas. We come now to the study of dilemmas, a class of arguments which combines in various ways characteristics of the three kinds of proposi-

tion already studied, the categorical, the disjunctive, and the hypothetical. The peculiar feature of a dilemmatic argument is that it presents a choice between two alternatives. When used for rhetorical purposes these alternatives are of such a kind that, whilst one of them must be accepted, both lead to disagreeable results. To force by argument the acceptance of one or other of a pair of alternatives is the chief function of the dilemma, and as the result of such an argument an opponent is said 'to be on the horns of a dilemma.' Arguments in which the choice rests among three alternatives are not dilemmas in the strict definition of the term, but trilemmas, just as those which force a choice among four alternatives are tetralemmas. The principles of all these arguments, however, are the same, and it will be sufficient to study them in connection with the simplest forms, — the dilemmas.

The dilemma may be defined as a syllogism with a compound hypothetical major premise and a disjunctive minor. By a compound hypothetical proposition is meant one in which there is a plurality of terms in either the antecedent or consequent, or in both. The simplest form of hypothetical proposition, we have seen, is 'If A is B, it is C.' Such a proposition may become compound by adding terms to antecedent and consequent. 'If A is B, it is C or D,' is a compound hypothetical proposition, and we must understand it to mean 'If A is B, it is C' and 'If A is B, it is also D.' That is to say, there is no true disjunction in this case, but only an ab-

breviated expression of two simple hypothetical propositions. The same remarks hold good also of the other forms that compound hypothetical propositions may assume; for example of, 'If A is B or C, it is D;' 'If A is B or C, it is D or E.' Propositions of this type form the major premises of dilemmatic arguments. The reason why propositions of this type constitute the major premise of dilemmatic arguments is quite clear. For if the dilemma is thought of as an argument which leads to a conclusion through the affirmation or denial of alternatives in the minor premise, the major premise of such an argument must provide the basis for the alternatives affirmed or denied in the minor premise, and this it can do only when there is a plurality of terms in the antecedent, consequent, or both, of the major premise. In view of these more general statements, the structure of the dilemma will be evident from the following analysis:

1. The major premise is a compound hypothetical proposition
 - a. with a one-term antecedent and a two-term consequent,
 - b. or with a two-term antecedent and a one-term consequent,
 - c. or with a two-term antecedent and a two-term consequent.
2. The minor premise is a disjunctive proposition.

3. The conclusion is
 - a. either a categorical proposition, when the major premise has a one-term antecedent or consequent.
 - b. or a disjunctive proposition, when the major premise has a two-term antecedent and consequent.

Classes of Dilemma. Dilemmas, which are governed by the laws of the hypothetical syllogism, are of two kinds, — Constructive and Destructive. A dilemma is said to be constructive when the minor premise affirms the antecedent of the major, and to be destructive when it denies the consequent of the major disjunctively. Within each of these kinds, there are two forms, — Simple and Complex. A dilemma is said to be simple when its major premise has a single antecedent or consequent, and it is said to be complex when both the antecedent and consequent of the major premise contain two terms. We proceed now to the study of each of these kinds of dilemma, and to an examination of the forms that they may assume.

1. **Constructive Dilemma**,—an argument which proceeds from the acceptance of the antecedent to the acceptance of the consequent of the major premise.

(1) **Simple Constructive Dilemma**,—an argument whose minor premise affirms disjunctively the antecedent of the major, and whose conclusion categorically affirms the consequent.

If either A or B, then C. If a science furnishes useful facts, or if the study of it exercises the reasoning powers, it is worthy of being cultivated;

Either A or B. But either a science furnishes useful facts, or its study exercises the reasoning powers;

Therefore, C. Therefore, it is worthy of being cultivated.

(2) Complex Constructive Dilemma,—an argument whose minor premise disjunctively affirms the antecedent, and whose conclusion disjunctively affirms the consequent.

If A then C, and if B then D. If Aeschines joined in the public rejoicings, he is inconsistent; if he did not he is unpatriotic;

Either A or B. But either he did or did not;

Therefore, either C or D. Therefore, he is either inconsistent or unpatriotic.

2. Destructive Dilemma,—an argument which proceeds from the denial of the consequent to the denial of the antecedent.

(1) Simple Destructive Dilemma,—an argument whose minor premise denies disjunctively the consequent of the major, and whose conclusion categorically denies the antecedent.

If A, then both B and C. If table-rappers are to be trusted, the departed are spirits, and they also exert mechanical energy;

Either not B or not C. But either the departed are not spirits or they do not exert mechanical energy;

Therefore, not A. Therefore, table-rappers are not to be trusted.

(2) **Complex Destructive Dilemma**,—a n a r g u m e n t whose minor premise disjunctively denies the consequent of the major, and whose conclusion disjunctively denies the antecedent.

If A then C, and if B then D. If the industry of the United States is well organised, there is work for every efficient labourer who seeks it, and if all labourers are industrious, they will seek work.

Either not C or not D. But either some labourers cannot get work, or they will not seek it;

Therefore, either not A or not B. Therefore, either the industry of the United States is not well organised, or some labourers are not industrious.

The Validity of the Dilemma. The logical value of dilemmatic arguments has frequently been called in question. Thus Jevons says, "Dilemmatic arguments are more often fallacious than not." It may, therefore, serve to emphasise the character of these arguments if we indicate where their use may involve us in error. The fallacious character of a dilemma must be sought either in the major or in the minor premise. In the major premise, the ante-

cedent or consequent may not express possibilities, in which case the error will be easily detected; or the asserted connection between them may be false, in which case the error is likely to be obvious. For example, if we do not admit that 'A is B' and 'A is C' are possibilities, then we are not likely to admit that 'If A is B, it is C' is a possible premise of an argument. But if we admit both as possibilities, we may not admit that the one follows as a consequence from the other, and this will preclude 'If A is B, it is C' from figuring as the major premise of an argument. We thus see that before we can accept a compound hypothetical proposition as the major premise of a syllogism, we are required to admit that both antecedent and consequence are possibilities, and that they are related to each other as condition and conditioned. For example, it is possible for a triangle to be equilateral, it is also possible for a triangle to be equiangular, but in this case wherever one of these possibilities is realised the other is also found, the one is conditioned by the other. These are the facts that we express in the statement, 'If a triangle is equilateral, it is equiangular.'

Error arises most frequently, however, in the minor premise. The reason for this is that the alternatives may not be either exhaustive or exclusive. The difficulty of getting a dichotomous alternative which is exhaustive is due partly to the complex nature of the subjects of discourse, and partly to the fact that the major premise of these arguments is concerned only with the relation of

antecedents and consequence, and not with disjunctive relations. The proposition 'If A is B, it is C or D' means, as we have seen, that 'A is B' is the condition of both 'A is C' and 'A is D.' When, therefore, we affirm these two disjunctively in the minor premise of an argument, we must do so on other grounds than that they are antecedents or consequent of a compound hypothetical major premise. Hence, to quote Jevons, "if we were to argue that 'if a pupil is fond of learning, he needs no stimulus, and that if he dislikes learning, no stimulus will be of any avail; but as he is either fond of learning or dislikes it, a stimulus is either needless or of no avail,' we evidently assume improperly the disjunctive minor premise. Fondness and dislike are not the only two possible alternatives, for there may be some who are neither fond of learning nor dislike it, and to these a stimulus in the shape of rewards may be desirable. Almost anything can be proved if we are allowed to pick out two of the possible alternatives which are in our favor, and argue from these alone."

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QUESTIONS

1. What kind of predication underlies the disjunctive proposition?
2. What relation has disjunctive assertion to the universe of discourse?
3. Discuss the question whether the alternatives of a logical disjunction are exclusive.
4. "The alternatives of a disjunctive proposition must be exhaustive." Explain.
5. What are the characteristics and significance of a distinctive syllogism?
6. Discuss and illustrate the main classes of disjunctive syllogism.
7. Why do we distinguish classes of disjunctive argument?
8. What are the functions of the minor premise and conclusion of a disjunctive syllogism?
9. In what Moods may a disjunctive syllogism be stated?

10. How will your interpretation of 'A is either B or C' be affected, if the alternatives are understood to be exclusive, and if they are understood to be not necessarily exclusive?
11. Why is the disjunctive syllogism in Modus Tollendo Ponens valid whichever view of alternation we take?
12. Why is a disjunctive syllogism in Modus Ponendo Tollens valid only if we give an exclusive reading of the major premise?
13. What is a hypothetical proposition?
14. Name and state the difference between the main classes of hypothetical proposition.
15. What is meant by saying that the material of the judgments which are expressed in hypothetical propositions is possible not actual?
16. "Hypothetical propositions state the connection of a supposition and what follows from it." Explain.
17. Into what two parts may a hypothetical proposition be analysed?
18. What are the characteristics of a hypothetical syllogism?
19. Why does the denial of the antecedents, and the affirmation of the consequents of the major premise of a hypothetical syllogism make it impossible to state any valid conclusion?
20. What are the rules of valid inference for hypothetical arguments?
21. In what Moods may a hypothetical argument be stated validly?
22. In what Mood is a hypothetical argument constructive?
23. Describe and illustrate the Modus Ponens, and state why hypothetical arguments in this Mood are valid.
24. State and illustrate the different forms of the Modus Ponens.
25. In what Mood is a hypothetical argument destructive?
26. Describe and illustrate the Modus Tollens, and state why hypothetical arguments in this Mood are valid.

27. State and illustrate the different forms of the Modus Tollens.
28. What is it to be 'on the horns of dilemma'?
29. How do dilemmas, trilemmas and tetralemmas differ from one another?
30. Define a dilemma.
31. What are the various theoretically possible forms of dilemma?
32. What is a constructive dilemma?
33. Describe and illustrate the various forms of constructive dilemma.
34. What is a constructive dilemma?
35. Describe and illustrate the various forms of destructive dilemma.
36. What must we admit with respect to the major premise of a dilemma before it can serve as the basis of a valid conclusion?
37. What sources of possible error are found in the minor premise of a dilemmatic argument?

EXERCISES.

1. Write three disjunctive propositions in which the alternatives are exhaustive and exclusive.
2. Construct on the basis of your answer to the previous question three arguments in Modus Ponens.
3. Construct three disjunctive syllogisms in Modus Tollens.
4. Write three hypothetical propositions, only one of which may be introduced by 'if.'
5. State the nature and Mood of the following arguments, and examine their validity:
 - (1) If the study of logic furnishes the mind with a multitude of useful facts, like other sciences, it would deserve to be cultivated; but it does not furnish the mind with a multitude of useful facts; therefore, it does not deserve cultivation.

- (2) Mars must have warmth sufficient for protoplasmic metabolism, if life exists there; but life is not possible there, since the planet has not sufficient warmth for the purpose named.
6. What are the nature and validity of the following arguments:
- (1) If you say what is just, men will hate you; and if you say what is unjust, the gods will hate you. you must say one or the other. Therefore, you will in either case be hated.
- (2) If he were clever, he would see his mistake; and if he were candid he would acknowledge it. But either he does not see his mistake, or will not acknowledge it. Therefore, he is either not clever or not candid.

CHAPTER XIII

INDUCTION AND METHODS PRELIMINARY TO INDUCTION

General Statement of the Problem. In the foregoing chapters, knowledge was said to have two sides, from one of which it is seen to be almost wholly identified with what we call truth, and from the other it appears to be almost wholly concerned with what are called facts. On whichever of these two sides of knowledge we may chance to look, we are exposed to the danger of neglecting the other, and thus of forgetting that the distinction of truth and fact is only relative. If we say that the distinction is one that concerns the contrast that is ordinarily made between our thoughts-about-objects (truth) and the objects-thought-about (facts), it will not require a great deal of analysis to show that truth and fact are not two separate things, but two ways of looking at the same thing; and consequently, that if we try to separate them, in order to give exclusive existence to either, we can get rid of the one only by losing our grasp also upon the other. Thoughts without things are empty, and things without thoughts are meaningless.

Our previous discussions have led us to lay emphasis mainly upon the various ways in which our thoughts-about-things are capable of constant improvement and expansion. We may be said to have taken for granted the existence of truth, and our

problem was to study the methods by which one truth is or may be connected with another; to expound in an elementary way the systematic character of truth. From this point of view, the syllogism appeared as an instrument for bringing particular truths under general rules; it enabled us to see what were the conditions of the connection of one truth with another. In definition and classification we were studying how to make truth more explicit, to rid it of ambiguity, and to bring out other important features. In brief, we have been studying the judgment as the distinctive element in knowledge, and expounding the relations that judgments may sustain to each other.

In the present chapter, we turn to another group of problems that is suggested when attention is given to facts rather than truths. Now about facts, the logical consideration of which is commonly referred to as induction, we are primarily interested to know how facts are constituted; what we mean by fact, and what conditions fact must satisfy if the world of fact, like the world of truth, is to become systematic. Broadly stated, the problem of induction is an inquiry into the process by which, when we start with particular facts, universally valid statements about reality can be established.

What is Meant by Fact. There are several features of this statement of the problem of induction that need elucidation. And first, what precisely do we mean by fact? The word is used in a narrower and a broader way. It is used in a narrow sense

to indicate the objects of our knowledge of the material world, such as trees, mountains, railroads, streets, houses, etc. In the broader sense, it is used to indicate any object of knowledge, whether material or not, of which we may be said to have knowledge. Used in this latter way, a sensation, an idea, a purpose, an emotion, when they are made the objects of knowledge, would also be facts. If we compare these two uses of the word, we shall see that a single meaning underlies both; they differ only in the range of its application. For both, a fact is any object of which we are said to possess knowledge.

We may observe further that by knowledge as a conscious process is meant a judging process; hence, we may say that whatever is the object of a judging process is a fact, and nothing else is. What a judging process is has been explained in earlier chapters, and it is important here merely to remark that it is only when objects are made the subjects of judgments, and are qualified in distinctive ways, that they can be spoken of as facts. Facts, then, are objects that have been qualified by our thoughts about them. Now there are a number of ways in which facts are qualified by our thoughts about them; for instance, we think about them as possessing certain qualities, as being in a certain place, as continuing to exist for a certain length of time, as related to other objects in numerous ways. The important words in these instances are quality,

place, time, relation. In logic these are called categories.

The problem of the categories is much too difficult a subject for discussion in an elementary book of logic, but the student is required to understand at least what the word means and how it is used. We may say, then, that the categories are names for the differences in the forms of our thought about things; and, because thoughts and things imply each other, they indicate also a distinction in the nature of the things about which we think. If we remember that etymologically category means predicate, the categories may be looked upon as a list of the classes of attributes that can be predicated of a subject in a logical proposition. Thus, if we say that 'Man is an animal,' the term animal tells us what kind of a substance man is; we are thinking about man under the category of substance. If, again, we say that 'The violets have a sweet odour,' we are thinking about violets under the category of quality. How many categories or generic differences in the forms of our thought about things there are is a matter that does not concern us here; the list varies with different writers, but Aristotle names ten. The point to emphasise is that the predicates of logical propositions can be classified under a number of main heads which denote the general forms of our thinking about things, and these forms of thought are called categories. Until we have thought the objects of consciousness under one or

more of the categories in an act of judgment, they are not properly called facts; a fact is, from this point of view, any object of consciousness which has been thought under one or more of the categories.

Nature of Reality. Our statement of the inductive problem contains, secondly, the word reality. It is, we said, about reality that we seek universally valid judgments. What is reality? The answer to this question, like that of the categories, is too difficult for an elementary discussion, and besides the subject itself belongs to another department of philosophy, namely, to metaphysics. But we may understand the term well enough for our present purposes, if we utilise the results of our examination of the meaning of fact. Broadly stated, by reality is meant the 'world of fact.' What such a phrase as that just employed is ordinarily understood to emphasise is that facts are not isolated, unrelated things; that they are, on the contrary, connected in numerous ways with other facts. This is a matter of our ordinary every day observation and experience. We put our hand in the fire and it is burned. We eat too much or eat the wrong things, and our bodies suffer. In the social world, we have a disagreement with one person, and the behaviour of a number of others toward us is affected thereby. The movements of the planetary bodies regulate our seasons, the rain fall determines the size of our crops. These instances are sufficient to suggest, what is true throughout our experience, that facts

do not exist apart from each other, but are, so to speak, bound up in bundles together. Such bundles or groups of fact we speak of as worlds of fact. If we carry our observation further, it will appear that as individual facts are related to each other so these bundles of facts are connected in numerous ways. The combinations of these more complex groups we may also call worlds of fact. Thus we have the world of commerce, the world of conduct, the world of religion, the world of science. Within each of these worlds, the facts, although diverse, are generically related, — they are facts of a given kind. Looked at in one way, it seems as if these several worlds had little if anything in common, and that there were consequently a pluralism of such worlds. A little closer study has convinced some men, however, that we can detect a likeness between some of these worlds on the ground that they are concerned with material things, and that a likeness exists between still others on the ground that they are concerned with mental things. If this is true, the many worlds are reduced to two, each of which is very complex, and between which there is said to exist no similarity at all. Whether this is true or not we need not discuss; it is a question that belongs to the philosopher, and there does not seem to be any general agreement about it. The point to emphasise is that whether there be one or many worlds of fact, every one is agreed that when we speak about reality we are speaking about those worlds and the facts therein contained. When,

therefore, we say that the problem of induction is an inquiry into the processes by which, starting with particular facts, universally valid statements about reality can be established, we mean that we are interested in finding out the truth about these worlds, what are the relations between the facts which give these worlds their characteristic features, and make them the permanent realities they appear to be.

Our study of this problem falls into two parts. In the first part, we shall consider a number of processes preliminary to induction, and with which induction frequently has been confused; in the second part, we shall expound the inductive process strictly so-called, a process, however, which we shall see emerging into a clearer and clearer light in our exposition of the first part. The present chapter is devoted to the former task; the next four to the latter.

Induction by Simple Enumeration. A prevalent notion identifies induction with an examination of a number of instances of a given phenomenon. Whenever we are comparing a number of particular facts for the purpose of discovering in what respects they are alike, we are supposed, by those who hold this view, to be engaged in an inductive procedure. In agreement with this supposition, the distinction has been drawn between perfect and imperfect induction. In the case of perfect induction, we are dealing with a group of facts all the members of which are known. Thus, we may make state-

ments about the days of the week or the months of the year, and be quite sure that no instances, other than those we have examined, will occur to make a revision of our judgment necessary. We may say, for example, that 'All the months of the year have less than thirty-two days,' and 'The week consists of seven days.' Whenever the group of facts we are examining is known to be limited, we may collect all the instances and observe each before making the general statement. The enrollment of students in a college class is such a definitely limited group, and about it we may, for example, make the statement that 'None of the students in this class is over six feet tall.' Imperfect induction is based upon the same principle of counting and comparing instances, but we have no assurance that all the cases that have been examined are all the cases that exist, and, therefore, a lower degree of certainty is supposed to belong to the statements that, under these circumstances, we are able to make. This was the view of induction commonly held during the Middle Ages. It consists in stating that all the known instances of a given kind possess a certain attribute, and, therefore, that all instances of this kind will be found to possess the same attribute. For example, the Negroes of Africa for centuries had seen no human beings who were not black; they would, therefore, suppose that all human beings were black. For many centuries each and every crow that had been observed by Europeans was black; it was, therefore, supposed that all crows were black. In

both these instances the induction has been found to be erroneous. The fault to which every imperfect induction is liable is that experience is likely at some time to prove it wrong.

Both of these kinds of induction have been rejected by writers in the modern era, and especially by Bacon and Mill. Perfect induction, Mill says, is of no scientific value; the conclusion is only a statement in a different form of the premises. The conclusion, in other words, is merely an enumerative judgment, — a judgment which summarises the data and does not lead beyond them. In induction proper, we are said to go beyond what is contained in our data; we proceed from the known particulars to the unknown universal, from what is true of a given case or number of cases to what is true always and everywhere of all cases whatsoever. Jevons, on the other hand, defends perfect induction, and in doing so states very well the value that we may attach to this process. He says: "If Perfect Induction were no more than a process of abbreviation, it is yet of great importance, and requires to be continually used in science and common life. Without it we could never make comprehensive statements, but should be obliged to enumerate every particular. . . . The fact is, that the power of expressing a great number of particular facts in a very brief space is essential to the progress of science. Just as the whole art of arithmetic consists in nothing but a series of processes for abbreviating addition and subtraction, and enabling us to

deal with a great number of units in a very short time, so Perfect Induction is absolutely necessary to enable us to deal with a great number of particular facts in a very brief space."

The case of imperfect induction is not in principle different. Imperfect induction, says Bacon, "proceeds by merely citing instances and is a childish affair (*res puerilis*), and being without any certain principle of inference it may be overthrown by a single negative instance." By a negative instance, Bacon means an instance which contradicts all the instances that previously have been examined. For example, a single instance of a grey crow overthrows the inference that all crows are black. Mill, referring to imperfect induction, says that "it consists in ascribing the character of general truths to all propositions which are true in every instance that we happen to know of. . . . In science it carries us but a little way. We are forced to begin with it; we must often rely on it provisionally, in the absence of means of more searching investigation. But, for the accurate study of nature, we require a surer and a more potent instrument." This more potent instrument is "to be found in interrogating nature" by experiment, so as to discover the conditions under which the phenomena occur.

The Value of Simple Enumeration. In the preceding section, we have examined the two forms of induction by simple enumeration, and we have seen that in perfect induction all the positive instances, and in imperfect induction a larger or smaller num-

ber of the positive instances of a phenomenon are available for observation. The conclusion that is based upon a complete enumeration of the instances does not go beyond these instances themselves, whereas the conclusion that is based upon an incomplete enumeration is understood to apply also to instances that have not been examined and that may not be known to exist. In the former case, the conclusion may be described as an empirical generalisation; in the latter, the conclusion has the character of an hypothesis which is held subject to revision or rejection in view of our examination of other instances that may come to light. These descriptions serve to distinguish the scientific value of these two forms of simple enumeration. Perfect induction belongs to the stage preliminary to induction, whilst imperfect induction must be classed among the steps of induction properly so-called. The latter, we shall reserve for future consideration, and turn our attention immediately to the former.

Empirical Generalisation. Induction by complete enumeration, which, as we said, leads to an empirical generalisation, is not entirely a simple process, and it suggests certain questions of importance for the study of induction. These questions we shall consider in this section. And first we should observe that if we ask why there is any such process of combining in single statements a number of facts, why we classify at all, we may perhaps say with Mill, who in this follows Hume, that there is a tendency of the mind "to generalise its experience, pro-

vided this points all in one direction." This abbreviation and summation of numerous facts and experiences takes place as a measure of protection against being overwhelmed with the diversity of the events which life daily presents to each of us. It is not a method that is peculiar to science; it is used by all of us in common life. It has a practical value; it does, as Mill points out, facilitate our handling a great number of data. But because we cannot escape using it, and because its employment is so general, the method is not, therefore, altogether simple. It presupposes the existence of ideas, — of the idea, at any rate, that all the instances we examine may possibly constitute a single class. When, however, we ask by virtue of what particular idea a number of facts constitute a class, we must remember that classes, as we saw in the chapter on classification, are not ready made things; there are, as we said, no 'natural kinds,' nor, on the other hand, are there any ready made ideas, except as those have been acquired in the course of experience, that can serve as principles of classification. The fact is that we classify on the basis of observed likenesses, and the members of a class are determined by the kind of likenesses with respect to which each instance is examined. In science, the principle of classification is consciously entertained; in common life, it is not brought into clear definition. In the latter case, we are guided by the accidental attributes of phenomena, and our generalisations have, as a rule, the value merely of shorthand mem-

oranda; by them we are able to get back to the facts at any time we wish, but our understanding of the facts is not thereby advanced. The process is not for this reason without value, but its particular worth is likely to be obscured if we fail to observe its limitations. We may, therefore, repeat that empirical generalisations do not make any group of facts, for science or for common life, more intelligible; they merely group together a number of data, and thereby suggest that the observed material may be connected in some more fundamental way. It is because they hint at some underlying principle of connection that empirical generalisations have a place in the method of scientific investigation.

Colligation of Facts. A method of dealing with facts which emphasises more directly the importance of ideas in the development of knowledge is that which is called by Whewell the "colligation of facts by means of an exact and appropriate conception." Whewell states his view, and defends it against the objections of Mill, by calling attention to the presence of ideas in all our perceptions. He says, "All perception of external objects and occurrences involves an active as well as passive process of mind;—includes not only Sensations, but also Ideas by which Sensations are bound together, and have a unity given to them. From this it follows, that there is a difficulty in separating in our perceptions what we receive from without, and what we ourselves contribute from within." He, there-

fore holds that, "we cannot obtain a sure basis of Facts, by rejecting all inferences and judgments of our own, for such inferences and judgments form an unavoidable element in all Facts. . . . We are not able, nor need we endeavour, to exclude Ideas from our Facts; but we may be able to discern, with perfect distinctness, the Ideas which we include." In agreement with this position, what Whewell means by colligation is quite clear; it consists in bringing together a number of observed facts under a common description, as, to use one of his own illustrations, when Kepler, having made a number of observations on the successive positions of Mars, brought them together under the one conception of an ellipse. Mill contends that the method of colligation does not differ from an empirical generalisation, and since it involves no inference, it is not an induction. We do not need to go into the dispute between Whewell and Mill on these points; it is clear, from the statements of what colligation means for Whewell, that ideas are essential to the existence of facts, and that a conception of the mind operates as a principle of connection between facts. The relation between fact and idea, however, is not always stated clearly by Whewell, for he sometimes speaks as if the inductive act consisted of a superinduction of conceptions upon facts, that is, as if facts and ideas were external to each other. Green, in making the same point, says that "Whewell . . . spoils his own case (against Mill) by often writing as if the antith-

esis between ideas and facts were a valid one; as if the 'superinduction of ideas' upon facts were merely an operation that had to be performed *ex parte nostra* in order to give science." The value of Whewell's insistence upon colligation lies in this: it asserts that all our knowledge of the world of fact depends upon a more or less conscious exercise of the judging activity which carries us beyond a merely empirical generalisation, and enables us to regard the facts of observation under a new point of view. These points of view, our author contends, are supplied by, but are not necessarily derived from, the mind itself. As we shall see later, the point of view under which a group of facts becomes intelligible is sometimes ascertained by a stroke of genius, sometimes as the result of a laborious and diligent search. But however it may have been discovered, ideas and the facts that they make intelligible are not two discrete and mutually exclusive things; they are, as we have seen, only the reverse sides of a single experience. It is only by an act of analysis, as Whewell himself believes, that they can be distinguished.

Parity of Reasoning. We often entertain beliefs about new facts on the ground of the similarity of these facts to other facts with which we are acquainted. For instance, if the bull dogs that we have known are unfriendly toward strangers, we shall shape our conduct toward any dog that has the general appearance of this breed in accordance with the belief that no bull dog likes strangers.

This is an example of an empirical generalisation. From it may be distinguished another type of reasoning which, while dealing with numbers of instances, affirms that we may entertain a belief about new facts, not because we have reason to entertain it about similar facts, but because we may apply to the new facts tests similar to those that were used on the old. That is, if we have shown that X is true of A, we may argue that it is true also of B, not because it is true of A, but for the same reason that it is true of A. This method of argument is called parity of reasoning. Mill's illustration is this: "Having shown that the three angles of the triangle ABC are together equal to two right angles, we conclude that this is true of every other triangle, not because it is true of ABC, but for the same reason which proved it to be true of ABC." Mill objects to calling this an induction on the ground that it "is not believed on the evidence of particular instances," that is, it is not proved in each of the instances, and, therefore, there exists no ground of inference from the known to the unknown cases. The importance of this type of reasoning at our present stage of inquiry is that it raises definitely the question whether numbers of instances are necessary for an inductive conclusion.

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QUESTIONS

1. What do you assign to the distinction of truth and fact?
2. In what two ways may the word fact be used, and what common element is present in both uses?
3. 'Facts are objects that have been qualified by our thoughts about them.' Discuss.
4. What is a category, and what is the relation of categories to the judgment?
5. What is a fact from the standpoint of the categories?
6. What is meant by saying that reality is the world of fact?

7. What light does the study of the worlds of fact throw upon the problem of monism and pluralism?
8. How must we interpret the inductive problem, if by reality is meant the world of fact?
9. What view of induction is implied in so-called perfect and imperfect induction?
10. State and illustrate what is meant by perfect induction.
11. State and illustrate what is meant by imperfect induction.
12. What is Mill's criticism of perfect induction?
13. What is Bacon's estimate of imperfect induction?
14. What is Bacon's estimate of perfect induction?
15. What, according to Mill, is the value of imperfect induction?
16. What is the value of perfect and imperfect induction as instances of simple enumeration?
17. What is an empirical generalisation?
18. Why, according to Mill, do we generalise our experience?
19. 'Generalisation presupposes the existence of ideas.' Explain.
20. What is the value of empirical generalisation for science and for common life?
21. State Whewell's conception and explanation of the colligation of facts.
22. State Mill's objections to colligation.
23. What is Green's criticism of Whewell's position?
24. What is the essential truth of the doctrine of colligation?
25. What is parity of reasoning, and how does it differ from empirical generalisation?
26. Why does Mill refuse to regard colligation as a truly inductive method?
27. What is the importance of Mill's objection?

EXERCISES.

1. Give at least six examples of what are known as facts, and distinguish the factual and truth elements involved in these examples.
2. Make six statements of fact, and determine under what category each statement is made.
3. Give three illustrations of what is known as reality as distinguished from fact.
4. Give three illustrations each of perfect and imperfect induction.
5. Write out two examples from common life and two from science of an empirical generalisation.
6. Illustrate, by a carefully considered example, the meaning of a colligation of facts.
7. Give three illustrations of parity of reasoning.

CHAPTER XIV

SOME DESCRIPTIVE METHODS OF INDUCTION

Relation of Numbers of Instances to an Induction. The question with which we closed the last chapter, namely, whether numbers of instances are necessary for an induction, may be made the starting point of the present one. When induction is thought of as an inference based upon the examination of numerous instances of a phenomenon, it assumes, as we have seen, two forms: one, when we have assurance that all the instances are known; the other, when no such assurance can possibly be entertained. The former, we have considered, and it was said to be an illustration of a method of organising experience that is not confined to any special kinds of fact, but is used over the whole extent of our life. The name that we gave to this method was empirical generalisation. The value of empirical generalisations lies in the fact that they enable us to deal with large numbers of data in a simple way; we can deal with collections of fact as single facts whenever we ignore the differences and are interested in the likenesses between the facts which enter into such collections. The assurance that every one feels in organising his experience in this manner is due to the limits within which it is consciously carried out. Empirical generalisations are held to apply only to the instances that have been examined.

Numerous instances are necessary for an empirical generalisation, but such generalisations, since they do not carry us beyond the instances we have examined, cannot be regarded as inductive inferences.

The other form of induction by simple enumeration differs from the one just considered in the fact that not all the instances of the phenomenon have been observed, and the conclusion which the known instances are found to warrant is extended to other instances that have not come under observation. The question we are concerned with in such a case is whether, as Mill seems to hold, the validity of our inference is in proportion to the number of the instances examined, that is, whether the ground of the certainty of such an induction is found in the number of the instances on which it is based; or whether it is due to some other characteristic of the instances, for example, to the nature of the instances themselves.

In attempting to answer this question, we may observe, in the first place, that the position held by Mill emphasises only one of the factors of an enumerative induction, and that, even if we hold, with him, that it is an essential factor, it nevertheless depends for its effectiveness on the co-operation of another factor of which no special mention has as yet been made. For no one is prepared to maintain that the mere fact of numbers, irrespective of the other qualities of the instances that an induction brings under review, can be the ground of an inference to other unknown instances. Mill's

position, therefore, must be understood to be that, although a greater degree of certainty attaches to any conclusion we reach respecting certain observed facts if the number of such observed facts is very great, and a lower degree of certainty if the number of such observed facts is relatively few, all the facts that enter into an induction must exhibit the qualities or relations with respect to which the induction is made; and that, if we extend to other instances the conclusion reached with respect to any one group of instances, this is possible because the new instances resemble the old ones in their essential nature. Whether, therefore, we are interested in arriving at an induction or in extending to new cases the conclusion of an induction already made, the fact of the common nature of all the instances must be established or assumed if the induction itself is sound, or if it is to make an inference possible. In either case, the instances in question must all resemble one another in essential respects.

We may remark, in the second place, that, in the light of what has now been shown, the question of the relation of numbers of instances to the validity of an induction is an interesting rather than the most important aspect of the inductive problem. There is no doubt, for example, that, within limits, our confidence in the conclusion of an induction is increased by the number of instances by which it is known to be supported. But we must distinguish between our confidence in a conclusion and the

ground on which the truth of that conclusion is supposed to rest; and, we must remember, it is with this latter question that we are here concerned. But we may say, even with respect to our confidence in the truth of inductive conclusions, that, beyond certain limits which vary for different phenomena, this is not affected in an appreciable manner by any considerable increase in the number of instances of a given phenomenon that may be brought to our attention. For example, should we not feel a justifiable impatience with any one who, during the hours of only a single day, counted the number of objects within his observation that fell to the ground, and who informed us that that additional number of observed cases should be regarded as an additional reason for our confidence in the law of gravitation? And with respect to the truth of an inductive process, we may observe that the number of instances that need to be observed before a conclusion is reached varies according to the nature of the instances themselves, and the ability of the particular individuals who examine them. If this is true, the number of instances is a somewhat uncertain factor in the establishment of an induction, and there seems to be no reason why, if the nature and conditions of the observation are sufficiently unambiguous, the truth about a whole class of cases may not be reached as the result of the study of a single example. Indeed, in the history of science there have been instances in which some striking event has arrested attention, and set the problem

for the scientist, and in which the study of other examples of the same phenomenon has been undertaken mainly for the purpose of verifying the conclusions reached with respect to the single instance. And, further, it may be said that, whether the truth of an induction has been reached as the result of an examination of one or many instances of a phenomenon, we may infer certain facts of other instances only when we assume that these also resemble the old one or ones in essential respects. Indeed, this is only to state the significance that belongs to all scientific statements whatever, and it gives the reason why all such statements are essentially hypothetical. Thus, if we suppose that P is a phenomenon under examination, and that X is the characteristic with respect to which it is finally determined, then we may say of any new example M that 'If M is P, it is X;' that is to say, X is predicated of M, because it is P. We may, therefore conclude, both with respect to the establishment of an induction and with respect to the possibility of extending an induction to further instances of a phenomenon, that "the strength of an induction is not proportional to the number of instances cited. The generalisations that lead to the most trustworthy hypotheses are based, not on the counting of the number of instances, but on the weighing of their quality and character. One crucial instance may be worth a hundred others."

Logical Significance of Statistical Methods. The progress of modern science has been associated so

intimately with the application of mathematical conceptions to concrete phenomena that the conclusion of the preceding paragraph may appear to be one-sided, if not erroneous. To meet this objection, we shall examine the logical character of the statistical methods, and ascertain their place in an inductive process. And, at the outset, we should observe that whenever statistical methods are employed, the data under observation can most often be given only in the form of aggregates and averages. These methods, as a rule, are applied distinctly for the purpose of ascertaining the numerical distribution of certain characteristics which have come under observation in a limited number of cases. Sometimes all that it is necessary or possible to do is to ascertain the total number of the recorded instances of the phenomenon. Usually, however, the interests we hope to satisfy by the use of the statistical methods are more specialised than this. For instance, we may wish to know what is the average occurrence of some irregular group character, as, for example, the average number of insane persons born per generation in a particular family. Or we may wish to ascertain the average amount of a constant group character, as, for example, the average amount of butter fat given per head by a particular breed of cattle. And even when these methods are applied to individuals, we still have in view the discovery of group characteristics. On this point Thorndike writes: "The sciences of human nature commonly use measures of individuals only in order to get

measures of groups. Not John Smith's spelling ability, but that of all fifth grade boys taught by a certain method; not A's delicacy of discrimination of weights, but that of all men; not B's wage, but that of all railroad engineers during a certain period; not C's children, but the productivity of the English race as a whole; not individuals, but groups, are commonly measured, compared and argued about." An examination of the individuals enumerated in the census returns as to age, sex, married, unmarried, etc., also illustrates that interest in the characteristics of groups which prompts the use of distinctly statistical methods of inquiry. Sociological and insurance tables, as well as those of the physical, biological and psychological sciences, are based upon the same interest. The particular service, therefore, that the employment of statistical methods is able to render is to bind together diverse numbers of fact of a particular kind, and thus to provide a basis for the comparison of otherwise incomparable aggregates. "The method of counting . . . agreements and comparing results," says Gibson, "constitutes the Method of Statistics."

The conclusions to which statistical inquiries lead are generalisations, but they are to be distinguished from the empirical generalisations spoken of above in two respects. In the first place, statistical conclusions may be taken as a type of scientific as contrasted with empirical generalisation on the ground that they are the products of a systematic investigation of numbers of fact. By systematic in

this connection we mean that the basis of the classification of data is in each case consciously chosen, and that it is applied as a test of what can and what cannot be included in the numeration. For example, in an enumeration of houses in India, as Scripture points out, great difficulty was experienced on account of the indefiniteness of what was meant by a house. Similarly, the Weather Bureau would find it impossible to classify the hot, temperate, and cold days during a single year because of the indefiniteness of the connotation of these terms. But if hot is defined as a shade temperature above 75° , cold as a shade temperature below 35° , and temperate any degree between these two, it is readily seen that there will be no difficulty in applying to the days of the year these tests, and generalising the results.

In the second place, statistical conclusions are expressed always in quantitative terms. This is, of course, not an accident; it is a result directly aimed at. In this way, certain features of the objects of our experience come to have a value and significance which, under any other method of observation, would be in danger of being overlooked altogether. For example, isolated events, and events that occur but rarely in our experience, unless they are very striking, are liable to receive but a passing notice, and to present but a small part of their meaning to such casual observation. It is also true that we are prone to overlook the import of those occurrences which are brought regularly to our notice. Familiarity breeds neglect as surely as do entire novelty

and strangeness. The average person seems to dwell in safety between these two extremes, with the result that the world presents him with few problems, and stimulates him but rarely to sustained intellectual effort. It is, however, a characteristic of the scientific mind that there is nothing so common that it does not deserve comprehension, and nothing so rare that it is beyond consideration. Out of this temper have been born those efforts to reduce the passing show of our daily experience to a definite system and order; and it has seemed, to those engaged in it, not the least worthy task, as it has been not the least profitable, to count the number of times a given fact occurs, and to make number a part of the facts themselves. Among the Greeks, Pythagoras interpreted the world in this mathematical way; and all our modern sciences, the physical, biological and social, are employing statistics in the belief that when the relative frequency of the occurrences of a phenomenon is known, the phenomenon itself has been more accurately observed, and can be more exactly described.

Qualitative Methods of Observation. Statistics, however, do not provide the only methods by which the observation of phenomena is carried on. There are classes of phenomena to which it cannot be fruitfully applied. We have said that whenever our material can be presented most advantageously in the form of aggregates and averages, statistics lend themselves as a means of bringing out features of this material that in no other way receives ade-

quate recognition. But we are not always or only interested in collective groups of objects. We are also interested in objects as instances of a genus, as instances in which the characteristics of a genus are given concrete expression. For instance, whenever we call a certain flower a primrose, we are not describing the flower as an individual thing; rather are we neglecting altogether what is peculiar to the flower as an individual, what makes it this primrose rather than that; and are recounting the characteristics which this object shares with an indefinite number of other specimens of the same species. The difference between this case and those that are amenable to statistical observation is that in the latter we need to collect together numbers of individuals before the important characteristics are open to observation, whereas in the former, the individual instance displays all the attributes that are distinctive of the genus to which it belongs. Field botany, as our illustration suggests, is a descriptive science in the sense we are now indicating. Natural history and descriptive geology also belong to the same class. In these sciences, individual objects display universal characteristics, that is, those which in our experience are found to belong to an indefinite number of other objects of the same kind or class. These common features of objects have their interest, and they set for us problems which tax our industry and challenge our efforts to solve. It is obvious, in the light of these statements, that the accurate knowledge which is the goal of our intel-

lectual life is not all of a single type, and that the methods of observation which we adopt or devise must be appropriate to the nature of the objects which are the material of our study. Sometimes we must avail ourselves of the quantitative methods of statistical observation, sometimes of the qualitative methods of immediate perception.

The Nature of Scientific Observation. Whatever the methods of observation may be, and however numerous, they are determined by the nature and purpose of observation itself. Now, in order to gain a clear understanding of the nature and purpose of observation, we may distinguish scientific observation from the attitude we are accustomed to maintain toward the ordinary events and circumstances of our every day life. Those adjustments with our environments which, during the course of our early years, we have succeeded in acquiring have been determined almost exclusively by practical considerations, and they have been maintained as the habits of our lives very largely on account of their value as instruments of physical and social survival. The earliest task that devolves upon the infant is to come into relations with the world around him, and his primary concern is with the world of material things from which he may receive either benefit or harm. It is only so far as it contributes to this end that, in its earliest beginning, the mind of the child is stimulated into activity, and his interest is aroused in the things and persons which constitute his world. Intelligence, that is to say, is a later

development than are those motor responses to environment through which the well-being of the individual is secured, and comes at first upon the scene of the individual's life, not in order to introduce any new interest or to initiate a new set of aims, but as the guide and servant of the practical which, through its means, is enlarged and rendered more efficient. These relations of the practical and intellectual in the order of their development throw an interesting light upon the function that intelligence plays in the maintenance of human existence; for, as it was in the beginning, so does it remain for the greater part of each individual's life, there is no independent intellectual interest, and the functions of human understanding are determined throughout by the practical purpose which they instrumentally advance. If, in view of these considerations, we observe the characteristics of our ordinary life, we shall understand why, instead of displaying a careful attention to details, the perceptual consciousness has a conspective range which enables it to take in a large amount of detail without losing its practical effectiveness. What is not of practical importance fails to be noticed; only those features of the environment which are practically important are gathered up and made objects of attention.

How far perception, as it is ordinarily employed, is from giving accurate knowledge of the commonplace facts and events of life is well known. As an

example of the unreliability of everyday observation, we may cite the contradictory accounts of eyewitnesses of a common street accident. But, further, the conclusions and inferences that we base on such observations are, as a rule, no more trustworthy than the observations themselves. In our daily life we exhibit constantly, what Whewell calls, "a vague and loose mode of looking at facts." As a result, to quote from the author just referred to, we remain "for a long time under the belief that a body, ten times as heavy as another, falls ten times as fast; that objects immersed in water are always magnified, without regard to the form of the surface; that the magnet exerts an irresistible force; that crystal is always found associated with ice; and the like. These and many others are examples of how blind and careless men can be, even in observation of the plainest and commonest appearances; and they show that the mere faculties of perception, although constantly exercised upon innumerable objects, may fail in leading to any exact knowledge."

If now we consider what may properly be called scientific observation, we shall see that this differs from immediate perception in important respects. Scientific observation, in the first place, has not the conspensive range of immediate perception, but is characterised by a close attention to detail within a limited field. This difference is conditioned by the difference in the purpose for which each exists. The purpose that controls the character and direc-

tion of our perceptions was seen to have its origin in the interests of our practical life. There is not, on the other hand, any such thing as observation in the strict meaning of the term, until our interest in the world of the practical life has failed to be satisfied, either because the environment is too complex for perception to cope with, or because the relevant features of the environment have been hastily overlooked. In either case, importance comes to be attached to the details of a situation, and the conditions are favourable for the development of an interest in details quite apart from the regulation of our ordinary practical activities. Scientific observation does not differ from immediate perception in the fact that the former is, and the latter is not, purposive, but in the character of the purpose which prescribes the goal and directs the course of each. In the case of scientific observation, the purpose that renders the details of an event or situation important is theoretical; it springs from the interest we have in gaining an intimate acquaintance with these situations and events. That is to say, scientific observation must needs be used when we are launched upon a search for truth; it is an instrument of our acquisition of knowledge. What things are, and how they come to be what they are, are the questions which emphasise the necessity of accurate observation.

In contrast with immediate perception, scientific observation is, in the second place, analytic; it seeks in the environment for those features which are

relevant to the theoretical purpose by which it is guided. The difficulty that one experiences in his earliest scientific efforts is due to this selective character of observation. If, for example, we placed an histological preparation under the microscope, and, without any previous acquaintance with the subject to which it relates, we were to attempt to observe the character of the tissue, we should very likely fail; and our failure would be due probably to the fact that we had no defined purpose to guide our selection of the relevant detail. The beginner in any science is subjected to much loss of time and unnecessary discouragement if the instructor does not understand that scientific observation does not consist in looking at a mass of detail, but in looking for certain relevant aspects in a mass of detail. But to look for something relevant implies that a purpose is functioning in our examination of objects, and it is this purpose or idea which the capable instructor provides as the guide to his pupil's observation. We observe only what we are prepared to observe, and mere collections of fact are useless until an idea illuminates and makes them intelligible. This view of scientific observation is illustrated in the work of all the great scientists. It was conspicuously true, for example, of Darwin, of whom his son writes: "He often said that no one could be a good observer unless he was an active theoriser. This brings me back to what I said about his instinct for arresting exceptions, it was as though he were charged with theorising power

ready to flow into any channel on the slightest disturbance, so that no fact, however small, could avoid releasing a stream of theory, and thus the fact became magnified into importance." In other words, we observe not with our senses but with our intelligence. Or, as Huxley put it, what one sees when he looks through a microscope depends as much upon what is behind the eye-piece as upon what is under the objective.

Experimental Observation. If we reduce the idea of a scientific experiment to its lowest terms, we may say that it consists in the interposition of an instrument of precision between our sense organs and the material we are studying. From this point of view, a scientific experiment secures an extension of the range of our perceptive organs, and, in so far as it does this, brings out more of the detail of the material under examination. The balances of the physicist, the microscope of the biologist, the telescope of the astronomer are devices for thus extending the field of scientific observation. But there are limitations to experimental observation, as thus understood, which often would hinder the progress of truth if they were not offset by observation under normal conditions. Hence, as Lotze says, "Observation often acquaints us with broad characteristics of phenomena, which in experiment would have been obscured by special conditions." The fact is that observation and experiment supplement each other, and there are cases when neither alone could lead us as far as when both are used in conjunction.

There are also some phenomena to which experimental observation cannot be applied at all, and for our examination of these we must rely upon other means of observation. In this connection, it may be recalled that Darwin's great work was achieved by the method of observation, in the broad sense just described, rather than by the methods of experimental observation.

A further consideration affecting our view of the range of scientific observation is that, however successful we may be in improving our present apparatus, and in devising and constructing new apparatus, there is, in all probability, a limit beyond which our sense organs are not capable of being supplemented in that way, and a point, consequently, at which the observations we make by the use of instruments of precision must stop. What the scientist does when he has reached his actual perceptual limits, and what he would attempt to do if he reached the theoretically possible limits of perception, is to supplement his actual observations by guesses as to what, if perception were possible, further observation would, in all probability, make known. In some such way as this have the luminiferous ether, atoms, ions, etc., become parts of the actual working data of science, data, be it remarked, that are regarded, by science, as truly material substances as are the grosser bodies that are now within the range of present perception. This kind of obser-

vation we may call, for the sake of convenience of reference, theorising, or theoretical observation.

Another characteristic of experimental observation is that, like all observation, it must be used in the service of the purpose which directs the observation. Hence the impossibility of any one being an experimental scientist who is not thoroughly acquainted with the mechanical structure of the apparatus which he either uses or designs. There is a quite prevalent notion, especially among students and inexperienced instructors, that one is necessarily an experimentalist if he works in a laboratory and manipulates apparatus. To these, no question could come with more pertinence than this: 'For what purpose are you using this particular piece of apparatus, and precisely in what way does it enable you to realise this purpose?' It is not apparatus that lends significance to our ideas, but ideas that make our apparatus intelligible instruments of observation. To lose sight of this truth is to miss entirely the true meaning of experimental science; for we may be as passively assimilative when looking through a microscope as we might be actively observant when using our natural organs of perception. But when our apparatus "passes into the service of an idea, and we proceed to put our object under such conditions as the idea requires for testing and verifying itself, our observation is no longer merely passive, but experimental. A definite, purposive interference of some sort with the production of the phenomenon to be

noted is the *desideratum*" of experimental observation.

At precisely what point can this interference with the phenomenon be introduced most advantageously? The answer to this question is given unanimously by all the workers in the field of science. Science seeks to control, not the nature, but the conditions of the occurrence of phenomena. "To observe with accuracy and convenience," says Jevons, "we must have agents under control, so as to raise or lower their intensity, to stop or set them in action at will." Welton writes: "Appeal to experiment is, then, necessary whenever simple observation alone will not make plain all the essentials of a phenomenon; and its object is to eliminate all the conditions which are not specially operative in the particular case under observation." In like strain Gibson says: "Control over the conditions under which the object shall present itself for study is the great desideratum and the factor in experimental observation *which most influences the precision and certainty of the results obtained*. Such control enables us to vary the circumstances, and to repeat the observation as often as we please by simply reproducing the concurrence of conditions requisite for bringing into play the desired effect."

REFERENCES

SEE THE END OF THE NEXT CHAPTER

QUESTIONS

1. What are nature, extent and value of empirical generalisations?
2. Does an empirical generalisation involve an inductive inference? What is your reason?
3. Are numerous instances of a phenomenon necessary for an empirical generalisation? Why?
4. Can imperfect induction be regarded as an example of an inductive inference? State reasons for your answer.
5. What problem is involved in the fact that the conclusions of imperfect induction are extended to unexamined instances of the phenomenon in question?
6. What is Mill's position on the validity of imperfect induction?
7. What influence does the fact that a large number of instances of a phenomenon have been examined have on the validity of an inductive inference?
8. What value may we give to number of instances in an inductive process?
9. What fact with respect to the nature of induction is involved in the hypothetical character of scientific statements?
10. What particular interest determines the use of statistical methods?
11. State and illustrate some of the questions that statistical methods enable us to answer?
12. What purpose does the application of statistical methods to individuals commonly serve?
13. In what respects do statistical conclusions differ from empirical generalisations?
14. Describe the particular method of observation employed by field botany, natural history and descriptive geology.
15. What are the characteristics of perception, and in what relation does perception stand to intelligence?

16. In the light of your answer to the previous question, what view must be taken of the logical function of perception?
17. In what respects does scientific observation differ from ordinary perception?
18. What do you understand by experimental observation?
19. What are the limits of experimental observation?
20. Explain the nature of theoretical observation.
21. Illustrate by reference to the relation of the scientist to his apparatus the place of purpose in experimental observation.
22. 'Experimental observation involves a purposive interference with the production of phenomena.' Explain.
23. State what a scientist may and may not do in interfering with the phenomenon he wishes to observe.

EXERCISES

1. Give at least six examples of empirical generalisation.
2. Give six illustrations of imperfect induction, and show how each may be used as the basis of an inference.
3. Illustrate from each of the sciences you have studied the hypothetical character of inductive inference.
4. Show, by a carefully worked out example, the nature and method of statistical observation.
5. Give one example of experimental observation.
6. Illustrate what is meant by theoretical observation.

CHAPTER XV

SOME EXPLANATORY METHODS OF INDUCTION

The Nature of Scientific Explanation. The methods that were examined in the last chapter may be classified roughly as methods of observation, and those that are to be examined in the present chapter may be classified approximately as methods of explanation. It will, perhaps, help us to understand these latter methods if in this section we inquire into the meaning of the term explanation. This is the more necessary because some confusion on this subject has been occasioned by the way in which scientists have sometimes expressed themselves with respect to the aims of science. For example, one declares that science consists of a complete description of the phenomena that are brought under observation, while another declares as emphatically that description is only a preliminary step in the work of the scientist, and that the goal of all science is explanation. If we were to take such statements absolutely, we should be led to suppose that there is between the scientists an irreconcilable difference of opinion as to the nature of the work in which they are engaged, and we might presume that, until they arrived at a settlement of these differences, it were useless for us to discuss the matter further. This, however, would be a false view to take; for not only must logic examine the

aims and methods of the sciences, but the differences already referred to are verbal rather than real, and are due in a large measure to the ways in which what is after all a substantial agreement has happened to get itself expressed.

Let us first get before ourselves what is meant ordinarily by a scientific explanation, and then, in the light of this information, we may consider the distinction between it and scientific observation. All scientific explanation conforms to a very simple type; it consists in stating what other things occur when the particular thing we are observing occurs. If, for example, I hold a book in my hand and, releasing my grasp upon it, it falls to the ground, we should ordinarily explain the falling of the book to the ground by saying that I had released my hold upon it. The book fell *because* I released my hold. If, now, experimenting with other objects than books, I find that pencils, stones, silverware, china-ware, etc., fall to the ground when the hand that held them releases its grasp, I may put the results of these observations into the statement that 'all objects fall to the ground when their support is removed.' If, further, the fall not only of the book but of all objects whatsoever is connected with the force of gravity, the process of explanation has been carried a step farther. But however far such a process is carried, it consists essentially in describing events that occur along with the event which, at the beginning, was the object of our observation. 'This occurs when that occurs' is the universal for-

mula which reveals the nature of scientific explanation.

The same view is sometimes expressed more precisely in the statement that scientific explanation is concerned with the discovery of the conditions of the occurrence of any object of observation. Let *P* be the phenomenon or object observed, then the question of explanation relates to the conditions, *C*, under which it occurs. When these conditions have been discovered and recorded, we are said to have explained the phenomenon. Out of the innumerable circumstances that accompany an event, the problem of scientific explanation is to select those which must be present whenever the event is present, and which when they are absent the event itself does not occur. The factors of explanation are always found among the concomitant circumstances of the phenomenon which comes before us for explanation. An examination and description of these is what is meant by a scientific explanation.

With these statements in mind, we may now make clearer the distinction between scientific observation and explanation. It is obvious, in the first place, that the phenomena which we include under each of these terms are continuous with each other. For instance, the falling of an object is continuous with the release of the hand's grasp upon it. Stated generally, a phenomenon and its conditions form an inter-connected series of events, no one of which is unnecessary to the series, and each of which is important if the series is to maintain its essential

character. From this point of view, the distinction between a phenomenon and its conditions is more or less accidental and artificial; and if we take such series as scientific units, regarding the objects which are the starting points of our observations as fragments which through scientific methods we are to supplement by their necessary concomitants, we shall see that the whole aim of science has been fulfilled when such a series has received a full and complete description. If, in the second place, we do not regard the object which provides us with the starting point of our observation as a fragment, but as a phenomenon which has a quasi-independent existence, we shall be inclined to consider the search for conditions as a distinct step in scientific investigation, and, laying emphasis upon this, the problem of explanation will assume larger proportions than it did in the other view. But whichever of these two ways of looking at the scientific problem we may take, the factors of each are the same, and between these views there is only a verbal difference. On either view, we have to supplement what is given — the phenomenon, with what is not given — the conditions; and to determine the conditions of a phenomenon is what we understand as the problem of scientific explanation.

The Meaning of Analogy. The word analogy is used in different senses, and these we shall proceed to distinguish. It is used, in the first place, to indicate an identity of relation. This is the meaning that it had for Aristotle with whom it corresponded

to mathematical proportion. In this sense, it assumes the form of $2:4::3:6$, where the relation between $2:4$ is identical with the relation between $3:6$, and the factors between which the relation obtains are homogenous. We may state the characteristics of this first meaning of analogy generally by saying that whenever a is related to b in the same way that c is related to d , then whatever follows from $a:b$ follows also from $c:d$, if the inference is based upon nothing but the identity of the relation between the two pairs of factors or terms.

The word analogy is used, in the second place, to indicate a similarity of relation. The difference between this and the preceding use of the term is not due to the absence of proportion, but to the difference in the character of the terms between which the proportion obtains. Although the relation between two pairs of terms is similar, the terms between which the relation holds may be heterogeneous. For example, the number of vibrations per second of the sound which is recognised by our musical sense as middle C (c') is 261, that of its octave (c'') is 522 (2×261). The similarity of relations involved in these facts is expressed in the proportion $261:522::c':c''$. But since the octave of any musical note always has twice as many vibrations as the note itself, we may express the law of octaves in the proportion: x vibrations of air: $2x$ vibrations of air :: a note: its octave. The analogy may be interpreted to mean that the relation between the series of physical stimuli is connected in our

knowledge with their corresponding impressions in such a way that given the former we can infer the latter.

The word analogy is used, in the third place, to indicate any resemblance between things. Here the idea of proportion which was present in the preceding uses has dropped out, and resemblance of any kind between objects is made the basis of analogical inference. For example, certain resemblances were early known to exist between sound and light: that both were capable of being reflected, that the direction of reflection of each obeyed the same law, that the angle of reflection is equal, in each case, to the angle of incidence. It was further known that sound travelled in waves. From these data it was inferred analogically that light also travelled in waves. If it had been known that the equality of the angles of reflection and incidence in the case of sound were due to its traveling in waves, the inference to a like mode of propagation in the case of light would have been more certain. The more important the resemblances, that is to say, the more certain the inference. From the example just used, the analogy between things may be expressed generally in the following manner: If *a* resembles *b* in certain respects, *x*, and *a* exhibits the character *y*, it is inferred that *b* will resemble *a* in possessing the character *y* also.

The Logical Character of Analogy. We have shown that analogy is a method of inferring certain possible events on the ground of a resemblance

between certain ascertained events. "In the syllogism of Analogy," says Hegel, "we conclude from the fact that some things of a certain kind possess a certain quality, that the same quality is possessed by other things of the same kind." In every argument from analogy an identity of some sort is assumed, and it is upon the character of the assumed identity that the validity of the inference is in every case said to depend. Let us look at the soundness of such arguments in the light of this statement.

In the first place, it is obvious that not any kind or degree of resemblance is a safe principle of inference. In common life, we meet with analogical arguments that are very superficial. For instance, the loyalty of college students to the university from which they were graduated may often lead to a mistaken estimate of a man who is a graduate of the same university. "It would be a very bad analogy," as Hegel remarks, "to argue that since the man Caius is a scholar, and Titus also is a man, Titus will probably be a scholar too; and it would be bad because a man's learning is not an unconditional consequence of his manhood." In public addresses on political and social questions we may often detect arguments of this sort. In the period of the Commonwealth in England, the demand for annual Parliaments is said to have been urged on the ground that the serpent — which is the wisest of beasts — sheds his skin every year. We also find analogical reasoning in the works of men of science, and sometimes the conclusion which these argu-

ments are intended to support rest upon a very weak and even false resemblance. Take, for instance, the conception of the human body as a machine. "When we desire to set machinery in operation," as Chittenden, commenting on this view, says, "we must get up steam, and so a fire is started under the boiler and steam is generated in proportion as fuel is burned. The source of the energy made use of in moving the machinery is the extraneous combustible material introduced into the fire-box, but the energy of muscular contraction, for example, comes not from the oxidizable food material in the stomach, but from the material of the muscle itself. In other words, in the animal body it is part of the tissue framework, or material that is closely incorporated with the framework, that is burned up, and the ability to endure continued muscular strain depends upon the nutritive condition of the muscles involved, and not upon the amount of food contained in, or introduced into, the stomach."

In the second place, the resemblances upon which analogical arguments are made to rest must be important resemblances. Accidental resemblances and superficial likenesses are at the basis of all fallacious reasoning by analogy. With this class of analogies in mind Heine was led to exclaim, "Heaven defend us from the Evil One and from metaphors." But if the resemblance is essential and important, there is no need to avoid the use of this method of reasoning. Now when we speak of a resemblance being essential, we mean that the characteristics

with respect to which objects are compared must be essential either to the existence of the objects or to the purpose which these objects realise. If, to illustrate the former case, we infer that certain things are true of diamonds because they are true of graphite, we, no doubt, do so with the thought in mind that these substances are different forms of carbon. To illustrate the latter, we may argue from the similarity in shape of certain flints found in the ground and those used by certain savage tribes of the present day that the former were used as weapons since this is the use that the latter are known to have.

In the third place, analogical arguments must take into account not only the importance of the resemblances but the non-importance of the differences between the objects compared. An interesting example is the following, borrowed from Gibson. He says, "we might enumerate many points of external resemblance between the Whale and the Shark, and found upon them an analogical argument to the effect that the respiration-processes in the two animals must be similar. The whale, we might say, resembles the shark not only in all the common characters of Vertebrates, but also in its submarine habitat and in being (as regard species) one of the very largest of marine animals. Like the shark, it is fish-like in external form, its fusiform body being well fitted for cleaving the water. Anteriorly its body passes into the head without any distinct neck, and posteriorly it is furnished with a

swimming-tail into which the body gradually tapers. It has no hairy covering. Like the shark, again, it has a wide mouth, and it is of predaceous habit, feeding only on living animal nutriment. Therefore we may with great probability conclude that its method of respiration is like that of the shark—i. e., that it breathes the oxygen dissolved in the water, and has no need to be supplied with atmospheric air.

“But this argument would be unsound. The points that we ought to have observed are the characters *connected with the function of respiration*. The presence of gill-slits in the shark and their absence in the whale is a difference so essential to the inquiry that its observation would at once have been sufficient to make our analogy fall to the ground. And among the still more obvious external differences there is a single character which also should alone outweigh all the above mentioned resemblances. The extremity of the shark’s tail is expanded vertically; in the whale the flukes of the tail are placed horizontally. From many points of view this difference might be regarded as unimportant; but from the point of view of our analogical argument it is very important indeed, for it is intimately connected with the problematic point of resemblance that we are endeavouring to establish. For sea-creatures which, like the whale and the shark, spend their life in swimming freely through the water, the direction of movement is chiefly determined by the presence or absence of the air-breathing habit.

Those creatures that have no need of atmospheric air move usually in straight-forward and lateral directions, and for effecting such movements a vertically expanded caudal fin is admirably fitted. But creatures that need to rise frequently to the surface of the water for the purpose of respiration are constantly moving upward and downward. To movements of *this* kind a tail with horizontally expanded fluxes is precisely adapted. Thus the whale's horizontally expanded tail affords a strong presumption in favour of the presence of the air-breathing habit; and this apparently trifling difference between the two creatures must be regarded as fatal to the cogency of the analogical argument."

The Function of Analogy in Induction. We have been considering analogy as a special kind of argument, the way in which Aristotle understood it. We proceed now to discuss the value that analogies have for advancing our knowledge of the world about us, the assistance they may give in the problem of explanation. Analogy, as our exposition has shown, is an argument from similar instances. It is, to quote Mellone, "any resemblance between things which enables us to believe of one what we know of the other." We advance from one instance to another on the basis of a perceived or imagined similarity between the instances. The necessity of advancing in an analogical argument from particular to particular, has led to the saying that analogy "sticks in the particular instances." Although this is true, it is not on that account worthless as an in-

strument or means of arriving at the law of the connection between resembling instances. For if there is a resemblance, or a group of resemblances, between P and P' , such that we feel warranted in affirming of P' what we know positively only of P , we may look upon the similarity, not merely as a bridge from the one instance (P) to the other (P'), but as a suggestion of a specific connection between the two instances which is not brought out explicitly by the analogy. It is this characteristic of suggesting more than it can possibly make clear, that renders analogy of service in an inductive process. The following remarks are intended to make this point clear.

Two things are characteristic of analogy. In the first place, the comparison of instances, as we have seen, rests upon what are called their essential attributes, but, in the second, the degree of the resemblance always remains more or less undetermined. When we have assured ourselves that the resemblance depends upon essential features of the compared phenomena, we are, however, still in doubt about the precise factor or factors in the resemblance upon which the inferred characteristic really depends. Thus any two phenomena, P and P' , may resemble each other in possessing the attribute x , a resemblance which seems to warrant us in expecting that if P also shows the characteristic y , P' will likewise show the same characteristic. To take a concrete example. The chemist classifies

a number of elements together under the head of alkaline metals because all these elements — potassium, sodium, caesium, rubidium, and lithium, — possess certain characteristics in common; for instance, their active combination with oxygen to decompose water at all temperatures; their forming strongly basic oxides, highly soluble in water, yielding powerful caustic and alkaline hydrates from which water can not be expelled by heat; the solubility of their carbonates in water; and the formation by each metal of only one chloride. Now, if in the progress of the chemistry of any one of these metals a new property is discovered, it is natural to infer by analogy that the other members of the group will display under appropriate conditions the same property. But beyond the suggestion of a connection of the new property with the characteristics of the alkaline group analogy cannot go; it cannot point out on which attribute, or combination of essential attributes, the new property really depends. Thus an argument by analogy presumes a connection between x and y , but it cannot show that y depends on x rather than on some other property in P which may not belong to what we have called its essential attributes. However strong the presumption may be that x and y are causally related, there always remains the possibility that y is conditioned in some other way. Analogical arguments for this reason are never conclusive, and in an inductive process they are valuable only because they suggest lines of further observation which may

either verify or discredit the analogical inference. Mill, therefore, is right when he regards an analogy as "a mere guide post, pointing out the direction in which more vigorous investigations should be prosecuted. It is in this last respect," says Mill, "that considerations of analogy have the highest scientific value. The cases in which analogical evidence affords in itself any very high degree of probability are, as we have observed, only those in which the resemblance is very close and extensive; but there is no analogy, however faint, which may not be of the utmost value in suggesting experiments or observations that may lead to more positive conclusions." With this view, to which Mill does not consistently adhere, Jevons is in agreement, and he has made it the foundation of his chapter on analogy in the *Principles of Science*.

Nature of Hypothesis. An hypothesis is an attempted explanation of an occurrence, a supposition about the connection of particular phenomena. To get the meaning of this general statement before us, let us consider a particular case. Malaria is a disease which has a set of characteristic symptoms, the chief of which are chills and fever occurring at more or less definite intervals of time. When the physician has become familiar with the characteristic symptom complex of the disease, he is able to diagnose the new cases that come under his observation by means of the similarity that they bear to this complex; all the cases that show the diagnostic marks of the disease he calls malaria. The logical

procedure in this instance is analogical. If, however, instead of taking the physician's attitude which requires him to diagnose only for the purpose of selecting the proper remedies, we approach the disease from the standpoint of the scientist who wants to know what malaria is, we shall have to go beyond the similarities which figure in the physician's diagnosis, and, by a series of suppositions or hypotheses, work our way among the antecedents and concomitants of the cases of the disease for some clue to the agent which is responsible for the production of the disease. It is not enough to say that similar symptoms mean the same disease, the question we are now asking is what produces the similar symptoms in all the cases that occur. We are looking for something more than similarity, we are searching for an identity among the conditions under which the disease is contracted. Thus, for example, night air, damp places, exhalations from marshy ground have been proposed as the antecedent condition which is capable of producing malarial fever. It is only within comparatively recent times that, for these hypotheses, another has been substituted which connects the disease with the bite of the *Anopheles* mosquito, and this is the view that is held at the present day. 'The bite of the *Anopheles* mosquito' is our hypothesis for the occurrence of malaria.

In the light of this illustration, we may observe that not any supposition about phenomena is an hypothesis. It is only when the supposition points us to phenomena of the same order as the event

that we are trying to explain that it satisfies the conditions of a tenable hypothesis. In medicine, for example, we no longer entertain theological explanations of disease, and for devils we are substituting bacteria and their toxins, and in those diseases which cannot be shown to be of bacterial origin, as for example many of the occupational diseases, we assign either a chemical cause, as in lead poisoning, or a mechanical one, as in the disease characteristic of the Sheffield steel grinders. When made with regard to the requirement we are now considering, namely, that the supposition should be stated in terms of the same order as the phenomenon to be explained, hypotheses differ from one another only in the degree of their probability; and it seems to us to be quite misleading to speak of them as guesses, as is done by some logicians, as if they were altogether uncontrolled by the nature of the phenomena for which an explanation is being sought. The truth of this statement is apparent when we recognise that the demand for an hypothesis is only a particular form of the demand for a careful analysis of the phenomena under observation. Our acquaintance with objects becomes more minute as well as more accurate under the stimulus that comes from the call for explanation; and if one step in the process of explanation is hypothetical, that step must still be controlled by the nature of the observed material if we are to safeguard the inductive process against useless and irresponsible guesses. The satisfaction of this requirement by

science may be briefly illustrated. To account for the phenomena of sound, the physicist, starting with the sound experience, connects this with the undulations of the air which occasion it, and these undulations, again, he connects with a vibrating body, — a stretched string, the vocal cords, an organ pipe. When the scientist asks what is the source of the phenomena of light which occasion the sensation of vision, although he cannot point definitely to the material bodies which produce the undulations in the ether, he forms his hypothesis in terms of the same order as the phenomena that he has under investigation; and he does so because the “scientific imagination,” to quote Tyndall, “demands, as the origin and cause of a series of ether-waves, a particular vibrating matter quite as definite, though it may be excessively minute, as that which gives origin to a musical sound. Such a particle we name an atom or a molecule. I think the intellect, when focussed so as to give definition without penumbral haze, is sure to realize this image at last.”

Objections to the Use of Hypothesis. Although the history of modern science furnishes so many illustrations of the value of hypotheses, and although the methods in use at the present day for the advancement of scientific knowledge depend also upon their employment, there is, nevertheless, among scientific men either only a half-hearted recognition of their services, or a misleading rejection of their claims to a place in scientific procedure. This interesting fact is not difficult to understand,

and we refer to it here because it enables us to explain some aspects of the formation of hypotheses which as yet have not been mentioned. The hesitancy of men of science to recognise that the advancement of scientific knowledge is, in any respect, dependent upon hypotheses, may be attributed to their effort to exclude from all parts of their work influences and additions that are derived from the nature of the mental processes through which that work is carried on. The aim of science is throughout objective in the sense that its task is to describe and explain the facts of observation, to arrive at judgments about these facts that are independent of the idiosyncrasies of the individual mind. "The scientific man," as Karl Pearson says, "has above all things to strive at self-elimination in his judgments, to provide an argument which is as true for each individual mind as for his own." The life of science depends upon loyalty to this aim. The intellectual integrity of every scientific man is involved in his efforts to maintain this aim. All this may be granted; it may even be admired and praised. And yet we cannot avoid the impression that if the spirit of loyalty to professional ideals which is displayed by workers in the fields of the particular sciences were supplemented by an acquaintance with the history of scientific achievements, the objections to hypothesis would be mitigated, and in time might entirely disappear. For the admission of hypothesis in no way interferes with keeping the work of science objective, and

their practical rejection may often render impossible of realisation the aims which stimulate scientific effort.

The situation that has just been described may be made more explicit by saying that without an active and fertile imagination, it is impossible to formulate a satisfactory hypothesis. It is, in part, because the scientist does not take the pains to ascertain what are the nature, conditions, and limits of the imagination that he looks askance upon the formation of hypotheses. This is not the place to state and develop a doctrine of the imagination; it is enough for our purpose to call attention to the fact that many of the leading men of science have borne eloquent testimony to the value of the imagination in advancing the interests of scientific truth. In his address on 'Scientific Use of the Imagination,' Tyndall writes: "Bounded and conditioned by co-operant reason, imagination becomes the mightiest instrument of the physical discoverer. Newton's passage from a falling apple to a falling moon was, at the outset, a leap of the imagination." Tyndall prefixed to his essay the following passage from an address to the Royal Society by Sir Benjamin Brodie. "Lastly, physical investigation, more than anything besides, helps to teach us the actual value and right use of the Imagination — of that wonderful faculty which, left to ramble uncontrolled leads us astray into a wilderness of perplexities and errors, a land of mists and shadows; but which, properly controlled

by experience and reflection, becomes the noblest attribute of man; the source of poetic genius, the instrument of discovery in Science, without the aid of which Newton would never have invented fluxions, nor Davy have decomposed the earths and alkalies, nor would Columbus have found another continent." In his *Methods and Results*, Huxley writes: "It is a favourite popular delusion that the scientific inquirer is under a sort of moral obligation to abstain from going beyond that generalisation of observed facts which is absurdly called 'Baconian' induction. But any one who is practically acquainted with scientific work is aware that those who refuse to go beyond fact rarely gets as far as fact; and any one who has studied the history of science knows that almost every great step therein has been made by 'anticipation of nature,' that is, by the invention of hypotheses which, though verifiable, often had little foundation to start with, and not infrequently, in spite of a long career of usefulness, turned out to be wholly erroneous in the long run."

These quotations are sufficient to illustrate the importance of the imagination in the construction of scientific hypotheses, and to indicate the reason why many men of science look with suspicion upon the use of hypotheses. "Scientific men fight shy of the word (imagination) because of its ultra-scientific connotations," because its use is often uncontrolled, and because under the best conditions it sometimes leads us astray. But to reject an instru-

ment of research because of its limitations is no more reasonable than would be a physician's refusal to employ strychnine in the cure of a patient, because that drug is on the commercial market in the form of 'Rough on Rats.' We are not advocating an indiscriminate exercise of the imagination, nor the substitution of it for the careful and painstaking observation of phenomena. What we are pointing out is that there are conditions of physical investigation which render all known instruments of precision inadequate and clumsy tools, and that if observation is not to be hindered its range must be extended as far beyond that of our instrumental world as this is beyond the world made known by our natural organs of perception. We cannot, for example, bring under observation by the most powerful microscope the particles of matter which are the media in our atmosphere of reflected light. By what means has the existence of such particles scattered throughout our space come to be believed in? Tyndall cites this example as an illustration of the use that science may make of the imagination, as he does also Darwin's hypothesis of a primordial germ as the source of all the variety of organisms now upon the earth's surface. These instances, and many others that are among the most brilliant achievements of science, lead us to look upon the imagination as itself an instrument of observation, and in the form of carefully stated hypothesis, it leads to an explanation of many ob-

served facts which without its aid would postpone indefinitely important lines of research.

Verification of Hypotheses. If we are to carry our explanations of events to a successful termination, the hypotheses by means of which the relations of particular phenomena are brought under an intelligible view need to be tested and not merely stated, to be verified, not merely formulated. There are two steps in the process of verification to which we may give attention. "To prove or disprove (an) induction we must have resort," says Tyndall, "to deduction and experiment." Mill's definition of an hypothesis, that it is "any supposition which we make (either without actual evidence, or on evidence avowedly insufficient) in order to endeavour to deduce from it conclusions in accordance with facts known to be real," points to the same two requirements. An hypothesis which is not capable of being verified would serve no immediate purpose, and whether it serves any purpose at all need not concern us. We are now studying living hypotheses, those which answer questions, and render intelligible the particular facts of observation. And it is about these that we are warned that it is not their intelligibility merely, and not the fact that they answer questions, which recommends them finally to our acceptance. It is only if our hypotheses are true, if they make us acquainted with the actual relations which objects sustain to one another, that they are received as part of our scientific knowledge. The formation of hypotheses takes us beyond what

is actually known and observed, and for this reason there is in every hypothesis an element of risk; but they must at the same time make it possible for us to return to fact and the observation of fact, and it is this that renders them useful in the advancement of human knowledge. The return to fact is made in the two steps of deduction and experiment.

Let us take as an illustration of these two stages in the process of verification the well-known fact that water rises to a certain height in a pump, and the explanation of this fact on the ground that "Nature abhors a vacuum." The story goes that in trying to raise water to a great elevation, some Florentine gardeners found that the column of water ceased at the height of thirty-two feet. The matter was referred to Galileo who, embittered by the reception that had been accorded to his science, said that he supposed that Nature abhorred a vacuum only at the height of thirty-two feet. It was left for Toricelli, Galileo's pupil, to provide an answer to the inquiry. He supposed that water might be forced into a tube by pressure on the surface of the liquid outside. After much thought, it occurred to Toricelli that the atmosphere might exert such pressure, and that the column of water thirty-two feet in height might be the exact weight necessary to equalise this atmospheric pressure. This supposition was Toricelli's hypothesis. Was it true? This was the next question that the physicist confronted, and it is exceedingly instructive to observe the way he went to work to verify his hypoth-

esis. He deduced certain consequences from the hypothesis. He argued that if thirty-two feet of water will hold the atmospheric pressure in equilibrium, a shorter column of a heavier liquid should do the same. According to his calculations, mercury, which is thirteen times heavier than water, should be able to balance the pressure of the atmosphere at thirty inches. The next step in the process of verifying the original hypothesis was an experiment. Toricelli filled a glass tube a yard or so in length, open at one end and closed at the other, with mercury. Placing his thumb over the open end, he inverted it in a bath filled with the same metal. When he removed his thumb, the mercury began to sink, and continued to sink until the height of thirty inches had been reached. The proof of the hypothesis was complete.

Hypothesis and Theory. The difference between an hypothesis and a theory is a relative one merely. To bring out this feature of the terms' meanings, we may say that a theory is a completely verified hypothesis, and that an hypothesis is an incompletely verified theory. This statement will have served its purpose if it remind us that hypothesis and theory both spring out of the demand for explanation, that they both involve the use of the creative imagination for their formation, and call for verification in the way that has been described. It is these similarities which justifies the popular use of the terms as synonymous. It will be useful for us, however, to look at the matter a little more closely. We

may observe, for instance, that we advance from hypothesis to theory, not by leaving our hypothesis behind, but by developing it in the light of further experiences so as to make it accord with facts hitherto not known or not understood. In this sense, the hypothesis becomes a theory. To illustrate, when ice-cold water is poured into a glass on a hot summer day, little drops of water collect and trickle down on the outside of the glass. How do we account for this phenomenon? The sudden cooling of the glass takes place by radiation of its heat, and the water-vapour of the atmosphere condenses on the cooled surface of the glass. The same hypothesis may be extended to account for the formation of dew, and a number of other phenomena. An hypothesis that can be used to explain a variety of facts which were not thought of when the hypothesis was formulated, can justly be regarded with greater confidence than one which explains only a single phenomenon. Every new fact which fits into an hypothesis strengthens it. At first, an hypothesis has an explanatory value only for the particular phenomenon with respect to which it is framed. But if an hypothesis is capable of growth, both in respect to the number and variety of the facts which it is capable of explaining, and in the simplicity of the form which it assumes, the conditions are present for the development of the hypothesis into what is known as a theory. We speak, for example, of the Darwinian hypothesis, because the supposition it states is still in a condition of growth, and because the modifications which are being introduced into Darwin's

statement are intended to make his view harmonise more fully with progressive experience and recent discovery. When, however, we regard the idea of growth throughout the known world as a confirmed doctrine of science, we no longer speak of the Darwinian hypothesis, but of the theory of evolution. In the same way, and for similar reasons, we speak of the theory of gravitation, the Copernican theory, the theory of the conservation of energy. What we mean by such designations is that the evidence for the particular hypotheses which lies back of them is ample and varied, that the exceptions within the fields of their application are either insignificant or are absent altogether, that newer experiences and discoveries tend to confirm these views. When lines of evidence tend to converge and to be focussed on an hypothesis, that hypothesis has no longer the tentative position of a supposition, but the firmer and surer foundation of a confirmed belief. The formulation of such a confirmed belief is called a theory.

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QUESTIONS

1. Is there necessarily any irreconcilable opposition between the view that the aim of science is complete observation, and that it is explanation? Explain.
2. To what simple type does all scientific explanation conform? Illustrate.
3. What is meant by saying that explanation consists in discovering the conditions of the occurrence of phenomena?
4. Are all the circumstances attending the occurrence of an event conditions of its occurrence?
5. What is meant by saying that the phenomena of observation and those of explanation are continuous with each other?
6. What bearing, if any, do the facts included in your answer to the previous question have upon the distinction between a phenomenon and its conditions?

7. In what sense can it be said that scientific explanation is a process by which what is given in observation is supplemented by what is not so given?
8. Can you connect the conception of scientific explanation implied in the previous question with the conception of knowledge stated in Chapter I?
9. State and illustrate three senses in which the word analogy is used.
10. Why is not any kind or degree of resemblance a safe principle of inference?
11. What is meant when it is said that analogical reasoning depends upon important resemblances?
12. In what way does the validity of reasoning by analogy depend upon the attitude one takes toward differences?
13. What light does the saying that analogy 'sticks in the particular instances' throw upon the character of analogical argument?
14. In what sense can analogy be regarded as a method of explanation?
15. What are the limits of analogical argument?
16. What does Mill mean by calling analogy 'a mere sign post'?
17. State and illustrate what you understand by an hypothesis.
18. Why must hypotheses be stated in terms of the same order as the phenomena they attempt to explain?
19. Is there any objection against calling hypotheses 'guesses,' and, if so, what is the ground on which it rests?
20. What reason may be given for the scientists' objection to the use of hypotheses?
21. What is the relation of imagination to the task of formulating scientific hypotheses?
22. What are some of the 'facts' which we owe to the exercise of imagination in the field of the natural sciences?

23. State and explain the two steps involved in the verification of hypotheses?
24. Illustrate by the use of concrete example the process of verifying an hypothesis.
25. How do hypothesis and theory agree with and differ from each other? Illustrate.

EXERCISES

1. Examine the explanations of three simple events with which you are acquainted, and state whether they conform or not to the fundamental form of explanation as stated in the text.
2. Give three instances in which explanation consists in the statement of the conditions of a phenomenon's occurrence.
3. Give two illustrations, one from literature and one from science, of analogy. Examine each with a view to showing to which meaning of the word it conforms, and whether it satisfies the conditions of a valid inference.
4. Upon what logical principle does each of the following statements depend, and what can you say as to their validity as arguments:
 - (1) It is said that a general resemblance of the hills near Ballarat in Australia to the California hills where gold had been found suggested the idea of digging for gold at Ballarat.
 - (2) There are no great nations of antiquity but have fallen to the hand of time; and England must join them to complete the analogy of the ages. Like them she has grown from a birth-time of weakness tutelage to a day of manhood and supremacy, but she has to face her setting. Everything that grows must also decay.
5. Construct an hypothesis to explain some fact of your experience, and explain how it may be verified or overthrown.

CHAPTER XVI

CAUSATION

General Statement of the Problem. The fact that underlies the problem of causation is the observed regularity in the order of the particular phenomena which occur in the course of our ordinary experience, and in the fields of the several sciences. That our ordinary and scientific experience is characterised, within ascertainable limits, by some kinds and degrees of regularity is a fact about which there is universal agreement. Indeed, this is a feature of experience so generally recognised that it constitutes the starting point not only of the theoretical interest which leads us to the more accurate forms of human knowledge, but also of the practical interests which lie at the foundation of the political, commercial, and social relations by which human individuals are held together in a community. A world from which all order were absent would be a world in which all thought and action would be impossible, and to which all thinking and active agents would ultimately succumb. The institutions of society and the organisation of the sciences presupposes that some sort of unity and order belong to the world; and it is the business of all our sciences to study the regularities of the phenomena and events of the world for the purpose of endowing our notion of such regularities with greater pre-

cision. The problem, then, that confronts us in view of this regularity of the world's order is: How shall we conceive this regularity so as to make intelligible to ourselves the principles or laws by which it is maintained and upon which it depends?

The Practical Aspect of the Question. There is no doubt that this is not a theoretical question merely, a question only of the sciences, and unimportant or superfluous in the absence of theoretical interests. The way in which the world in which we live may be expected to behave is a broadly practical interest; and the man who in the morning starts off to his office, as well as the one who goes to his laboratory, begins the day's work with the expectation that the uniformities upon which he has relied in the past will be maintained in the present. Indeed, we may say that, in the history of the individual and of the race, interest in the regularity of the course of observed events was practical before it was theoretical; and it would not be difficult to show that the practical motive has never been divorced from the theoretical when, in the progress of the sciences, these uniformities have become the subject of very specialised study. No one, for instance, who has the least knowledge of primitive peoples can fail to be impressed with the fact that the origin and development of their institutions is closely interwoven with their struggle with the forces of nature. Upon his understanding of the physical world has depended the primitive man's ability to maintain himself in existence. We

see the relationship thus indicated between the growth of intelligence and the economic struggle illustrated in a broad way if we contrast, for example, the peoples of the northern and southern latitudes. Whenever, as in the latter case, the food supply is plentiful and relatively independent of the cultivation of the domestic arts, intelligence is comparatively low; but when, as in the former case, the food supply is scanty or precarious, it becomes increasingly important to understand the physical environment from which has to be won the means of physical support, and, if the community is to perpetuate itself, to socialise this knowledge in the domestic arts.

There are many concrete illustrations of the way in which, at the present time, observation of natural occurrences, and inferences drawn from such observation, are allowed to affect our practical life. From the character of the sunset we infer that it will be a fair or stormy day tomorrow, and, therefore, that it will, or will not, be possible to take a day's outing in the country. What the farmer calls 'good growing weather' is an inference, based upon observed connections in the past, of the same practical sort. Now about the facts that underlie such practical inferences as these, we may observe that they are selected, often by the coöperative labours of many generations of observant men, from a very complex mass of details, and that no effort is made, as a rule, either to determine the precise nature and connection of the facts themselves,

or to enumerate exhaustively the other factors with which, in our experience, they are associated. For example, what constitutes 'good growing weather' may be for one man a matter of temperature, for another a matter of humidity, and for a third a matter of both; but neither of these factors, nor both together, can be regarded as a sufficient explanation of the term, and many other elements of the situation in which it is relevant to talk about growing weather must be taken account of — such, for instance, as the nature of the particular crop, the particular stage of its growth, its condition at that stage, the nature of the soil, the location of the field in which the crop is growing, etc. — if the phrase is to have for us any practical significance. When, therefore, we examine the character of those uniformities which our practical interests lead us to observe, we find that they are made up of certain striking or important features of a complex situation, which are connected together in such a way that, on their recurrence in our experience, we are led to expect the recurrence of other features of the situation with which they have been associated in our past experience. In other words, one part of an experience is taken as a sign of the recurrence of another part with which it is usually connected. Beyond such empirical connections as are found in experience, the practical view of causation does not enable us to advance.

The Theoretical Aspect of the Question. If we compare the theoretical with the practical way of

dealing with the observed regularities in the occurrence of natural events, we shall see that it differs from the latter in two respects. In the first place, the theoretical view, in Venn's suggestive phrase, 'screws up' the antecedent or cause and the consequent or effect of an event into close juxtaposition, so that they come to be regarded as nearly as possible concurrent. The possibility of doing this, it is obvious, depends upon our ability to eliminate from the phenomenon we are examining all the disturbing or irrelevant elements. If, therefore, it is remembered that no two natural events are exactly alike in all their details, that some of the attendant circumstances are peculiar to the instance of the phenomenon we are examining, and form no part of the phenomenon itself, it is evident that the omission of these, by whatever means it is brought about, can serve only to give greater precision to the factors that are involved in the orderly sequence, and to make it possible to determine with greater accuracy the details which are involved in the causal relation. We pass from the practical to the more theoretical view of causation, that is to say, when we devise means of eliminating from our observation of natural events what is irrelevant to our search for the conditions of the occurrence of such events.

In the second place, the theoretical view insists upon the importance of a careful analysis of the antecedents of an event as a condition of establishing a causal relation. Mill insisted on this point

with some emphasis, and he criticised the practical view for the unregulated way in which it picks out some one event or circumstance, and calls it the cause. There is just ground for Mill's contention, if our study of the observed regularities in the occurrences of natural phenomena is made from the point of view of the requirements of accurate knowledge; but if we admit the legitimacy of the practical point of view, if, that is, the order of nature's changes are taken as a clue to the kinds of conduct that are likely to be efficient, we may say that the more refined analysis that Mill had in mind, and which is required by science, would in large measure defeat this latter purpose. It is because the interests of science are so intimately bound up with a detailed knowledge of the conditions of phenomenal occurrences that, in contrast with those of the practical life, its requirements are satisfied only by the minute analysis of the phenomena with which it is particularly concerned. In emphasising this point Mill has directed attention to an important difference between the practical and theoretical views of the causal relation.

The Genetic Relation of the Practical and Theoretical Views. While the theoretical view of causation differs, as we have said, from the practical in important respects, we should be departing from the truth if we supposed that the differences between these views expressed the whole truth about them. There is between the two views no such opposition in fact as an examination of their differences alone

would lead one to think. We must regard them, rather, as earlier and later stages in the progressive efforts of the race to make clear to itself the relation in which it stands to the changing environment under which its life has to be lived. It was doubtless useful for the theoretical view that practical interests had led to the observation of large numbers of uniformity in the occurrence of natural events. In this way, material was already prepared for it, and to this material it could apply the stricter methods which a change in interests was the means of defining. But in developing and applying these methods the sciences have not freed themselves altogether from practical considerations and, moreover, have themselves contributed very largely toward a new and broader conception of the practical itself. By this statement we do not merely mean that, in its history, scientific progress frequently has been determined by difficulties encountered in a narrowly practical way, nor merely that scientific truth has so often been found capable of a like practical application; the more important fact is that, as scientific aims and methods have become established, our conception of what is practical has been broadened to include any kind of activity that is directed toward the attainment of an end. In the sense in which nowadays we use the word, any regulated or consciously controlled process is practical, and it would seem that science has not only contributed to this conception but is itself, in this meaning of the term, preeminently a practical affair. And just be-

cause the scientific man approaches his problems with definite questions in his mind, because, as Bacon said, he interrogates nature, there is a limit set, by the nature of the questions themselves, to the amount of analysis that is necessary in arriving at the solution of particular problems. It is not, therefore, in the amount of analysis performed that we can find the distinguishing mark of a theoretical view of the causal question. For if we insist that our analysis in any field whatever must be relevant, in kind and amount, to the purpose for which it is undertaken, it becomes at once obvious that the practical and theoretical views are brought together under a single view, and that the differences enumerated above must be understood, not as placing the views in question in different classes, but as indicating different species of activity which belong to a single class.

The Origin and Nature of the Causal Problem. In view of the foregoing considerations, two facts may be said to be fairly well established. In the first place, there is good reason for believing that the causal problem arises from the importance that the presence of order in human experience has for the practical no less than for the theoretical life. The problem of causation is of such general interest, as we have seen, not because the world in which we live is unregulated and disordered, but because, being regulated and ordered, we can adjust ourselves to it better if we know the principles and laws according to which, in the future as in the

past, things and events may be expected to happen. It is well to be perfectly clear on this point, because frequently its neglect has led to a somewhat one-sided, if not misleading, view of the problem itself. We have called attention to the impossibility of considering the causal problem as in any essential respect different for the scientist than for the practical man. For both, it is the fact that experience presents us with certain regularities in the order of our experience that directs attention to the problem, and that leads to a study of these regularities. We do not study the causal problem, that is to say, to escape from a world of chaos to a world of order, but, starting with the world of order, we seek to understand its laws, and to ascertain how far these laws are capable of being extended to new and larger areas of life.

In the second place, it is clear that the problem of causation has nothing whatever to do with the question why things happen as they do, if by that question is meant an inquiry into the reasons why our experience has the distinctive characteristics that it is known to have. Science, and the same thing is equally true of philosophy, starts with the acceptance of the world as it is known in human experience, a world, as we have seen, that has a certain determinate character. Now about such a world, it would be futile to ask ourselves why it has this rather than some other character, since the very problems which confront us in our everyday life, and in our sciences and philosophy as well, arise out

of the world's character as made known by human observation. The facts about the world in which we live, as these are observed and reported in increasing numbers and with improving accuracy, constitute the material of all human knowledge, and if there are any further questions that arise, these must concern the way in which such facts are connected in the unity of an ordered system. Instead, therefore, of conceiving the causal problem as an inquiry into the reason why facts are what they are, we must view it rather as demanding of us an exposition of the nature of the connection that binds the various parts of human experience together. Put simply, the causal problem is concerned, not with the question why things are what they are, but with the question how things happen as they do.

The Course of Our Further Discussion. Our further study of the causal problem will bring before us particularly three closely related questions. We shall examine, in the first place, the time relations of causal events; in the second place, the complex character of such events will demand attention; and in the third place, we must try to make clear what is meant by the necessity that is always recognised as belonging to such events. In our discussion of each of these topics, we shall make Mill's treatment, in Book III., Chapter V., of the *Logic*, our point of departure. It will be advantageous, therefore, for the student to read the whole of Mill's

chapter in connection with the following paragraphs.

The Time Relations of Causal Events. The first characteristic of a causal relation that Mill enumerates is the 'invariable antecedence' of certain of the factors involved in the causal relation. He says that "invariability of succession is found by observation to obtain between every fact in nature and some other fact which preceded it. . . . The invariable antecedent is termed the cause; the invariable consequent, the effect. And the universal law of causation consists in this, that every consequent is connected in this manner with some antecedent or set of antecedents. . . . For every event there exists some combination of objects or events, some given concurrence of circumstances, positive and negative, the occurrence of which is always followed by that phenomenon." He points out that we may not have discovered what the particular concurrence of circumstances may be in a given case, "but," he adds, "we never doubt that there is such a one, and that it never occurs without having the phenomenon in question as its effect or consequence."

In this passage, Mill definitely raises the question of the time relations of causal events. In laying emphasis on the antecedence of the cause, Mill takes the view that for any event to be considered as a cause at all, it must exist in time before the event which is, or may be regarded as, its effect. This, doubtless, is the conception of the time rela-

tions of cause and effect that prevails generally at the present day, and it may be worth while to ask how far it is an accurate description of the facts that are involved in causal events. There are, without question, many examples of the causal relation that lend themselves quite naturally and easily to this interpretation, examples, that is to say, in which the fact that we call the cause and the fact that we call the effect appear to be temporally distinct events. A blow on the head, for instance, may be followed by the loss of consciousness, and may be said to be the cause of the loss of consciousness. The explosion of a powder magazine may follow the application of a lighted match or fuse to the powder. Diphtheria makes its appearance ten days after the ingestion of the Klebs-Loeffler bacillus which, the physicians tell us, is the cause of the disease. In all these cases we have, or seem to have, undoubted examples of the necessary temporal priority of the cause, and the necessary posteriority of the effect, in causally related phenomena. It may be doubted, however, whether all causally related events can be made to fall so easily and naturally within this scheme or view. It would be difficult, for example, to detect in the explanation offered by the chemist of the formation of rust the temporal discontinuity of cause and effect which is an essential part of Mill's doctrine of causation. Rust formation, so we are told, is due to the combination of iron with the free oxygen of the air; but, if that is so, it is quite obvious that the chemical

combination of oxygen and iron does not precede, but is contemporaneous with, the formation of rust itself. There are innumerable instances of a similar character, and we may even say that the example chosen is typical of most of the chemical processes with which we are at all familiar. The case is important because of the large field of application of chemical laws, not only in the inorganic, but in the organic world as well. To take only one such example, we now know that bacterial action is largely, if not wholly, due to the toxins generated by microscopic organisms; and, if that is true, the final explanation of their action and results is, as is coming to be seen, a highly specialised branch of the science of chemistry. If, as seems probable, the results of bacterial action are not separable in time from the production of their toxins, an overwhelming list of examples can be appealed to to throw doubt on the necessary temporal antecedence and consequence of the events involved in the causal relation.

It is owing to some such considerations as these that the position has sometimes been taken that there is no distinction whatever between the cause and the effect of a phenomenon, in other words, that the cause and effect are identical. There seems no sufficient reason in the nature of causal phenomena why we should adopt so paradoxical a view. Cause and effect do stand for distinct things in our experience of causal events, and if, as we have said, this distinction is not necessarily one of temporal

discontinuity, we need to inquire further what the distinction is.

When we say that the relation of cause and effect is not necessarily one of antecedence and consequence in time, we must not be understood to affirm that causal phenomena have no relation at all to time. That would be not only a paradox, but exceedingly difficult to maintain. It is no doubt true that all causal phenomena are events in time, that they are constituted by a number of facts which come before and after one another in a series. That the whole number of facts in a causal phenomenon have a definite sequence and order in time there is no need to deny, and that the order and sequence can be determined is the permanent expectation and belief of science. From this fact, however, it does not follow that, within the series, the cause and effect are separated by a necessary temporal interval. What rather we seem warranted in saying with respect to the causal series is that their factors are continuous, — temporal continuity, not temporal discontinuity, seems to be the characteristic of all such events. But whether within the causal series the cause antedates the effect, or is contemporaneous, at least in part, with it, as it must be, for example, in the circular reactions of chemistry, is a fact to be determined in the particular instance. But whatever may be true of any particular instance, temporal discontinuity cannot be regarded as an essential feature of the causal relation. The utmost that we can say, from the

standpoint of their temporal relations, is that the cause and effect are continuous with one another.

The Complex Character of Causal Events. It is seldom, if ever, according to Mill, that an invariable sequence subsists between a consequent and a single antecedent. Usually, the causal relation exists between "a consequent and the sum of several antecedents; the concurrence of all of them being requisite to produce, that is, to be certain of being followed by, the consequent." If, for example, "a person eats of a particular dish and dies in consequence, that is, would not have died if he had not eaten of it," the death is usually attributed to eating of the dish although, as Mill points out, there were other factors contributing to the result, such as the bodily constitution of the individual, the particular state of his health, and, perhaps, a certain state of the atmosphere. These taken in conjunction with eating of the dish are, according to Mill's view, the 'invariable antecedent.' When we select one among the antecedents of a phenomenon and call this the cause, our selection, Mill says, falls generally on the one which came last into existence. But, he also points out, there is hardly any one among the number of antecedents that may not serve in this special sense as the cause "according to the purpose of our immediate discourse." This tendency "to associate the idea of causation with the proximate antecedent *event*, rather than with any of the antecedent *states*, or permanent facts, which may happen also

to be conditions of the phenomenon," Mill regards as not at all necessary to the common notion of cause. The conclusion, then, at which he arrives is that by a cause is meant "the sum total of the conditions, positive and negative, taken together; the whole of the contingencies of every description which being realised, the consequent invariably follows. The negative conditions . . . may be all summed up under one head, namely, the absence of preventing or counteracting causes."

The essential contentions of Mill in this passage are, first, that the antecedent of any fact is a complex group of facts, and, second, that the cause, popularly considered, is taken to be some one fact among the antecedents selected arbitrarily or for some special purpose. With regard to the first point we may quote Jevons. He says: "By an antecedent we mean any thing, condition, or circumstance which exists before or, it may be, at the same time with an event or phenomenon. By a consequent we mean any thing, circumstance, event, or phenomenon, which is different from any of the antecedents and follows after their conjunction or putting together. It does not follow that an antecedent is a cause, for the effect might have happened without it. Thus the sun's light may be an antecedent to the burning of a house, but not the cause, because the house would burn equally well in the night; but a necessary or indispensable antecedent is identical with a cause, being that without which the event would not take place. . . . There

are usually many different things, conditions, or circumstances necessary to the production of an effect, and all of them must be considered causes or necessary parts of the cause. Thus the cause of the loud explosion in a gun is not simply the pulling of the trigger, which is only the last apparent cause or occasion of the explosion; the qualities of the powder, the proper form of the barrel, the proper arrangement of the percussion-cap and powder, the existence of the surrounding atmosphere, are among the circumstances necessary to the loud report of a gun; any of them being absent it would not have occurred." In this passage it may be observed that what is popularly regarded as the cause, the second point mentioned by Mill, Jevons calls the occasion of the event, that is, that which, in connection with the other necessary antecedent or coexisting conditions, is effective in bringing these latter into active operation. What Jevons here speaks of as the occasion is sometimes called the immediate cause, the other necessary attendant circumstances being referred to as the conditions or causal conditions of the event in question.

That the cause of any event is some complex group of facts or circumstances is undoubtedly true. Indeed, this seems to follow as a matter of necessity from the general character of human experience. As we have pointed out frequently, objects and events are experienced by us, not as separate and individual things, but as subsisting with other objects and events with which they are connected in

a variety of ways. It is this feature of experience that was said above to be the starting point of the casual problem. Now it is no doubt true that within any experienced complex we can, and in fact do, isolate, by an act of discriminative attention, a given fact or event which we regard, for reasons special to the case, as the effect or result of other perceived or discoverable facts, and that, in this way, a certain uniqueness, a certain independence, comes to be regarded as the distinguishing mark of all effects whatever. But while no objection can be brought against thus isolating an effect, since in this way it is rendered more capable of exact description, and the particular causal relation that has to be determined is made more capable of exact definition, a similar line of remark does not apply to what we understand as a cause. For, in the first place, it is only because what we call the effect has a definite character that the search for a cause becomes a problem at all. Unless the fact or event of which we wished an explanation were made definite, there would be no point of starting for the establishment of a causal relation. It is owing to the fact that some part of the experience of which the effect is only a fragment remains indefinite that the cause is characterised at first by an indeterminateness that would be quite fatal if it attached to the effect.

But, not to press this point, we cannot be sure, in the second place, that, in any particular case, all the conditions which will make the oc-

currence of an effect intelligible fall within our experience of that case. As we have seen above, experimental and theoretical observation are valuable chiefly because they bring within experience factors, material to our understanding of a phenomenon, that are not included in our ordinary perception of it. An example, usually cited, is the discovery of the planet Neptune by Galileo in 1846. The observed fact which led to its discovery was the deviation of the planet Uranus from the course determined for it by the attractive forces of the sun, Jupiter, and Saturn. It was suggested, first by Adams and later by LeVerrier, that this deviation could be accounted for if we supposed that some hitherto unknown body, situated external to it in a particular position and at a given distance, were affecting the movement of the planet. Such a body, situated where Adams and LeVerrier had suggested, was later discovered by Galileo, and exists in our solar system as the planet Neptune.

In the third place, we may observe that, whether all the factors necessary to a causal relation are or are not parts of our ordinary perception of it, as indeed the illustration just used is sufficient to show, the effectiveness of any given fact, — the planet Neptune, for example, — seems to depend as much on its relations to other facts, — the sun, Jupiter and Saturn, — as it does on its own existence. It is no doubt true that if the fact, — the planet Neptune, — did not exist, it could be neither a cause nor any part of a cause; but if it is a cause or part

of a cause at all, it owes this character, not merely to its existence, but also to the way it acts and is acted upon by other facts in the system of facts in which the effect, — the course of Uranus, — is included as an important, and, indeed, a necessary element. To take another illustration, we cannot regard death as the result of a gun shot, poison, or any other single fact, but as the result of a network of facts which includes the gun shot or what not in a system which extends to and embraces the stated effect. From this point of view, it is evident that the problem of causation is one in which, starting with some fact or event, we endeavour to build up by observation and experiment a system of facts within which the fact or event with which we started is necessarily included. Causes, consequently, must always be complex, and the difficulty of determining causes is the difficulty of including within the system only those factors that are material to it.

Few words are necessary to make our position clear on the second point raised by Mill, namely, the importance usually given to one of the conditions of an event and naming this the cause. Practically, as Mill pointed out, the fact selected is usually the one which is nearest in time to the effect. But, as we have seen, the immediate temporal antecedence of the cause is not necessarily an essential feature of the causal relation. What rather, even for the practical view, is needed is the selection of an agent on which reliance can be placed to bring about

the desired result. Certainty as to the cause seems to be the desideratum, whether the causal factor is the immediate temporal antecedent of the effect or not. When, for example, we wish to rid the premises of rats, to borrow an illustration, we may choose either traps, strychnine, phosphorus, or terriers, and doubtless the one selected will be the one that, under the circumstances, is regarded as most efficient. And by efficient, in this connection, we mean the one that, with due regard to the circumstances of the case, realises best the purpose for which it is chosen. Now when the purposes, for the satisfaction of which causal relations are sought, become specialised as they do in science, when, that is, they are directed toward the attainment of knowledge, and not toward the improvement of the material conditions of life, the situation, so far as the causal relation is concerned, is not greatly different, for in science the search for a cause is determined by a purpose which the ascertainment of the cause itself is expected to satisfy. In the field of science, as was pointed out before, the purpose that directs an inquiry does not lie outside of, but is one with, the attainment of scientific truth. In this the chief difference between the theoretical and practical life is to be found. But when we have said that, it remains true that the conditions of knowledge are more rigid than those of practice, and the goal of inquiry requires more strict definition. Hence we find that what passes for a cause in practical life may seem, from the scientific point of

view, altogether inadequate, if not false, and, consequently, that a more thorough analysis of the antecedent and concurrent circumstances has to be undertaken if the end of scientific truth is to be reached. Now, whether or not, for the purpose of science, all or only part of the circumstances and conditions of an event are included in its cause will depend on the particular and immediate purpose it is intended to satisfy. No doubt, the ultimate goal of science is to discover laws of connection which are capable of extension to other than the observed phenomena, but, at the same time, we should observe that it can reach this end only by the determination of the particular connections of one empirical event with another in human experience. But even in this more restricted field of scientific effort, a certain rigid demand is usually made and observed, namely, that all and only those factors which can be regarded as essential conditions of a fact or event should be enumerated as its cause. It is only when there is a lapse from strict scientific precision that any thing less is possible; and even when such lapse takes place, the scientist feels himself obliged to fill out, on demand, the scantier statement by restoring all the enumerated elements that are of material importance to our understanding of the case.

The Necessity of Causal Events. The two factors of Mill's view of Causation that we have considered are, first, that causes are invariable antecedents of the facts or events we call effects, and

second, that causes are always complex groups of fact, the sum total of conditions, as Mill says, requisite to bring the effect into existence. There is still a third factor to which we may now give attention, a factor, it would seem, that occupies, in Mill's total view, a place which gives it a peculiarly special prominence. For, according to Mill, you may have a sequence displaying the features of complexity and invariability and not have one that is definitely causal. That is to say, B may have followed A invariably in our experience, and yet the ground or cause of the sequence A-B may really not be in A at all. The illustration, borrowed from Reid, that Mill uses is the sequence of night and day, in neither of which can the cause of the other be found, although there is no doubt that the empirical connection extends throughout human experience. Some other factor, therefore, is required to complete the idea of cause, for, as Mill says, "it is necessary to our using the word cause that we should believe not only that the antecedent always has been followed by the consequent, but that as long as the present constitution of things endures it always *will* be so." Unless the sequence can also be regarded as 'unconditional,' we are not warranted, by Mill's view, in considering it to be causally determined. "This is what writers mean when they say that the notion of cause involves the idea of necessity. . . . That which is necessary, that which *must* be, means that which will be, whatever supposition we may make in regard to all other

things. . . . That which will be followed by a given consequence when, and only when, some third circumstance also exists, is not the cause, even though no case should ever have occurred in which the phenomenon took place without it. Invariable sequence, therefore, is not synonymous with causation, unless the sequence besides being invariable, is unconditional."

It will be seen, at once, that in this statement, and only here, do we come, in Mill's view, face to face with the question, crucial for our comprehension of causal relations, of what we must mean when we say that one event is necessarily connected with another. Now it should be remarked that this statement of Mill's raises questions which for their adequate discussion would require us to go outside the limits of inductive logic into the field of general metaphysics; and this is true, not only for a student of the theory Mill proposes, but it is admitted as necessarily so by Mill himself. For, as he says, it is only so long as we can believe that the "present constitution of things endures," only so long, that is, as "the ultimate laws of nature (whatever they may be) as distinguished from the derivative laws and the collocations" remain unchanged, that what happens in experience can become a clue to what is true outside experience. In other words, it is only so long as the nature of reality is and remains what it is that we can argue from what has been to what will be in human experience. But the determination of the nature of reality, and of the

relation that we as thinking and active individuals sustain toward it, is the definite problem of metaphysics, and only indirectly does this problem come into view in the discussion of logical questions. But even incidentally it is worth while to remark that the relation of logic to metaphysics that the position of Mill raises is such that, unless we can presuppose an ordered system of reality of some sort, our confidence in human knowledge is ultimately shaken, and the distinction between truth and error to a large extent is obliterated. For, as we have seen throughout, the nerve of human knowledge is to be found in the confidence we have that in all science and philosophy, and, we should say, in all art and religion as well, we are interpreting, not the empty shadows of a merely temporal experience, but the real nature of the world which is not only the substance of all our knowing, but the very centre and core of the being which we as individuals possess.

Turning, now, from these rather abstruse considerations, we must try to interpret Mill's position with respect to its logical implications. In order that we may understand exactly what Mill had in mind to say, it will be necessary to gain and secure a correct notion of the term in which he translates the necessity that belongs, as he and Hume before him admit, to our notion of cause. According to Mill, the necessity that we ascribe to a causal sequence has its foundation in the unconditional character of the causal sequence. What, then, does Mill mean by unconditional? The meaning that Mill attaches,

in a general way, to this word is perhaps not difficult to detect, although he does not state definitely the sense in which it is understood. Most of his statements about it are negative. He says, for instance, that "that which will be followed by a given consequent when, and only when, some third circumstance exists, is not the cause," the meaning evidently being that it is a cause only when the consequent follows without such mediation. And, again, he says that "if we adopt the convenient modification of the meaning of the word cause which confines it to the assemblage of positive conditions without the negative, then instead of "unconditionally," we must say, "subject to no other than negative conditions," where, again, the absence of some third circumstance is necessary for determining an unconditional sequence. The point that Mill seems to make is that a sequence is unconditional when the consequent is seen to follow from the totality of conditions which constitute its cause, without any aid from any other circumstance which falls outside its cause as thus determined.

Two remarks may help to make the meaning of this statement clearer. In the first place, Mill's view leaves out of account, as indeed it has a right to, such metaphysical considerations as were referred to a little while ago. However impossible such a procedure would be if Mill were discussing the causal problem from the standpoint of the philosophy of knowledge, it is quite possible and even necessary when he is concerned merely with the

invariable connections that are built up in the course of human experience. The denial, therefore, that any "third circumstance" operates in an unconditional sequence cannot be meant, by Mill, to refer to the "constitution of things" which, as we saw, is in some sort the foundation on which all human knowledge in the long run depends. In the second place, we may remark that the "third circumstance," on the absence of which from any sequence the unconditionality of the cause is determined, must be interpreted to mean some third phenomenal circumstance, some further fact or event of human experience the same in kind as those that enter into the causal relation. What this statement amounts to is the declaration that whenever any causal sequence has been determined, it will be found to include all those positive factors on which the occurrence of the phenomenon depends, and that it is only when all such factors are present that the sequence is necessary, or, in Mill's term, unconditional. It may help to bring out the significance of such a statement if we say that when the stated conditions are satisfied in any given case, we can argue not only from the occurrence of the cause to the occurrence of the effect, but also from the occurrence of the effect to the occurrence of the cause. The presence of either the cause or effect necessarily implies the presence also of the effect or the cause.

The Plurality of Causes. In view of the fact that the aim of science is to point out a connection between a particular event and the totality of condi-

tions on the presence of which the event takes place, and on the absence of which it does not take place, we may ask how this conception of the aim of science is affected by the fact that the same event may occur for very different reasons. Now about such a fact there is no question. It was fully recognised, for instance, by Mill who tersely remarks that "it is not true that the same phenomenon is always produced by the same cause." The fact, then, is that an event M may be caused now by A, and, again, by B or C or D. This fact is known, in the terminology of Mill, as the plurality of causes. What effect, we may ask, does the recognition of such a plurality of causes have upon our view of the nature of causation as expounded in the preceding paragraphs?

If we consider this question strictly from the point of view of what we have seen the relations between cause and effect in a causal sequence must be, it is obvious that we can not regard those events whose occurrence depends on a possible plurality of antecedents as causally related events at all. It was said just now that in a causal sequence we must be able, not only to argue from the occurrence of the antecedent to that of the consequent, but also to argue from the occurrence of the consequent to that of the antecedent. Now the significance of such a connection between antecedent and consequent in a causal sequence is that it goes far beyond what, in an earlier chapter, we saw was formally possible when we were considering merely hypo-

thetical statements of fact. The rule concerning hypothetical statements, 'If A then B' for example, made it clear that from the occurrence of B we could say nothing with respect to the occurrence of A, and the reason assigned was that B might have occurred for some other reason than A. In other words, we had to recognise the possibility of what Mill calls the plurality of causes. Now the necessity for such a rule for dealing with the facts of our general experience is quite obvious when we consider that, with the development of science, there has come to be recognised a marked difference between empirically connected events and causally related events, and, consequently that until the causal character of a particular sequence has been established, we cannot be certain that one rather than another group of possible antecedent conditions, as a matter of fact, has been the occasion of any given consequent.

We may also point out that the recognition of a plurality of causes, if we emphasise the word cause in Mill's phrase, would necessarily throw our whole conception of cause into confusion, and there would remain for us no means of distinguishing between those events which were really connected as cause and effect, and those whose connections depended, in Mill's words, on some "third circumstance," events, that is to say, which are connected merely after the manner of an empirical juxtaposition. The distinction between causal sequences and empirical ones is important, it marks one of the gains

that have been acquired as the result of much labour, and it serves the distinctive purpose of mapping out the fields with respect to which we may claim to have different degrees of knowledge. It is certainly true, for instance, that the inferences drawn from our ordinary experience are entertained much less confidently than are those that rest on the foundation of a scientific induction. In the latter case alone do we have certainty; in the former, we reach only a higher or lower degree of probability. And the difference between certainty and probability rests, in part at least, on the fact that in the case of certain knowledge all the factors material to it are known, and in the case of probability we have no reason for believing that this requirement has been satisfied. And this statement enables us to see why it is that analysis plays so large a part in the method not only of the sciences but of philosophy as well, and why it is that, from the times of the Greeks, the distinction, based on the application of a method of analysis to the facts of experience, between opinion and knowledge, between the popular beliefs of an age on the one hand, and carefully established truths on the other, has been regarded as a condition of intellectual culture and progress. We cannot, therefore, entertain any suggestion of treating the two classes of connection, the causal and the empirical, as if no distinction had been made between them, and we cannot do so because all scientific advancement depends on it, and

because it would leave us without any adequate conception of what we should mean by a cause.

The result of our consideration of the conception of a plurality of causes amounts to this, that when we look at the conception from the standpoint of a carefully stated view of what causation implies, it seems to be a quite misleading, if not mischievous, conception. But, it may be objected, this result can be reached and maintained only if we allow a somewhat strict interpretation of the causal relation to determine the view we take as to the possibility of a plurality of causes. That is exactly true. But the recognition of its truth does not require us to alter our notion of the causal relation, but to indicate the field within which the operation of more than a single set of antecedents as a condition of the occurrence of a given fact or event may be said to occur. What we have seen is that the field of causally established relations is not such a field. For science in the strict sense of that term there is only one group of conditions that can be inferred from the occurrence of any given event. But when we remark that, as compared with the broad extent of our ordinary experience, the causal field is only a very small area, we see at once that, when we go outside this field, there must be possible some other than a causal interpretation of the sequences that occur. It follows, therefore, that we must go outside the field of the natural sciences if, looking at our experiences from the standpoint of their temporal rela-

tions, we wish to emphasise and illustrate the possibility and nature of a plurality of causes. Nowhere, except in the sciences, can we argue from consequent to antecedent. The relations of antecedent and consequent, therefore, that occur in our ordinary experience, are not, in the strict sense, causal relations at all. And because they are not, it seems altogether misleading to speak of these relations as involving a plurality of causes. What a discussion of the possibility of a plurality of causes forces on us is, then, the recognition of a type of experience which implies no more than a plurality of antecedent or concurrent conditions. To call these conditions, in their relation to the event with which they are connected in our experience, causal is to go beyond what we know about them; for, as we have said, the test of a causal relation is that we can argue, not only from a given set of conditions to the conditioned, but also from the conditioned to a particular set of conditions. It is exactly the impossibility of doing this latter that led Mill, in the class of cases we are now considering, to speak of a plurality of causes, and which leads us to speak of a plurality of conditions. If, by our terminology, we can keep clearly before ourselves the fact that the relations which we are now considering are not causal, in the strict sense of that word, but that they are connected empirically merely in our experience, it will be possible to set over against each other the two types of experience, and to entertain the problem, which the non-causal relations certainly sug-

gest, whether these latter can all be reduced to a single causal type. Although it takes us outside the limits of our present inquiry, we may hazard the remark that perhaps human life and experience are too rich and complex to be capable of interpretation throughout in terms of any single class of relations.

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QUESTIONS

1. In what general fact of observation does the problem of causation originate?
2. How, in view of this fact, may the causal problem be stated?
3. Explain what is meant by saying that the causal problem is practical before it is theoretical. Illustrate.
4. In what way does the practical aspect of the problem of causation tend to develop human intelligence?
5. Describe as accurately as you can the precise nature of the regularities in experience which we are led to observe from the practical point of view.
6. In what two respects does the theoretical differ from the practical view of causation, and what advantages are thereby secured?
7. State and illustrate Mill's criticism of the practical and theoretical conceptions of cause.
8. Show, by means of illustrations, that the practical and theoretical conceptions of cause are genetically related.
9. In what way has the theoretical notion of cause reacted upon the practical to give it a more definite and extended meaning?
10. What is meant by saying that the causal problem arises from the importance of order in human experience?
11. 'The causal problem is concerned with the question of how things happen as they do.' Explain.
12. What, according to Mill, is the first characteristic of a causal relation?
13. What is meant by the time relations of causal events?
14. How far do you agree with Mill that the cause of an event is its invariable antecedent?

15. What can you say with respect to the view that there is no essential difference between a cause and an effect.
16. Explain what is meant by the temporal continuity of causal events.
17. What, according to Mill, is the second characteristic of causal relations, and to what two facts of importance does it direct attention?
18. What does Jevons mean by the occasion of an event as contrasted with its cause? What other term is sometimes used for Jevons' 'occasion'?
19. Give three reasons for emphasising the complex character of causes.
20. What can be said for and against selecting one of the conditions of an event and calling that the cause?
21. In what way would your answer to the previous question be affected by the consideration that one's interest in the causal relation is practical and not theoretical?
22. What, according to Mill, is the third characteristic of causal relations?
23. Does Mill recognise the 'necessity' of causal relations, and, if so, what does he mean by necessity?
24. Does Mill's view of causation imply an appeal to metaphysics, and, if so, in what respect?
25. Explain what Mill means by unconditional.
26. Is it possible, on Mill's view of causation, to argue from the occurrence of an effect to that of the cause?
27. What is meant by the plurality of causes?
28. If in any particular instance we admit a possible plurality of causes, how does that admission affect our conception of its strictly causal character?
29. What is the difference between causally and hypothetically connected series of events?
80. Why would it be preferable to speak of a plurality of conditions instead of a plurality of causes?

EXERCISES

1. Enumerate six instances of regularity in the order of particular events which are of importance for the practical life.
2. Develop, from the theoretical point of view, the instances mentioned in your answer to the previous question in order to show to what question of scientific interest each may lead.
3. Examine six instances of scientific causation for the purpose of showing how each satisfies the three conditions of a cause mentioned in the text.
4. Examine the following and show in what respects they are, and in what respects they are not, statements of strictly causal relations.
 - (1) In my experience A has been invariably preceded by B, and we may therefore conclude that B is the cause of A.
 - (2) Scarlet poppies, scarlet verbenas, the scarlet hawthorne and honeysuckle are all odourless, therefore we may conclude that all scarlet flowers are destitute of odour.
6. Mention six instances which illustrate, what Mill calls, a plurality of causes. State for each instance the possible causes.

CHAPTER XVII

MILL'S EXPERIMENTAL METHODS

Their Historical Position. The publication of Mill's *System of Logic* in 1843 gave, no doubt, a great impetus to the study of what is commonly known as inductive logic, and especially to the consideration of the principles that underlie the methods by which, in the several sciences, causal relations are determined. Prior to Mill, however, interest in the inductive inquiry had had a somewhat long history, and the reader will find in Minto's *Logic* (pp. 243-272) an account of the course that this interest took from the time of the Franciscan monk, Roger Bacon (1214-1292), to the modern era which was ushered in, and whose development was determined by the progress of experimental science. That the advancement of learning, of which Francis Bacon (1561-1626) may be taken as the modern herald, should have maintained itself successfully for over two centuries, and that during that period natural science should have extended its scope as well as secured its position, could not be without its influence upon the conception that was held of the nature and value of logical inquiries. In the presence of such a solid body of achievement as came rapidly to the credit of the natural sciences, it was inevitable that interest should be aroused in the methods by which such results were gained. Con-

sequently, we find that men of learning in this period gave considerable attention to the study of scientific methods, and in the first half of the nineteenth century Herschel's discourse *On the Study of Natural Philosophy*, (1831), and Whewell's two large volumes on the *History of the Inductive Sciences*, (1837), and his *Philosophy of the Inductive Sciences*, (1840), stand out prominently as contributions of capital importance to our understanding of the inductive procedure. Based as they are upon the facts of experimental science, and containing a description and analysis of the methods by which conclusions had been reached in the sciences, they constitute a repository of great value for any one who would study the theory of inductive science. With these books before him, Mill set out in his *Logic*, as he says, to "generalise the modes of investigating truth and estimating evidence, by which so many important and recondite laws of nature have, in the various sciences, been aggregated to the stock of human knowledge." In the chapter on 'The Four Methods of Experimental Inquiry,' Mill gives an account of the various ways in which causal relations are or may be determined. In what follows, we propose to describe these methods briefly, and shall limit ourselves chiefly to the task of stating and illustrating them. To determine the view that Mill took of them as a whole, or to estimate their value in detail, falls outside the scope of an elementary study of logical problems.

The Nature of the Problem to which these Methods Apply. In the foregoing chapter, we have seen that the causal relation, strictly interpreted, is reciprocal, that is, it is a relation between phenomena of such a kind that from the presence or absence of either term of the relation we can argue to the presence or absence of the other. The rise and fall of the mercury in a thermometer, for example, are reciprocally related to the varying degrees of temperature. To the establishment of such relations the sciences doubtless aspire in their investigation of the processes of nature. And that a considerable degree of success in this particular has been achieved in those sciences, such as physics and astronomy, which rest upon a mathematical foundation is sufficient to suggest, to those who are interested in the logic of the sciences, an inquiry to which the methods of Mill are the direct answer. Mill's methods, in other words, are a statement of the processes which have been found useful in the establishment of those causal relations in which the antecedent and consequent reciprocally imply each other. But since reciprocal relations have not been established between the antecedent and the consequent of all known phenomena, we should be putting an unnecessary strain upon these methods if we did not regard them as applying, in the strict sense, only to those special cases in which a relation of the kind in question had been determined. We may also remark that these methods are not offered by Mill as a substitute for the specialised modes of investigation which are

developed by each special science according to its needs. Indeed, the methods are quite general in nature, and could not have been formulated had not relations of the kind they imply been already successfully established. They must be understood, therefore, as revealing the logic of causal inquiries, when these inquiries have led to the determination of reciprocal relations between causes and effects.

Elimination as a Method of Causal Inquiry. We have just seen that Mill's methods aim to state the general modes of procedure in those instances in which reciprocal causal relations have been established. In the light of this statement, it might seem superfluous to ask whether the causal inquiry rests upon the process of eliminating from the cause, or effect, those data whose presence or absence from a phenomenon makes no difference either to its occurrence or non-occurrence. This is, indeed, the position that we should be inclined to adopt were it not for the fact that a difference of opinion exists as to whether, and in what sense, the methods, as Mill expounds them, depend upon such process of elimination.

Joseph, for example, takes the view that "the causal circumstances are indicated by a process of exhaustive elimination." His position, more fully stated, is that, in view of the nature of causal relations, certain particular requirements must be satisfied by any instance which claims to involve a causal relation, and that the failure to satisfy any one of these requirements disproves that relation between two given phenomena. These requirements he enu-

merates as grounds of elimination, and are as follows:

1. Nothing is the cause of a phenomenon in the absence of which it nevertheless occurs.

2. Nothing is the cause of a phenomenon in the presence of which it nevertheless fails to occur.

3. Nothing is the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it.

4. Nothing is the cause of a phenomenon which is known to be the cause of a different phenomenon.

These grounds of elimination are said by Joseph to be the foundations of Mill's inductive methods. The first is the foundation of the Method of Agreement; the second, of the Method of Difference; the first and second combined, of the Joint Method of Agreement and difference; the third, of the Method of Concomitant Variations; and the fourth, of the Method of Residues. It thus comes about, according to Joseph, that "Mill has not formulated four (or five) but one "Method of Experimental Inquiry" . . . of which the essence is that you establish a particular hypothesis about the cause of a phenomenon by showing that, consistently with the nature of the relation of cause and effect, the facts do not permit you to regard it as the effect of anything else (and *mutatis mutandis* if you are inquiring into the effect of anything)." Thus if I know that S must be caused by L or M or P or Q, because these are the instances in which the phenomenon occurs, then by testing each instance by the canons

of elimination I am able to discover the one relation in which S stands that is reciprocally causal.

Against this position Gibson argues that there is in Joseph's account a confusion between physical and logical elimination, and that when Mill speaks of the Methods of Agreement and Difference as methods of elimination he must be understood to mean the former and not the latter kind of elimination. The meaning of this contention is quite clear if by physical elimination is understood the method of excluding from a phenomenon certain elements which are found not to have an effect upon the occurrence or non-occurrence of the phenomenon. By logical elimination, on the other hand, is meant the rejection of certain hypotheses — as, for example, some of the alternatives in a disjunctive proposition — with respect to the causal connection. In this case, we are rejecting the non-causes as a means of discovering the true causes. There is a certain presumption in favour of Gibson's contention, for Mill himself says that elimination is a term "well suited to express the operation . . . which has been understood since the time of Bacon to be the foundation of experimental inquiry — namely, the successive exclusion of the various circumstances which are found to accompany a phenomenon in a given instance, in order to ascertain what are those among them which can be absent consistently with the existence of the phenomenon." But whether any or all of Mill's Methods are based upon a single type of elimination can be decided only by exam-

ining the methods themselves. For the present, it is enough to have stated the nature of the difference between the views that are held of the relation of elimination to the Methods that we are about to study.

The Method of Agreement. Mill states the principle upon which this Method rests as follows: If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon. The requirement that the instances in question shall have only one circumstance in common can not be satisfied unless all the other circumstances that are relevant to the phenomenon are different. Not any enumeration of instances, therefore, would satisfy the conditions of this Method, but only one in which all the instances, however much they may differ from one another, are alike in only a single particular. When, then, two or more instances of a phenomenon are shown to have only one circumstance in common, that circumstance, according to Mill, is the cause or the effect of the phenomenon. For this reason, the Method has been called the Method of Single Agreement, and is formulated by Jevons as follows: The sole invariable antecedent of a phenomenon is probably its cause. An illustration borrowed from Bain, may be given. "The North-East wind," Bain writes, "is generally detested in this country; as long as it blows, few people are at their best. Occasional well known causes of a wind being

injurious are violence, excessive heat or cold, excessive dryness or moisture, electrical condition, the being laden with dust or exhalations. Let the hypothesis be that the last is the cause of the North-East wind's unwholesome quality; since we know it is a ground current setting from the pole toward the equator and bent westward by the rotation of the earth; so that, reaching us over thousands of miles of land, it may well be fraught with dust, effluvia, and microbes. Now, examining many cases of North-East wind, we find that this is the only circumstance in which all the instances agree; for it is sometimes cold, sometimes hot; generally dry; but sometimes wet; sometimes light, sometimes violent; and of all electrical conditions. Each of the other circumstances, then, can be omitted without the North-East wind ceasing to be noxious; but one circumstance is never absent — namely, that it is a ground current. That circumstance, therefore, is probably the cause of its injuriousness."

It will be observed from this illustration that the Method is essentially a method of elimination; its use depends upon the possibility of analysing the phenomenon into its constituent parts, and of selecting from among them the one that is causal. It is evident, therefore, that the method cannot begin to be put into operation until we have before us at least two instances of the phenomenon, and that the reliability of the Method is much increased with an increase in the number and variety of the instances that are examined. In view of such facts

as these, Mellone has formulated the method as follows: When observation shows that two events accompany one another (either simultaneously or in succession), it is probable that they are causally connected; and the probability increases with the number and variety of the instances.

When this Method speaks of causes as single antecedents, it is evident that it presupposes that the phenomenon under investigation has been subjected to analysis, and that in this way its elements have been distinguished. Now, with respect to the supposition that causes are to be discovered among the elements into which the phenomenon is analysed, there are two questions to be asked. The first is, are we certain, in view of the inter-connection of all parts of the known universe, that single events ever do act in isolation from all the rest? But not to press the point in that form, are we certain that within particular universes, that is, with reference to particular groups of events, we can analyse a point for point concomitance between causes and effects? For instance, is the "ground current" of the illustration used above, on any supposition, a single antecedent? Is it not merely another name for "dust, effluvia, and microbes"? If, however, it be replied that the object of science is to establish causal connections between single antecedents and consequents, we may ask, in the second place, whether there exists any means of assuring ourselves that the analysis of a particular phenomenon, however complete it may seem, has succeeded in

detecting all of the factors present in the event that we call its cause? For a long time, for example, air was regarded as an element; and even since its analysis into oxygen, nitrogen, hydrogen, carbon dioxide, etc., modern chemistry has discovered still other elements in certain quantities. It may very well happen, therefore, that any supposed single antecedent is really complex. In most cases, it is only a relative simplicity that can be attained, a simplicity that is met with at the point where our methods of analysis are unable to carry us any further. We conclude, therefore, that the Method states an ideal that, in practice, is difficult to attain, and, when attained, to definitely certify.

What degree of confidence, we may ask, may be reposed in the results that are attained by the use of this Method? In other words, what are the limits of its usefulness? To be more specific still, is it distinctively a method by which we are able to explain the occurrence of phenomena? This question we must answer in the negative. The Method of Agreement is, as one has said, "a mode of reconnoitering," a method for delimiting the area within which causal relations may, with some degree of confidence, be expected to exist. But for the establishment of such relations we should have to pass beyond the method itself. Let us suppose, for example, that three instances of a phenomenon may be analysed in the following schematic way:

Antecedents	$\left\{ \begin{array}{l} A \ B \ C \\ A \ B' \ C \\ A \ E \ F \end{array} \right.$	$\left\{ \begin{array}{l} x \ j \ z \\ x \ y' \ w \\ x \ v \ u \end{array} \right.$	Consequents
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Are we certain, in such a case, that A; the circumstance in which all the antecedents agree, is the cause of the phenomenon? Starting with the observation of x, does it follow, because our analysis of all the instances of x shows A to be the one element that is common to these instances, that A is, therefore, the cause of x? Mill has pointed out in answer to such questions that their affirmative answer would take us beyond the limits of this Method, when and so long as we are obliged to consider, in our interpretation of the instances, the possibility of a plurality of causes, the possibility, that is, that x may be produced by more than one cause. If the instances of the phenomenon were known to be reciprocal, or if we were certain that we had all the instances before us, the situation would be different; but so long as neither condition is guaranteed, it only remains probable, as Jevons' and Mellone's formulations of the Method indicate, that the "one circumstance in common" is the cause of the phenomenon in question. It thus appears, to quote Mill, that the Method of Agreement "leads only . . . to unifomities, which either are not laws of causation, or in which the question of causation must for the present remain undecided. The Method of Agreement is chiefly to be resorted to as a means of suggesting applications of the Method of Differ-

ence, . . . or as an inferior resource in case the Method of Difference is impracticable; which . . . generally arises from the impossibility of artificially producing the phenomena," that is, when experimentation is not possible.

The Method of Difference. The Method of Agreement, as we saw, is a method of systematic observation, and its object, negatively stated, is to eliminate from the occurrences of a particular phenomenon those elements which, because they are not constant in all the instances, cannot be regarded as being the cause, or part of the cause of the phenomenon, and thus, affirmatively stated, to map out the area within which the cause may probably be found. Turning now to the Method of Difference, we find that it is a method of scientific experiment, and the formulation given it by Mill is as follows: If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former, the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon. Let us suppose, for example, that two instances of a phenomenon can be analysed schematically as follows:

$$\text{Antecedents } \left\{ \begin{array}{l} A \ B \ C \\ \quad B \ C \end{array} \right. \qquad \left. \begin{array}{l} x \ y \ z \\ \quad y \ z \end{array} \right\} \text{Consequents}$$

then we may say, according to this Method, that A is the cause of x. Thus, let us suppose that in two glasses equal quantities of lemon juice and water are mixed, and that in one of them a given quantity of sugar is dissolved. The taste of the liquid which contains the sugar, we shall say, is pleasant, and the taste of the liquid which does not contain sugar is unpleasant. What is the cause of the pleasant taste? The circumstance, says Mill, in which alone the two instances differ, namely, the sugar.

In calling the Method of Difference a method of scientific experiment, we may observe that scientific experimentation, as this was defined above, involves a definite control of the conditions under which a phenomenon occurs, and a definite inquiry to which the experiment is expected to give an affirmative or negative answer. Scientific experiment is like a shafted arrow, it is aimed at a mark, and its course toward that mark is controlled, as is the arrow which is sped from the bow. To illustrate, let us suppose that we wished to know why a feather, dropped simultaneously from the same height as a coin, reaches the ground after the coin. Our question will be, Is the greater resistance offered by the air to the feather the cause of the observed phenomenon? To answer this question, we must institute an experiment, and to do this, as we said, it is necessary to bring the phenomenon under controlled conditions. Thus, we first drop the two objects from the same level and at the same time in, let us say, the receiver of an air-pump. The feather is

observed to flutter to the base of the pump after the coin. We perform the same experiment again, with the exception that we take care to remove all the air from the receiver. Under these conditions, the coin and the feather reach the base of the receiver at the same time. The conclusion is that the only circumstance in which the two instances differ, namely, the presence of air in the former, is the cause of the phenomenon. Because, as in this example, the Method proceeds systematically by changing one condition at a time, it is known as the Method of Single Difference, and the following formulation has been given by Mellone: When the addition of an agent is followed by the appearance, or its subtraction by the disappearance, of a certain event, other circumstances remaining the same, that agent is causally connected with the event. Or, to state the principle on which any formulation of the Method relies, we may say that the circumstance in the absence of which a phenomenon fails to occur is the cause, or part of the cause, of the phenomenon.

It will add to the clearness of our apprehension of the meaning of this Method, if we distinguish the instances of the phenomenon that are successively observed. For this purpose, let us call the instance in which the suspected cause occurs the positive instance, and the one in which it does not occur the negative instance. By this terminology is meant that, in what is called the positive instance, we have the occurrence of the observed phe-

nomenon along with the occurrence of its supposed cause, and, in the negative instance, the absence of the phenomenon together with the absence of the supposed cause.

In view of this distinction, it will be seen that the Method depends, for its validity, on the negative instances through which a suspected cause is either proved or disproved. And, for this reason, it rightly insists on the importance of varying only one condition of the phenomenon at a time. For if we were to vary simultaneously two of a phenomenon's conditions, and found that some change resulted, we should be unable to tell whether this change was due to one of the altered conditions, or to the other, or to the joint action of both. And if no change ensued, we could not safely conclude that neither condition was affective, for the action of the one may have been neutralised by that of the other. And even when only a single condition is varied at any one time, it is important, as Mill pointed out, that the experiment should be conducted with dispatch, and its entire process brought as rapidly as possible to an end. The reason for this caution is that if the experiment is to serve the purpose for which it was instituted, the effect must be the effect solely of the one altered condition. If, on the contrary, during the course of an unusually prolonged experiment, the original conditions of the experiment were modified by the development of changes which determined in part the character of the observed effect we should have, what Mill called, an inter-mixture of

effects; and this would be as destructive of the value of the Method as any unsystematic variation in the conditions themselves.

It is a well known fact that not all problems are capable of experimental handling, and that those that are, are liable to have their results affected by the operation of conditions which we can in no wise eliminate. In the former case, the Method of Difference cannot be used. In the latter, the conditions referred to are those that belong to the structure of the universe to which the particular phenomenon belongs. Such conditions Mill called permanent causes. "The pendulum, for example, has its oscillations disturbed by the vicinity of a mountain." What, in such a case, can be done to make the Method applicable? It is evident that we cannot remove the mountain, but, as Mill points out, we can take the pendulum to a locality where the mountain's interference is reduced to a minimum. In this way, it ceases to be an "influencing agent," and becomes merely a "co-existing fact." When an experiment cannot be removed outside the sphere of the operation of such permanent causes, recourse may be had to the Method of Concomitant Variations which is discussed below.

The Joint Method of Agreement and Difference. There is, no doubt, something ideal in the requirements that are laid down as conditions of the use of the Methods of Agreement and of Difference. It is rarely that in the course of common life or scientific investigation a number of instances are given which

agree or differ in only a single instance. There seems to be, therefore, a breach between experience, on the one hand, and these modes of establishing causal relations, on the other. Either, then, we shall have to say that the Methods of Agreement and of Difference are of only limited application, and that causal connections cannot be established in a great deal of our experience, or that, with respect to that part of experience which does not come directly within the scope of these Methods, some other means of determining causal connections must be used. It is to meet the latter of these alternatives that the Joint Method of Agreement and Difference is particularly useful. The formulation of this Method by Mill was made necessary in view of the fact, as he says, that there are "many cases in which, though our power of producing the phenomenon is complete, the Method of Difference either cannot be made available at all, or not without a previous employment of the Method of Agreement. This occurs," says Mill, "when the agency by which we can produce the phenomenon is not that of one single antecedent, but a combination of antecedents, which we have no power of separating from each other and exhibiting apart." Besides those cases in the natural sciences which Mill had in mind, the Joint Method will be found applicable, as Creighton has pointed out, when we try "to reach generalizations regarding the behaviour of human individuals or human societies — in looking for moral, or social or economic laws."

The Joint Method is formulated by Mill as follows: If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon. Thus, we are concerned, in this Method, with two sets of instances, one set of positive instances, in which the phenomenon with its suspected cause is present, and one of negative instances, in which the phenomenon with its suspected cause is absent. When these conditions are satisfied, the Method affirms that the circumstances in which the two sets of instances differ is causally connected with the phenomenon. Let us suppose, for example, that two sets of instances, positive and negative, can be, schematically, analysed as follows:

POSITIVE INSTANCES

Antecedents	{	A B C D	x y z w	}	Consequents
		A B E F	x y v u		
		A C F G	x z u t		

NEGATIVE INSTANCES

Antecedents	{	B C D E	y z w v	}	Consequents
		C F G H	z u t s		
		E H J K	v s r q		

A comparison of these instances shows that when x is present A is present, and that when x is absent

A is absent, and this suggests that A and x are causally connected. But we cannot prove this by the Method of Agreement, because there are no two instances which agree in having only a single circumstance in common; and we cannot prove it by the Method of Difference, because there are no two instances which differ in no respect except in the presence of A and x in the positive instances, and their absence in the negative ones. Moreover, it will be observed that the instances supposed do not conform strictly to the requirements of Mill's statement, and that it is exactly this divergence that sets the problem which this Method is intended to solve. In other words, the agreements and differences are greater in the instances we have supposed than they have any right to be on a strict interpretation of Mill's statement. The reason for this divergence is that Mill's statement of the Method, as Mellone remarks, "is vague, and it is incorrect in more than one point; two positive instances would never be enough, still less could two negative instances; and it is not necessary that the negative instances should have 'nothing in common,'" and, as our illustration shows, it is not necessary that the positive instances should agree in only a single circumstance. With these amendments in view, we may quote Mellone's proposed formulation of the Method: Whatever is present in numerous observed instances of the presence of the phenomenon, and absent in observed instances of its absence, is probably connected causally with the phenomenon.

How, now, can we proceed to apply the Method? If we examine the positive instances, employing for that purpose the general principle of the Method of Agreement, we shall see that in the first two cases A or B or AB is the cause of the phenomenon x, since A and B are the only conditions present in both. For a similar reason, we see that A or C or AC is the probable cause of the phenomenon in the first and third cases; and that the cause in the second and third cases is probably A. Now if the probable cause in all these instances is the same, that cause must be A, since that is the only particular in which they all agree. If, now, we examine the negative instances, employing for that purpose the general principle of the Method of Difference, we note that they all agree in the absence of A and x, and that this is the only relevant circumstance in which they do agree. The absence in the negative instances of the supposed cause of x along with the absence of x, when taken in connection with the relative unimportance of the other circumstances, greatly strengthens the supposition that was reached as the result of examining the positive instance, namely, that A is the cause of x. Such a Method as the one here outlined has been called by various names. Mill spoke of it sometimes as "a double employment of the Method of Agreement," whence it is often called the Method of Double Agreement. Mill also spoke of it as the Indirect Method of Difference. But the name by which it is commonly known is the Joint Method

of Agreement and Difference, a name which Mill also used in referring to the method.

The following application of the Joint Method to a social problem illustrates so well, not only its usefulness in this class of problem, but the nature of the Method itself, that we shall quote it entire. "We may suppose," Creighton writes, "that in a certain part of the country it was noticed that a considerable difference existed in the number of criminal offenses committed, in proportion to the number of inhabitants, in the various towns. In several towns the percentage was high, while in others it was relatively small. This being so, a question naturally arose as to the cause of the high percentage. Now there were among the people various opinions concerning the matter. . . . Not being able to agree about the matter, it was decided to appoint a committee to investigate the circumstances existing in various towns where the same general conditions prevailed, and upon the basis of this comparison to decide. The towns with a high criminal percentage were examined first. The report of conditions there was as follows:

- Town A: Small police force—efficient schools—severe penalties—inactive churches—licensed saloon.
- Town B: Small police force—efficient schools—light penalties—active churches—licensed saloons.
- Town C: Large police force—inefficient schools—severe penalties—active churches—licensed saloons.
- Town D: Large police force—inefficient schools—light penalties—inactive churches—licensed saloons.

"This report revealed the fact that in each of these towns having a high criminal percentage there was one circumstance, and only one, invariably present,—the licensed saloon. This rendered it probable that the saloon was the cause of the high percentage of crime. Still, before finally deciding, it was thought well to investigate negative instances as well; that is, towns in which the high percentage of crime did not occur. The report of conditions there was as follows:

- Town E: Large police force—efficient schools—light penalties—active churches—no licensed saloons.
- Town F: Large police force—inefficient schools—light penalties—active churches—no licensed saloons.
- Town G: Small police force—efficient schools—light penalties—inactive churches—no licensed saloons.
- Town H: Small police force—inefficient schools—severe penalties—active churches—no licensed saloons.

"This table showed that in the absence of the phenomenon (high criminal percentage) one and only one of the conditions concerned was invariably absent; namely, the licensed saloon. This confirmed the previous report and established to the satisfaction of all that the saloon was, at least, the main cause of the high criminal percentage in the cities concerned."

The Method of Concomitant Variations. Mill states this Method as follows: Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either

a cause or an effect of that phenomenon, or is connected with it through some fact of causation. The Method presupposes two classes of cases. Whenever, for example, any variation in one phenomenon accompanies a variation in another, these phenomena may either be related directly as cause and effect, or they may be "two different effects of a common cause." The Method itself, however, does not enable us to decide, in any particular instance, which of these alternative possibilities is realised. In order to decide that point, as Mill says, we should have to determine whether one set of the variations in question were capable of being produced by means of the other. In that case, the relation is one of cause and effect. If, on the other hand, the variations in question were not capable of being produced in the manner indicated, we should have to regard them as effects of a cause that is common to both. In both cases the principle on which the Method rests is that "anything on whose modifications, modifications of an effect are invariably consequent, must be the cause (or connected with the cause) of that effect," As an example, "let us suppose," to quote Gibson, "that an electric bell is placed ringing under the receiver of an air-pump. The air is now gradually exhausted, and it is noticed that, *parri passu* with the exhaustion of the air, the sound of the bell grows fainter and fainter, until a point is reached at which it is no longer heard at all. The air is now allowed to pass back gradually into the receiver, and, as it does so, the sound of the bell is heard, at first faintly, then more and more loudly,

until the clearness of the original note is reproduced."

The Method of Concomitant Variations, as Mill himself indicated, is applicable particularly to those cases in which the Method of Difference, on account of the impossibility of removing the phenomenon under observation from the influence of permanent causes, entirely fails. Let us suppose, to take one of Mill's illustrations, that we wished to know what influence the moon exerts on the surface of the earth. "We cannot try an experiment in the absence of the moon, so as to observe what terrestrial phenomena her annihilation would put an end to; but when we find that all the variations in the position of the moon are followed by corresponding variations in the time and place of high water, the place being always either the part of the earth which is nearest to, or that which is most remote from, the moon, we have ample evidence that the moon is, wholly or partially, the cause which determines the tides."

Sometimes it is not the mere concomitance of varying phenomena, but the varying quantities of the associated phenomena, that leads to the determination of a causal relation between these phenomena. If, for example, a connection between heat and expansion is suspected, we can increase or diminish the amount of heat, and observe whether, under the varying conditions, any corresponding variations occur in the expansions of a heated body. That increase of heat is followed by increased expansion, and decreased heat is followed by decreased expan-

sion, is all the evidence we need that there is a causal connection between the two occurrences. It is by the use of a method such as this in cases like the last that precise quantitative variations between phenomena may often be made out. We may, that is to say, measure the increases and decreases in amounts of accompanying phenomena, and determine the exact correlation between them. Sometimes only one of the quantities can be measured, as, for instance, the correlation between the intensity of a stimulus and the resulting sensation which is formulated in what is known as Weber's law. Whenever we experiment with variations of quantity in accompanying phenomena, it should be observed, in general, that the greater the number and variety of the instances in which the correlation is known to hold, the greater the certainty that attaches to a given conclusion. The desirability of extending our observation of accompanying variable phenomena over a considerable range of instances is further emphasised by the fact that only in this way can we determine the limits within which their correlation holds good. In this way, for example, the upper and lower limits of the correlation between intensity of stimulus and sensation in each of the sense fields has been ascertained. Beyond these limits, the correlation no longer holds good either because the phenomenon changes its character or ceases altogether. Notwithstanding the necessity for exercising care in the use of this Method, it is particularly serviceable in those cases in which the amounts of varia-

tion can be measured. "Although," as Mill remarks, "the most striking applications of the Method of Concomitant Variations takes place in the cases in which the Method of Difference, strictly so called, is impossible, its use is not confined to those cases; it may often usefully follow after the Method of Difference, to give additional precision to a solution which that has found. When by the Method of Difference it has first been ascertained that a certain object produces a certain effect, the Method of Concomitant Variations may be usefully called in to determine according to what law the quantity or the different relations of the effect follow those of the cause."

The Method of Residues. Most of the phenomena with which the sciences have to deal are complex. The experimental methods, as they have been formulated by Mill, are, as we have seen, statements of ways that have been devised for the purpose of analysing various kinds and degrees of complexity in the material facts that constitute the subject-matter of the particular sciences. We have seen that when particular phenomena do not conform to the requirements of either of the fundamental Methods of Agreement or of Difference, the joint use of these Methods is then often necessary if we are to determine whether and, if so, in what respects, phenomena are causally connected. Sometimes our analysis of phenomena by this Method enables us merely to show that complexes are probably con-

nected together in a causal way. We may find, for example, in the series of instances:

POSITIVE INSTANCES

Antecedents	{ A B C D				x y z w				} Consequents
	{ A B E F				x y v u				

NEGATIVE INSTANCES

Antecedents	{ B C D E				y z w u				} Consequents
	{ C F G H				z u t s				

that the negative instances confirm what the positive instances suggest, namely, that AB and xy are, in all probability, causally related. But the Method itself does not, in this case, enable us to analyse the phenomenon further to determine whether A is causally connected with x or not. If, however, we supplement the Joint Method by the Method of Concomitant Variations, we may be able, by varying A or x, to ascertain whether the two phenomena are related as cause and effect. It is obvious, further, that we may employ the same Method for the purpose of ascertaining whether, in the above instances, B is causally connected with y, and C with z. Now, if experiment should show that A and x, B and y, C and z are causally connected then, the Method of Residues, which we have to discuss, assures us that the remaining phenomena, namely D and w, are also causally connected. Thus,

the Method virtually says that any remainder which is left over after other portions of a complex phenomenon have been explained, are themselves causally connected. Mill's statement is as follows: Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect if the remaining antecedents.

The Method of Residues, as it has just been stated, is constantly employed in the sciences. To take one illustration only, as this is given by Jevons. "In chemical analysis," he writes, "this method is constantly employed to determine the proportional weight of substances which combine together. Thus the composition of water is ascertained by taking a known weight of oxide of copper, passing hydrogen over it in a heated tube, and condensing the water produced in a tube containing sulphuric acid. If we subtract the original weight of the condensing tube from its final weight, we learn how much water is produced; the quantity of oxygen in it is found by subtracting the final weight of the oxide of copper from its original weight. If we then subtract the weight of the oxygen from that of the water, we learn the weight of the hydrogen which we have combined with the oxygen. When the experiment is carefully performed . . . we find that 88.9 parts by weight of oxygen unite with 11.11 parts of hydrogen to form 100 parts of water."

Mill calls our attention to the fact that the Method of Residues is specially serviceable in those

cases where several causes acting together produce an effect which is the joint result of all the causes taken together. In such cases, we have, what Mill calls, an "intermixture of effects." What is meant by an intermixture of effect has been explained by Jevons as follows: "If in one experiment friction, combustion, compression, and electric action are all going on at once, each of these causes will produce quantities of heat which will be added together, and it will be difficult or impossible to say how much is due to each cause separately. We may call this a case of the homogeneous intermixture of effects, the name indicating that the joint effect is of the same kind as the separate effects. There are several causes, each producing a part of the effect, and we want to know how much is due each." What is here called by Jevons a homogeneous intermixture of effects, Mill also called a Composition of Causes. He writes: "I shall give the name of the Composition of Causes to the principle which is exemplified in all cases in which the joint effect of several causes is identical with the sum of their separate effects." In contradistinction to the case just mentioned, the intermixture of effects may also be heterogeneous when, as Mill says, "the agencies which are brought together cease entirely, and a different set of phenomena arise: as in the experiment of two liquids which, when mixed in certain proportions, instantly become, not a larger amount of liquid, but a solid mass."

We may remark, further, that the Method of Residues is significant because it operates in the interests of thoroughness in scientific observation. It sets a barrier against carrying over into science the popular attitude towards what is small and insignificant, and its use has taught the scientist that what appears small and insignificant is often the hinge upon which scientific discoveries of the first important turn. "Almost all the greatest discoveries in astronomy," says Herschel, "have resulted from the consideration of residual phenomena of a quantitative or numerical kind. . . It was thus that the grand discovery of the Procession of the Equinoxes resulted as a residual phenomenon, from the imperfect explanation of the return of the seasons by the return of the sun to the same apparent place among the fixed stars." The discovery of the planet Uranus, to which we have referred in another connection, and the invention of the steam engine, are other instances of the importance of paying attention to residual phenomena. What one calls 'luck' in ordinary life, what one calls 'genius' in scientific life, and what one calls 'success' in commercial life are all of them very often merely the result of giving heed to details that the average person passes lightly by. In this regard, the following quotation from Darwin's 'Life and Letters' has its value. "A point apparently slight and unconnected with his present work is passed over by many a man almost unconsciously, with some half-considered explanation, which is really no explana-

tion. It was just these things that he (Darwin) seized upon to make a start."

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QUESTIONS

1. State briefly the relation of Mill's 'Logic' to the history of inductive logic in the modern era.
2. With what problem does Mill's chapter on 'The Four Methods of Experimental Inquiry' deal?
3. What is meant by calling causal relations reciprocal?
4. Are all causal relations reciprocal? Explain.
5. What relation have Mill's Methods to the special modes of inquiry developed by each science to meet its own problems?

6. Explain as well as you can exactly what Mill's Methods undertake to do.
7. In what sense can it be said that Mill's Methods rest upon a process of elimination?
8. What is Joseph's view of elimination as applied to causal problems?
9. What view, according to Gibson, does Mill take of the nature of elimination?
10. What is the characteristic feature of the Method of Agreement?
11. What is Jevons' formulation of Mill's Method of Agreement?
12. What criticisms may be passed on this Method for demanding a point of single agreement among the phenomena which it examines?
13. Is the Method of Agreement one of observation or explanation? Give reasons for your answer.
14. What degree of certainty attaches to the conclusions reached by the Method of Agreement?
15. What are the chief features of the Method of Difference as formulated by Mill?
16. What is meant by calling the Method of Difference a method of scientific experiment?
17. How does Mellone formulate the Method of Difference?
18. What two kinds of instances are presupposed by the Method of Difference, and by what names are they called?
19. Why is it necessary to vary only one condition at a time in a series of scientific experiments?
20. What does Mill mean by a permanent cause?
21. How does the operation of permanent causes affect the applicability of the Method of Difference?
22. Under what conditions does the Joint Method of Agreement and Difference become effective?
23. To what two classes of problems is the Joint Method applicable?

24. What are the chief features of the Joint Method as formulated by Mills?
25. Show, by the enumeration of supposed instances to which the Methods of Agreement and of Difference do not apply, the conditions which call for the use of the Joint Method.
26. What criticism can be made of the way in which Mill formulates the Joint Method?
27. What statement has been given by Mellone of this Method so as to avoid such criticism?
28. Explain carefully and in detail how you would apply the Joint Method.
29. Verify each step of your answer to the previous question by using the analysis of the social example given in the text.
30. What is the Method of Concomitant Variations?
31. What two classes of cases are presupposed by Mill's statement of the Method of Concomitant Variations?
32. How can you determine which of these two cases is present in any particular instance?
33. On what general principle does the Method of Concomitant Variations rest?
34. What is the relation of this Method to that of Difference? How is it affected by the operation of permanent causes?
35. In what way does the Method of Concomitant Variations enable us to determine causal relations between quantitative phenomena?
37. Illustrate how the measurements of quantitative phenomena enable us to determine causal relations, first, in instances where both antecedent and consequent can be measured, and, second, where only antecedent can be.
38. What, according to Mill, are the chief uses of the Method of Concomitant Variations?

39. State briefly the relations of the Methods of Agreement and Differences, the Joint Method, and the Method of Concomitant Variations to one another with a view to shrow what Mill means by the Method of Residues.
40. Illustrate the Method of Residues.
41. What does Mill mean by the intermixture of effects?
42. Why should the Method of Residues be specially applicable to cases which involve an intermixture of effects?
43. In what way does the Method of Residues operate in the interests of thoroughness in scientific observation?

EXERCISES

1. Examine each of the examples of inductive reasoning given below, and state by what method or combination of methods the conclusion in each case is reached.
 - (1) It is found that sun-lit air has been the means of eliminating some diseases, and of diminishing the severity of a number of others; whence it seems to follow that for maintaining a high degree of health sun-light air is essential.
 - (2) Other conditions remaining the same, children who are taught in classes which average from twenty to twenty-five in number attain in the same amount of time a higher degree of proficiency than those taught in classes which average from forty to fifty, or attain the same degree of proficiency in a shorter amount of time.
 - (3) Sachs maintained, in 1862, that starch is formed by the decomposition in chlorophyl of carbon-dioxide gas under the influence of light. He found that when all other conditions were constant, and light was excluded from a plant, no starch was formed; the single circumstance of readmitting light was accompanied by renewed formation of starch. Further, he found that if certain portions

of the leaves of an illuminated plant were covered with black paper, no starch was found in these portions.

- (4) Tyndall found that of twenty-seven sterilised flasks containing infusion of organic matter, and opened in pure Alpine air, not one showed putrefaction; while of twenty-three similar flasks, opened in a hay loft, only two remained free from putrefaction in three days. He concluded that putrefaction is due to floating particles in the air.
- (5) In 1675 Leeuwenhoek discovered infusoria, or animalcules under the microscope, and it was thought that such minute organisms as these might be spontaneously generated, even if the larger were not. About 1745 Needham performed a number of experiment to test this conclusion. He extracted the juices of meat by boiling, enclosed them in bottles, which were carefully corked and sealed with mastic, then subjected the closed bottles to heat and set them away to cool. In due course of time, the fluids thus treated became infected with microscopic life, and inasmuch as he believed that he had killed all living germs by repeated heating, he concluded that the living forms had been produced by spontaneous generation.
- (6) Spallanzy, however, thought that Needham's experiments had not been conducted with sufficient care. He, therefore, made a great number of similar experiments, using different kinds of infusions. But he placed them in thin flasks with slender necks, which were then hermetically sealed in flame, after which he immersed the flasks in boiling water for three-quarters of an hour, in order to destroy all germs that might be contained in them. Under these conditions no infusoria ap-

peared in them. Needham was not satisfied with these results, however, and objected that such prolonged boiling would destroy not only germs, but the generative force of the infusion itself. Spallanzani easily disposed of this objection by showing that when the infusions were again exposed to the air, no matter how severe or prolonged the boiling to which they had been subjected, the infusoria reappeared.

- (7) Goldschneider proved that muscular sensations play no considerable part in our consciousness of the movement of our limbs, by having his arm suspended in a frame and moved by an attendant. Under these circumstances, where no work devolved on his muscles, he found that he could distinguish as small an angular movement of the arm as when he moved and supported it himself.
- (8) The case has been supposed of a man who, having a taste for cucumber, attributes his chronic indigestion now to the salmon, now to the cheese, now to the pastry—no one of which is an invariable feature of his evening meal—but never to the cucumber, which he takes every evening. However, after having dined without cucumber on several consecutive evenings, whilst taking salmon one evening, pastry the next, and so on through the whole list of suspicious dishes, he cannot but notice that on no one of these evenings did any indigestion occur, and is thus brought to confess that the cucumber, after all, must have been the offending cause.
- (9) Ferri, in his *Criminal Sociology*, writes: I have shown that in France there is a manifest correspondence of increase and decrease between the number of homicides, assaults and malicious

wounding, and the more or less abundant vintage, especially in the years of extraordinary variations, whether of failure of vintage (1853-5, 1859, 1867, 1873, 1878-80), attended by a remarkable diminution of crime (assault and woundings), or of abundant vintages (1850, 1856-8, 1862-3, 1865, 1874-5), attended by an increase of crime.

CHAPTER XVIII

FALLACIES

Fallacy Defined. The earliest systematic account of fallacies was given by Aristotle in his treatise was on sophistical arguments (*περὶ σοφιστικῶν ἐλέγχων*). The aim of this treatise was to name and examine the various tricks that were relied on by many of the sophists to win the applause of their audience. We may say, therefore, that it contains an examination of the various forms of 'sophistry' current among the Greeks of the fourth century B. C. To this practical employment of the term corresponds, in a general way, the meaning that must be assigned to it when, at the present day, it is used in a popular sense. From this point of view, a fallacy may be said to be any error of statement or belief. If, for example, any one should believe that the sun revolves around the earth, such a belief would be called fallacious, and the statement of that belief would be called a fallacy. This meaning of the word, however, is too indefinite for the purposes of logic, and it covers too miscellaneous a group of cases to be of much service in directing its inquiries. It is customary, therefore, to narrow the interpretation of the term by confining its application to any breach of the principles and rules of logical procedure. In the foregoing pages, we have become acquainted with a number of the principles and

rules to which we must submit our thought and our speech if what we assert and what we express are to carry logical conviction. And, doubtless, our study of these principles and rules, and our effort to put them into practice, have convinced us both of the necessity and of the difficulty of conforming our thought and speech to their requirements. Now whenever our thinking fails to conform to such principles and rules, we are said, in the strict sense in which the term is used in logic, to be guilty of a fallacy. A fallacy, therefore, may be defined as any breach of a logical principle or rule.

This statement of what we understand in logic by a fallacy needs to be safeguarded in one particular. As it stands, and especially if one does not regard strictly the context in which it occurs, it will seem capable of including, not only those breaches of logical procedure to which the definition is intended to confine it, but also those errors which arise from other than logical sources, and to which the term fallacy can be applied only in a popular and conventional sense. There are, no doubt, natural limitations of mind which prevent all of us at some time from attaining, in the character of our intellectual life, anything that can be called with propriety logical exactness. Indeed, the ordinary course of life, as we have so often said, does not depend for the attainment of its objects on the strictness with which logical requirements are met in the field of the interests by which that life is controlled. And, doubtless, mistakes occur here

which have their roots in the mental process by means of which that life is carried on. We have pointed out, for instance, that perception does not provide a secure foundation of knowledge, and if we were to add that the organisation of our ordinary experience proceeds very largely upon principles of association, it would be evident, in the light of our previous studies, that association cannot guarantee against logical error the conclusions to which it is the means of conducting us. In so far, then, as we have regard to the mistakes that arise in this field, it would be quite misleading to apply the term fallacy to these mistakes, and they must be considered as falling outside the scope of our definition. We may point out also that lapses into this more usual method of procedure are likely to occur in the course of our efforts to reach more strictly scientific results. The exclusion of these cases from the field of the fallacies is made on the principle that such cases do not involve ignorance of the methods of logical procedure, but carelessness in their application; just as, for example, ignorance of a mathematical rule would not be attributed to one whose mistake was obviously due to haste or inattention in its application to a given case. We must regard as logical fallacies, therefore, only those instances of erroneous conclusions which rest upon grounds which are demonstrably false. It is in this sense, then, that a fallacy was defined above as any breach of a logical principle or rule.

Classification of Fallacies. Although Aristotle was limited in his treatment of fallacies by the practical purpose to which reference has been made, and although he had before him chiefly those errors to which the Sophists were exposed by their desire for popular favour, his classification of fallacies has remained the basis of what may be regarded in logic as the traditional position on this subject. In the work referred to, Aristotle divided fallacies into two main classes, a division which corresponds, in a general way, to the distinction on which we have insisted in our discussions of logical doctrine, the distinction, namely, between truth in the form of judgments, and truth expressed in the form of propositions. According to this distinction, fallacies may arise from the way in which we express or interpret propositions, or, as Aristotle said, they may be due directly to language (*παρὰ τὴν λέξιν; in dictione*); or they may exist in the thoughts or judgments that we assert, or, as Aristotle expressed it negatively, they may not be due to the language we employ (*ἔξω τῆς λέξεως; extra dictionem*). Under the first class Aristotle enumerates six kinds of fallacy, some of which are trifling and dependent wholly on the syntax of the Greek language; and under the second class, he enumerates seven. Beyond the distinction of these two main classes we are not directly interested in Aristotle's classification.

There is, perhaps, no entirely satisfactory classification of fallacies, and the most that can be attempted here is to bring under some practical and

intelligible scheme the various errors to which logical thinking is exposed. It will help us to do this if we recall, in the first place, that the proposition was said to be an expression in words of the truth asserted in a judgment. We saw that, for the person speaking, the problem was to find words which express adequately the thought that he desires to convey, and, for the person spoken to, it was to interpret the words in which a judgment is expressed in the sense that they had for the person who first used them as the embodiment of an idea. In either case, the logical value of a proposition depends to a large extent on language, on the fitness of the verbal forms which are chosen as the medium of ideas. If one realises the great difficulty, which the masters of literature alone have at all adequately overcome, of choosing words that reveal the meaning their employment is intended to convey, he will understand why logicians, since Aristotle, have continued to regard language as one main source of logical fallacy. The study of the fallacies which beset our interpretation of propositions through the ambiguous use of language will form for us the first group of fallacies.

Our second main group of fallacies will comprise those mistakes which threaten our comprehension of the full meaning of propositions. It will be remembered that we pointed out that our relation to propositions is two fold. In the first place, propositions need interpreting, and, as we have just now intimated, the difficulties in the way of interpreting

propositions constitutes the first main class of fallacies. But, in the second place, it was pointed out that propositions contain implications which limit in certain determinate directions what we can think about the subject-matter of their assertion. For instance, we said that if we admit the truth or falsity of a given proposition we are thereby committed to the truth or falsity of a number of other propositions. An examination of the possible sources of error in this region, the region of logical implication, will enable us to avoid one of the most frequent pitfalls in the way of that accuracy and precision that is the desideratum of the intellectual life.

The fallacies that are included under the two heads just mentioned refer to the meaning of propositions taken singly, they are the fallacies which arise in the field of simple, unmediated assertion. When, however, we go beyond this region, and our assertions take the more complex forms that we have studied in the syllogism and induction, a special group of fallacies has to be recognised, because here thought is guided in its efforts to reach logically sound conclusions by special principles and rules. The mistakes that occur in connection with the control of thought by these principles and rules will constitute a third main group of fallacies.

A fourth group of fallacies may be recognised if we recall that no logical assertion, whether simple or complex, is ever a wholly detached unit, and

that, if such assertions are to be interpreted correctly, they must be brought into relation to the contexts out of which their original meaning was forged. Every assertion, we have said, implies a certain universe of discourse. Now we often find that the plausibility of statements and arguments depends on suggestions derived from their contexts which are not directly related to the truth character of the statements and arguments themselves, and for these extrinsic reasons are frequently, though fallaciously, accepted as true. They are cases of this kind that we shall classify under the present head.

For the sake of convenience, we shall refer to the four main classes of fallacy that have just been distinguished, in the order in which we have considered them, as fallacies of interpretation, of implication, of inference, and of the context. A certain latitude must be allowed in the use of this classification, as indeed it must of any classification, of fallacies. The only justification that any scheme can receive is that it brings together, in a way that connects with the development of logical doctrine, the errors to which we are exposed in our efforts to think accurately and convincingly. If, in addition, it groups the fallacies in a way that is easily remembered, it has a still further advantage.

Fallacies of Interpretation. These are due, as we have said, to the deceptiveness of the language in which propositions are expressed. They all, therefore, fall under the defect of ambiguity. But since

ambiguity may attach either to the words or to the structure of our statements, we may distinguish two classes of error to which our interpretation of propositions is exposed. Among the fallacies due to verbal ambiguity we may include equivocation, accent, composition, division, accident, and converse fallacy of accident; and among those due to structural ambiguity, amphibole and false parenthesis.

Verbal Ambiguity. In discussing the fallacies that are due to verbal ambiguity, we may remind ourselves that ambiguity is not identical with mere indefiniteness in the meaning of words, and that, in view of a certain unavoidable indefiniteness in the meanings of the words we employ, propositions can attain only a non-ambiguous signification. The principle of non-ambiguity, we said, controlled all our efforts after logical exactness both with respect to terms and propositions. We cannot, therefore, convict of any of the verbal fallacies a proposition whose terms are indefinite, if this indefiniteness does not spread to the meaning of the proposition taken as a whole. When, however, the indefiniteness of the parts affects the meaning of the whole proposition, we have a case that falls under the class of fallacies that we are now considering. With this reminder, we take up *seriatim* the several fallacies that originate in verbal ambiguity.

Equivocation, or indefiniteness in the denotation or connotation of a term, gives rise to a fallacy when either term of a proposition is interpreted in a different sense by the speaker and by the person

spoken to, and this affects the meaning of the proposition as a whole; or when, for the same individual, any term which appears in a continuous discourse fails to retain the same meaning in the various propositions in which it occurs, and this affects the conclusion to which one is led. The latter is, perhaps, the more usual, as it is, certainly, the more insidious, form of the fallacy of equivocation. Any form of continuous address, and especially of public address, exposes one to the danger of equivocation, and the most careful writers sometimes fall into it. A familiar example is the following taken from Mill's argument in support of the position that happiness is the chief aim or good of life. He says, "The only proof capable of being given that an object is visible is that people actually see it. The only proof that a sound is audible is that people hear it. And so of the other sources of our experience. In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable is that people do actually desire it." For the purpose of his argument, the word desirable must have the meaning 'worthy to be desired,' but this is a meaning that is not analogous to visible and audible in the context. Visible and audible mean able to be seen or heard, and, therefore, all that Mill has a right to argue is that desirable means 'able to be desired.' But from this meaning he could not argue that happiness is the *summum bonum* or chief good of life, for the fact that a person desires an object is no proof that it is, in the

sense Mill uses the word, the most desirable. Against committing or being imposed on by the fallacy of equivocation we may submit the meanings of our terms to the methods of translation and definition, as these terms have been explained above.

Accent. The fallacy of accent is any misconception due to the emphasis placed on a word or group of words in a sentence. It was for Aristotle the mistaken accentuation of a word in writing Greek. "A ludicrous instance of this fallacy," to quote Jevons, "is likely to occur in reading Chapter XIII of the First Book of Kings, verse 27, where it is said of the prophet, 'And he spake to his sons, saying, Saddle me the ass. And they saddled *him*.' The italics indicate that the word *him* was supplied by the translators of the authorised version, but it may suggest another meaning. The commandment, 'Thou shalt not bear false witness against thy neighbour,' may be made by a slight emphasis of the voice on the last word to imply that we are at liberty to bear false witness against other persons." We may, however, give certain words or groups of words a prominence that does not really belong to them in another way. For example, in quoting an author we may separate a word from its context, italicise a word which was not italicised in the original, or, by leaving out qualifying words and phrases, we may give certain ideas a prominence which they did not have for the author himself. Connected with this latter interpretation, the fallacy, as Creighton

observes, "appears in connected arguments of any kind in which, while the facts are not actually misstated, certain aspects of them are so disproportionately dwelt upon and emphasised, at the expense of the rest, that a false idea of the subject in its entirety is the result. In this wider form, this fallacy is one that may be described as the particular vice of *special pleading*; and the caution that may be suggested against it is, in the language of the astronomer, to make allowances for the 'personal equation' both in one's own thinking and in that of others."

Composition. The fallacy of composition, like its converse, the fallacy of division, turns on the distinction between the distributive and collective use of terms. This distinction, it will be remembered, is based on the fact that we may predicate qualities and relations of a class without at the same time predicating those qualities or relations of the individuals that constitute the class; and, conversely, we may predicate qualities and relations of the individuals that constitute a class without at the same time predicating those qualities or relations of the class. Now, we are guilty of the fallacy of composition when, having predicated something of a term used distributively, that is, of each of the individuals that fall under the term, we suppose that the predicate belongs to the term in its collective sense, that is, of the individuals taken as a group. "We must not argue," to use Jevons' illus-

trations, "that because every member of a jury is very likely to judge erroneously, the jury as a whole is also very likely to judge erroneously; nor that because each of the witnesses in a law case is liable to give false or mistaken evidence, no confidence can be reposed in the concurrent testimony of a number of witnesses. It is by a fallacy of Composition that protective duties are still sometimes upheld. Because any one or any few trades which enjoy protective duties are benefited thereby, it is supposed that all trades at once might be benefited similarly; but this is impossible, because the protection of one trade by raising prices injures others."

Division. The fallacy of division is, as was said, the converse of the fallacy of composition. It was so considered by Aristotle. It occurs when we suppose that a statement that is true of a group is true of the individuals of the group. If, for example, it is true that a particular class of college students are bright, it does not follow that this is true of each or any particular one of the members of the class, any more than it follows that because a jury can be relied on to reach a just verdict that a similar confidence can be placed in any one of the jurymen. Gibson has called attention to an interesting illustration of this fallacy to be found in Thomas à Kempis' *Imitatio Christi*. In the paragraph referred to, the writer is exhorting the 'good monk' not to seek any earthly delight, but to remain alone in his cell. The argument by which this advice is supported reads: "What canst thou see elsewhere

which here thou seest not? Behold the sky and the earth, and all the elements, for of these are all things made." Expressing this argument syllogistically, Gibson detects the fallacy of division in the middle term. Thus:

The elementary substances (in organised combination)
are the whole material world.

The objects included in the prospect from your cell are
the elementary substances (not so combined).

Therefore, the objects included in the prospect from your
cell are the whole material world.

As this illustration suggests, the fallacy of division, in common with many others, is most liable to occur in continuous discourse, and the value of putting the subject-matter of such discourse into the logical forms suitable to them is that we may the more easily detect any fallacy lurking in our thought, or in the forms in which that thought is expressed.

Accident. The fallacy of accident occurs when we suppose that what is true of a term used in a relatively indeterminate or unconditioned sense, is true of the term when used in a relatively determinate or conditioned sense. The Latin name of the fallacy is, *Argumentum a dicto simpliciter ad dictum secundum quid*. It is, that is to say, a transition from what is true of an indeterminate statement to what is true of the same statement 'with a modification.' In other words, it consists in securing assent to a statement in a general form, and then proceeding as if it held under some special

circumstance. If we supposed, for example, that because it is right for a man to inculcate his own opinions, it is, therefore, right for a magistrate to use his position to enforce his own political views, we should be guilty of this fallacy. For, as Mellone, from whom this illustration is taken, remarks, "we cannot infer of his special powers as a magistrate what is only true of his general rights as a man." We may often detect the fallacy in our thoughts about moral questions. Thus, in discussing such questions, "one may insist that all men by nature seeks after the good, meaning that each man acts *sub specie boni*, and is therefore seeking after some kind of good, even when he is pursuing his own pleasure. But, in continuing the argument, he may use the term 'good' in a differentiated sense — e. g., in the sense of the common good, or the good of humanity — and come thereby, through a fallacy of accident, to the conclusion that all men by nature seek the general good."

The Converse Fallacy of Accident occurs when we suppose that what is true of a term used in a determinate or conditioned sense is true of the term when used in an indeterminate or unconditioned sense. The Latin name of the fallacy is, *Argumentum a dicto secundum quid ad dictum simpliciter*. The following illustrations will be sufficient to make the nature of this fallacy obvious. For example, let us suppose that it can be shown that the syllogism is of no use in the discovery of new truth, are we warranted in supposing, as the early modern

critics of the syllogism seem to have done, that it is, therefore, of no use whatever? Or, if philosophy 'bakes no bread,' is it, therefore, useless? The affirmative answer to both these questions involves the converse fallacy of accident.

Structural Ambiguity. In the chapter on the proposition, it was pointed out that particular statements are often ambiguous, not because the terms in which they are expressed are ambiguous, but because the structure of the sentences in which ideas are expressed are so. It is to the grammatical structure of sentences, for example, that we must attribute, in part, the difficulty sometimes experienced in determining the subject and predicate terms of given propositions. Whenever syntactical relations obscure the meaning of sentences or paragraphs, there is presented a hindrance to the correct interpretation of propositions. Difficulties of the kind in question are more possible in the classical languages where the word order in sentences is more variable, and where oblique constructions are more common, than in English; but something of the same sort is seen in English poetry in which a departure from the relative simplicity of the ordinary prose sentence is often made necessary by the demands of metrical composition.

Amphibole. Now, whenever the structure of the sentences in which ideas are expressed, in prose or in poetry, leads to opposed interpretations of such sentences, we have the typical fallacy of amphibole. Sometimes sentences whose structure renders

their interpretation difficult, although not necessarily ambiguous, are spoken of as amphibolic. This is a view similar to that sometimes taken of words, when any degree of indefiniteness is thought sufficient to render them equivocal. This seems to us too loose an interpretation of these fallacies. As we have pointed out in regard to words, there is also a certain freedom in the use of syntactical relations that is allowable and necessary if particular shades of meaning are to get expressed in the sentences we employ. The object that the translation of sentences into propositions keeps in view is, not to give all statements an equal clearness, but to bring out the exact degree of clearness that belongs to each of our statements. A certain indefiniteness of meaning may be characteristic of propositions, therefore, without involving them in the fallacy of amphibole. Before we can convict any statement of being amphibolic, we must have exhausted all the means at our disposal to interpret the statement, and to give it an expression which truly represents its meaning. If it then remains ambiguous, if, that is, our attempt to give the statement a propositional form is defeated on account of its equivocal syntax, we may properly regard it as an example of the fallacy of amphibole. An illustration of this statement may be found in the often quoted lines of Shakespeare, 'The Duke yet lives that Henry shall depose.' We are baffled in any attempt to put this sentence into the form of a proposition, because its structure does not enable

us to determine whether it means, 'The Duke yet lives who shall depose Henry,' or, 'The Duke yet lives who shall be deposed by Henry.' Such a statement, therefore, is essentially amphibolic. The student will recall, in further illustration, many of the utterances of the oracles, for instance, 'If Croesus should wage war against the Persians, he would destroy a mighty empire.' As further examples we may cite, 'It is possible for thee the enemy to overcome,' 'The lion David slew.'

False Parenthesis. Closely connected with the foregoing, is the fallacy of false parenthesis. It consists essentially in an uncertainty with regard to the relations of the main and subordinate clauses of a sentence. For example, 'I ruined the cause and injured my own prospects which I deeply regret,' a statement that may mean either, 'I ruined the cause, and injured my own prospects which I deeply regret,' or, 'I ruined the cause and injured my own prospects, which I deeply regret.' A similar fallacious statement is 'I will begin and finish the task tomorrow.' The advertising columns of our newspapers are sometimes said to provide us with humorous illustrations of the fallacy of parenthesis. For instance:

LOST — A valuable silk umbrella belonging to a gentleman with a curiously carved head.

WANTED — A groom to look after two horses of a pious frame of mind.

FOR SALE — A Newfoundland dog; will eat anything, particularly fond of children.

We introduce these instances, not in order to subscribe to the view that they are amphibolic in character, but to point out that if these are to be admitted as examples of amphiboly, then any badly constructed sentence would have to be included whether or not their meaning was involved in ambiguity. This is a view to which, as we said above, we cannot subscribe.

Fallacies of Implication. The second main class of fallacies which we have now to consider comprises those that occur when for any reasons we mistake the implications of propositional statements. The difficulties that confront our efforts to give to any statement a propositional form have been considered already, and in contrast to these, the ones with which we are now concerned presuppose that those difficulties have been overcome, and that we know unequivocally the significance that particular statements have. Until we are sure what the asserted meaning of a proposition is, we cannot raise the question whether, and if so what, further or implied meaning it can have.

It will be remembered that by implication is meant the unexpressed signification of a logical proposition, and that implications may assume one or the other of two forms. In the first form, the unexpressed or implied propositions may be related to the given proposition in the way of opposition, in the second form, they may be related to the given proposition in the way of eduction. These

distinctions will provide us a natural and easy method of classifying the fallacies of implication.

Fallacies of Opposition occur when, from statements accepted as true or false, other statements dealing with the same subject-matter are derived, the truth or falsity of which is, mistakenly, supposed to rest upon the principles that determine the relations of opposed propositions. It has been shown above that there are four kinds of relation between propositions in opposition, — contrariety, subcontrariety, subalternation, and contradiction. These relations have been studied, and the propositions between which each obtains have been indicated. In view, therefore, of what we have learned with respect to this class of implication, it is readily seen that, in respect to this class, our thought becomes fallacious when we suppose that the principles of opposition justify a statement which, as a matter of logic, they are incapable of supporting. Or, to put it another way, any statement which avowedly rests upon the principles of opposition is fallacious if it can be shown that those principles fail to give it the necessary support.

In general, it is quite obvious that there are as many fallacies of opposition as there are pairs of propositions between which relations of opposition may hold. Some of these relations are in themselves so obvious, the relation of contradiction, for instance; and others lose so readily whatever obscurity they may have when they occur between certain pairs of propositions, that it is not necessary to

consider all the possible fallacies of opposition. All, therefore, that we shall attempt is to point out some of the difficulties that may be experienced in the effort to conform our thinking to the principles of opposition.

And, in the first place, we may consider the *fallacy of contrariety* which occurs when we suppose that the falsity of one universal proposition, A or E, involves the truth or falsity of the other, E or A. It may be false, for example, that 'None of the members of the team are *bona fide* students,' but it does not, therefore, follow that 'All the members of the team are *bona fide* students,' and it does not follow because, it might still be true that 'Some of the members of the team are not *bona fide* students.' It is certainly very easy for the untrained mind to proceed from the falsity of a universal to the truth of its logical contrary, and one is particularly exposed to this error when the latter statement is separated in our discourse from the former by an appreciable lapse of time. But, as we saw above, the statement of what is false in a universal form does not enable us to say anything with respect either to the truth or falsity of its contrary. In such cases the contrary is always involved in doubt.

In the second place, we are exposed to a *fallacy of subcontrariety* when we attempt to argue from the truth of a particular proposition, I or O, to the falsity of another particular proposition, O or I. We run into this error when we forget

that two particular propositions dealing with the same subject-matter, one of which is affirmative and the other negative, may both be true. From the statement, for example, that 'Some of those who have given their word are keeping it faithfully,' it does not follow that 'Some of those who have given their word are not keeping it faithfully' is false, since this latter statement might be true of any of those not included in the original statement.

In the third place, *fallacies of subalternation* may arise when we argue either from the falsity of a universal proposition, or from the truth of a particular proposition. The falsity of a universal proposition, 'All that I have done for you has gone for nought,' might very well seem to involve the falsity of the particular subaltern, 'Some of the things I have done for you have gone for nought;' but this would not necessarily be the case, because the truth or falsity of the subaltern proposition depends in this case upon the degree of falsity of the original proposition, and it is exactly this that we cannot determine from that proposition itself. Unless it were so common, one would hardly think it necessary to call attention to the other fallacy of subalternation, the fallacy, namely, that is involved in proceeding from the truth of a particular statement to the truth of its corresponding universal. It may, for example, be quite clear that from the statement that 'Some books are dull,' we cannot proceed to the statement that 'All books are dull,' but we dare say, it is not equally obvious that we cannot proceed from the

statement, 'Some planets move in their own orbits', to, 'All planets move in their own orbits.' The reason why we cannot argue from the truth of a particular statement to either the truth or falsity of the universal of the same quality seems to be that we have no means of determining, by an examination of the original statement, the precise amount of truth that is claimed by it, that is, whether it states the whole truth about its subject, or only part of it.

Fallacies of Eduction. Eductions were defined above as those relations which propositions dealing with the same subject-matter have to one another when they express what is true about this subject-matter from different points of view. There are, as we said, two fundamental methods for determining these relations, obversion and conversion, and it is with respect to these methods, therefore, that fallacies are likely to occur. An eductive fallacy may, consequently, be defined as an error of implication due to a misunderstanding of obversion, of conversion, or of both.

The fallacy of obversion is most likely to occur when for the contradictory of the predicate of an original proposition we substitute its contrary in the obverse proposition. Now by contrary terms we mean those which, although opposed to one another, do not exhaust between them the particular universe of discourse to which they both belong. Black and white are, in this sense, contrary, but they are not contradictory, terms. By a contra-

dictory term we mean one that, together with the original term, does exhaust the universe of discourse to which both terms belong. Thus, black and not-black are contradictory, but they are not contrary, terms. It is, then, a confusion of contrary with contradictory terms, in the sense in which these have been defined, that gives rise to the fallacy of obversion. Thus, it would be quite fallacious, for the reason already assigned, if we supposed that we were warranted in saying of a particular individual that he was bad, if all we knew was that he is not good; good and bad are contrary, they are not contradictory, terms. Again, if we supposed that the obverse of the proposition 'He is not happy,' was 'He is unhappy,' there would be as complete misunderstanding of what was implied in the first statement as if we supposed that because a certain object was not red that it was, therefore, green.

Fallacies of Conversion are most likely to occur in the case of the universal affirmative, and the particular negative propositions. In the latter case, the fallacy occurs when we suppose that from a particular negative proposition anything can be asserted merely by converting it. It is quite true, for example, that 'Some elements are not liquid,' and also that 'Some liquids are not elements,' but the one statement does not follow from the other by simple conversion. Again, a fallacy of conversion occurs when we suppose that the universal affirmative proposition can be simply converted, a supposition that is based on the false assumption

that both terms of the proposition are distributed. In universal affirmative propositions, as we have seen, the predicate terms are undistributed; and, consequently, when, through conversion, these terms become the subjects of new propositions, these propositions must be particular in quantity. The neglect of this consideration leads directly to a fallacy of conversion.

Fallacies of Inference. The class of fallacies to which we now direct attention includes the errors to which our thinking is exposed when it is confronted by the difficulties that are involved in the more complex logical processes known as arguments. The two classes of fallacy already considered, the fallacies of interpretation and implication, are concerned with the errors to which thought is liable when it deals with single statements; but it is not until these simpler errors have been avoided, not until single statements have been given a strictly logical form and content, that there comes into view the class of fallacy that is connected with our attempt to put into their logical relations such logically determined propositions. For, in an argument, not only must the constituent propositions be free from ambiguity, but the relations between these propositions must be precisely determined. It is perfectly obvious, therefore, that the avoidance of the fallacies of interpretation and implication is essential, if we are to meet and overcome the difficulties that are occasioned by the more complex character of our thinking in the process of argument. We may,

therefore, say that just as the avoidance of the fallacies of interpretation are presupposed by any attempt to avoid the fallacies of implication, so the avoidance of both these classes of fallacy is presupposed by any attempt to overcome the fallacies of inference.

Arguments, from the time of Aristotle, have been recognised to be of two kinds: those, namely, that start with the recognition of some general principle, and those that start with the collection of a number of particular facts. In the former, the aim is to bring, through the mediation of a middle term, some particular fact under the principle which constitutes what is called the major premise of the argument; in the latter, it is, through an analysis of particular facts, to state a principle that shall exhibit what is common to all the facts examined, and, if possible, to extend that principle to other facts which either have not been examined, or which are not known to exist. It is usual to call the former kind of argument deductive, and the latter, inductive; and, although there is a danger of supposing, falsely, that they constitute quite separate logical processes, we may use these names, deductive and inductive, for the purpose of classifying the fallacies of inference. The deductive fallacies will include those errors that arise in connection with the syllogism, and the inductive fallacies will include those that arise in connection with the various steps through which it is necessary to pass in our efforts to establish causal connections.

The Deductive Fallacies. Whenever an argument starts from the conscious acceptance of a general truth about its subject-matter, and proceeds to show that some particular fact is a special instance of that truth, it is said to be a deductive argument. Now, since we accept such general truths as are the starting points of deductive arguments in either one or the other of three forms, in the forms, namely, of categorical, hypothetical, or disjunctive propositions, these arguments, as we have seen, assume a form and character which corresponds to the difference in the method and principle by which thought, in each case, proceeds to its conclusion. In consequence of this fact, we found it necessary to consider categorical arguments, hypothetical arguments, and disjunctive arguments separately, and to give special attention to the rules by which thought, in each type of argument, is controlled. It follows, from this statement with respect to the various forms of deductive arguments, that the deductive fallacies can be classified most conveniently by following the line of cleavage between the arguments in connection with which they are likely to occur. In the following sections, therefore, we shall pass in review what we may call the categorical, hypothetical, and disjunctive fallacies, the fallacies which occur, or are liable to occur, in those types of argument to which similar descriptive names have been given.

Categorical Fallacies may be defined as those mistakes in reasoning which render the conclusions of

categorical arguments erroneous. These fallacies fall conveniently into two groups, according as the error lies in the terms, or in the character of the proposition of the syllogism. To the former group we may assign the fallacies of four terms, and to the latter, the fallacies of insufficient premises.

Fallacies of Four Terms occur as a breach of the rule which states that a syllogism must contain three, and only three, terms. Their occurrence is due to a failure to rid the terms of a syllogism of ambiguity. Now, since ambiguity may occur either in the connotation or in the denotation of a term, and may attach to any one of the terms of a syllogism, it is evident that the fallacies of four terms are of two kinds; those, namely, which occur when the meaning of any of the terms of a syllogism is ambiguous, and those which are due to a mistaken distribution of any of the terms of a syllogism. Among the former, it is usual to call special attention to the case of the middle term, but it should be remembered that the ambiguity of the major or of the minor term may also invalidate an argument by giving rise to the fallacy of four terms. When the fallacy of four terms invalidates a syllogism on account of the connotative ambiguity of the middle term, it takes the special name of *ambiguous middle*; when an argument is invalid on account of a similar ambiguity of the major term, the fallacy is called *ambiguous major*; and when ambiguity in the meaning of the minor term destroys the validity of an argument, the fallacy is called *ambiguous minor*.

It is not necessary to give illustrations of all these fallacies. The following will serve to show the form in which they are likely to occur. If, as Jevons remarks, we argue "that 'all metals are elements and brass is a metal, therefore it is an element,' we should be using the middle term 'metal' in two different senses, in one of which it means the pure simple substances known to chemists as metals, and in the other a mixture of metals commonly called metal in the arts, but known to chemists by the name alloy." Or, if we argued "that 'what is right should be enforced by law, and that charity is right and should be enforced by law' . . . it is evident that 'right' is applied in one case to what the conscience approves, and in another use to what public opinion holds to be necessary for the good of society."

The other type of fallacy which results in breaking an argument under the weight of four terms, is due to a disregard of the denotation or distribution of the terms of a syllogism. In these cases, the erroneous conclusion results from a neglect of the extent of a term's meaning, rather than to a disregard of the meaning itself. In other words, it is a fallacy of the division, not a fallacy of the definition, of a term. Now, as we have seen, the distribution of the terms of a syllogism is subject to special rules. In respect of the middle term, the rule states that this term must be distributed in one, at least, of the premises; and in respect of the major and minor terms, the rule states that if either of these terms is distributed in the conclusion, it must

have been distributed in the corresponding premise. When the rule relating to the middle term is disregarded, the resulting fallacy is known as *undistributed middle*; and when the rule relating to the terms of the conclusion is disregarded, the resulting fallacy, in the case of the major term, is known as *illicit process of the major*, and, in the case of the minor, as *illicit process of the minor*. The following illustrations will throw light on these statements. If we should argue, for example, that because "all rash men are confident, and all brave men are confident," therefore, "all rash men are brave," we should be guilty of the fallacy of undistributed middle. Again, if we argued "that 'because many nations are capable of self-government, and that nations capable of self-government should not receive laws from a despotic government, therefore no nation should receive laws from a despotic government,' we should be clearly exceeding the contents of our premises. The minor term, many nations, was undistributed in the minor premise," and must not, consequently, be used in a distributed sense in the conclusion. While it is comparatively easy to detect an illicit process of the minor, it is much more difficult as a rule to detect an illicit process of the major. The following from Creighton is an example of the latter fallacy. "All rational beings are responsible for their actions; brutes are not rational beings; therefore, brutes are not responsible for their actions."

Fallacies of Insufficient Premises. These constitute the second class of categorical fallacies, and they may be described as breaches of those rules of the syllogism that were considered and illustrated above as rules of quality and rules of quantity. The rules of quality refer to the grounds on which affirmative and negative conclusions can be based, and the rules of quantity to the grounds on which universal and particular conclusions can be based. In the main, we apprehend that these rules have caused the student no great difficulty, and that he will have become familiar with their application. But because limiting instances are liable to cause more or less confusion, because, that is, we find it difficult to keep strictly within the boundaries within which any set of rules applies, we direct attention again to the fact that there are conditions which preclude certain propositional statements from rendering valid conclusions. It is a general fact that conclusions do not follow from any combination of propositions. If we wish to support a conclusion by premises or grounds, those premises must have a character from which the conclusion can be shown to follow, and that means that a certain other character is strictly forbidden. Now, if this is true of particular conclusions, we may go further and ask whether there are any combinations of propositions which support no conclusion whatever, that is, whether there are any limits of strictly categorical argumentation. Now such limits to all categorical inference we find in the two cases that are included here for con-

sideration, the cases, namely, of negative and particular premises. We have shown above that any attempted reasoning on the grounds of particular or negative premises involves us in one or other of the fallacies of distribution; but what is more important to remark now is that the study of these two cases enables us to insist upon certain characteristics of syllogistic reasoning which are fundamental to our understanding and employment of this type of argument. The syllogism, it should be observed, rests upon the possibility of making with respect to the subject-matter of the argument an affirmative and universal statement. The universal statement need not be affirmative, and the affirmative statement need not be universal; but unless the premises contain the statement of universal affirmative relations, no conclusion whatever can be drawn. Conversely, if we wish to ground a belief on premises from which it can be shown to follow, these premises, considered as a combination of propositions, must be universal and affirmative. Indeed, this is only to say that any categorical argument must conform to the character of syllogistic reasoning. For, to speak of syllogistic reasoning at all, is to imply that conclusions are inferred from, or rest upon, statements about a given subject-matter which are affirmative and universal in the sense in which those terms have been explained. Now, if this is the true character of the syllogism, it follows that particular premises, whether they be both negative, or both affirmative, or one negative and the other affirm-

ative, cannot express the structure of a syllogistic argument, because they do not contain the note of universality; and that negative premises also fail to do so, whether both premises are universal or only one is, since they fail to express the note of affirmation. To ground a conclusion upon premises which fulfil only one of the conditions of a categorical argument, that is, upon propositions which are affirmative without being universal, or upon propositions which, although universal, are not affirmative, is to fall into one or the other of the fallacies of insufficient premises. In the former case, we have the *fallacy of particular premises*, and in the latter, *the fallacy of negative premises*.

Hypothetical Fallacies may be defined as those mistakes in reasoning which render the conclusions of hypothetical arguments erroneous. Hypothetical arguments, as we have seen, derive their character from the relation of antecedence and consequence which is expressed in their major premise. Two observations only need to be made to remind ourselves of the nature and limits of hypothetical arguments. The first is that the major premises of such arguments are always hypothetical propositions, that is, propositions which state a relation between two possibilities. From the fact that a relation of antecedence and consequence is always affirmed between these two possibilities in hypothetical propositions, it follows that the acceptance of a hypothetical statement commits us to the belief that the possibility which is expressed as the consequent

can be regarded as an actual fact, only when the possibility expressed as the antecedent states a condition which is known to be fulfilled. We cannot argue in the reverse order, that is, from the known existence of the consequent to the existence of the antecedent. The second remark is that the major premise affords no guarantee that the condition it states is anywhere fulfilled, and, consequently, no guarantee that the fact stated in the consequent is any where to be met with. Whether the antecedent or consequent is a fact that the minor premise can affirm or deny, depends upon whether either has been observed. Thus, the possibility of a hypothetical argument rests upon our ability to affirm or deny, for some other reason than the major premise itself affords, one or other of the possibilities between which the major premise postulates a relation; but within these limits the validity of such arguments depends upon a proper interpretation of the relation between antecedent and consequent asserted by the major premise. It is, consequently, to a misunderstanding of this relation that we must look for the source of the fallacies which render the conclusions of hypothetical arguments erroneous.

The hypothetical fallacies are due, as we said just now, to a wrong interpretation of the relation between antecedent and consequent asserted in a hypothetical proposition. Now, since there are two terms in a hypothetical proposition each of which we may affirm or deny, the relation between antecedent and consequent may be interpreted, theo-

retically, in any one of four possible ways. It might be interpreted to mean that (1) if the antecedent is affirmed, the consequent may also be affirmed; (2) if the antecedent is denied, the consequent may also be denied; (3) if the consequent is affirmed, the antecedent may also be affirmed; (4) if the consequent is denied, the antecedent may also be denied. Now, as we pointed out before, it is only when the minor premise either affirms the antecedent or denies the consequent that the relation asserted in the major premise is interpreted correctly; only two, therefore, of the alternatives enumerated above represent truly the relations involved in a hypothetical proposition. When either of the other relations is asserted in the minor premise of an argument, that is, when the minor either affirms the consequent or denies the antecedent, the argument must be rejected as fallacious. The *fallacy of denying the antecedent* leads to an erroneous denial of the consequent, and the *fallacy of affirming the consequent* leads to an erroneous affirmation of the antecedent. Both fallacies, as we shall see, arise from the neglect of an underlying material fact of considerable importance for a correct understanding of the hypothetical syllogism.

In general, it may be remarked that the hypothetical fallacies are due, on the material side, to a failure to recognize the complexity and interconnection of natural events. It is true that the object of our study of nature is to determine, in the strict sense, causal connections, that is, relations between

phenomena of such a nature that, not only does the consequent follow from the antecedent, but the stated antecedent is implied in the occurrence of the consequent. Our knowledge of natural events, however, frequently falls below this standard, and we have to be satisfied with the affirmation of a one-sided relation, that is, with the statement of a relation between antecedent and consequent of such a nature that while the former is always followed by the latter, it does not always and necessarily precede the latter. There are large areas of knowledge, in other words, in which the relations involved fail to display the characteristics of causal connections, and in reference to them we are bound to recognise what Mill called a plurality of causes. Now it seems to be the neglect of this consideration that is really at the foundation of the fallacies which befall hypothetical arguments. For, as Mel-lone states, "if we deny the antecedent, we cannot, therefore, deny the consequent, for the latter may be true for other reasons, and if we affirm the consequent, we cannot affirm the antecedent, for the consequent may result from other reasons." In either case, that is to say, we must recognise the possible existence of more connections between antecedent and consequent than the one which is made the subject of statement by the major premise, and this possibility places limitations upon the interpretation which we can give to hypothetical statements. It is true that, if the major premise states a causal connection between the

antecedent and consequent, we may argue from the affirmation or denial of the former to the affirmation or denial of the latter, and vice versa. In any other case, the possible operation of a plurality of causes must be allowed to determine the limits which we place upon hypothetical arguments. From this point of view, it seems quite clear that when we argue, "If a man's character is avaracious, he will refuse to give money for useful purposes; this man refuses to give money for useful purposes, therefore, this man's character is avaracious," the conclusion is erroneous, not merely because the minor premise affirms the consequent, but because, as we all know, there are other reasons, not at all connected with a man's character, which may make a refusal of the kind in question the necessary and right course for him to pursue. And it seems equally obvious that when we argue, 'If some agreement is not speedily arrived at between employers and workmen, the trade of the country will be ruined, an agreement will not be arrived at and therefore the trade of the country will be ruined,' the conclusion is erroneous, not merely because the antecedent is denied, but because the prosperity of a country depends upon other conditions than the one specified in the antecedent of the major premise. In both these instances we see that the breach of the formal rule of hypothetical arguments is tantamount, on the material side, to the neglect of the plurality of causes, that is, to the failure to recog-

nise the complexity and interconnection of natural events.

Disjunctive Fallacies may be defined as those mistakes in reasoning which render the conclusions of disjunctive arguments erroneous. Disjunctive arguments, as we have seen, derive their character from the fact that the major premises of such arguments state a series of alternative predicates any one or more of which, as a matter of fact, may be found to qualify their subjects. Any conclusion which is reached on the basis of a disjunctive major premise is valid or erroneous according as the enumerated alternatives do or do not conform to the conditions of disjunctive statements, and as the inference is or is not warranted by the premises on which it is made to rest. With respect to the nature of disjunctive statements, it is sufficient to recall that these must satisfy three conditions. First, the alternatives must all fall within a single universe; not any set of terms that we can state in the form of 'either — or' are, in the logical sense, disjunctive. Second, the terms of the disjunction must be exhaustive; all the species that fall within the universe must be enumerated disjunctively. Third, we saw that the alternatives do not necessarily, although they sometimes do, exclude each other; it does not follow, therefore, that only one of the predicates can belong to the subject under the conditions that determine a particular argument. Whenever we fail to observe either one of these requirements of disjunctive statements in the process of argument,

we are guilty of one or another of the disjunctive fallacies.

The fallacy of false disjunction occurs when we proceed to a conclusion on the ground of a major premise in which the presented alternatives do not all belong to a single genus. The neglect of the first positive condition of disjunctive statements leads necessarily to fallacious arguments, because, where it is not satisfied, there is no common field within which the systematic connections of things, as set forth in disjunctive statements, may be said to fall. A failure to keep clearly before oneself the fact that disjunctive alternatives are definitely alternatives within a single field of knowledge, reveals an entire misapprehension of the nature of disjunctive statements, and leads unavoidably, therefore, to erroneous conclusions. When, for example, we are prepared to find in a stranger to whom we have been introduced all sorts of social virtues merely on the ground that he has come from our home town, or when race prejudices make us blind to the virtues that individuals of a particular race display, we are, in all probability, guilty of the fallacy of false disjunction. The logical process underlying social attitudes of the kind in question may be illustrated as follows: 'Either from my home town or devoid of all or certain current social virtues; from my home town, therefore not devoid, etc.'

The fallacy of incomplete disjunction occurs when we proceed to a conclusion on the ground of a major

premise in which the presented alternatives are not exhaustively enumerated. It is true, of course, that the observation of single facts may lead to a disjunctive statement which does not meet the condition of complete enumeration. We may say, for example, 'This tree is an oak or an ash.' But propositions of the kind just illustrated, while they may serve as minor premises of disjunctive arguments, cannot serve as major premises, for the reason that, although the universe within which the alternatives fall has been accurately determined, the alternatives within that universe have not been completely enumerated. The necessity of a complete enumeration of the alternatives is due to the fact that the disjunctive syllogism is intended to show, not only what a thing is, but also to determine, within the genus to which it belongs, what it is not. Let us suppose that a particular tree is either an oak or an ash. Then, if we argue, 'This tree is an oak or an ash, it is an oak, and therefore it is not an ash,' the argument would be formally valid, and the conclusion would be true; but if we supposed that the conclusion stated the whole truth about the subject, we should fall into error due to the incomplete disjunction of the major premise. Or, to take another illustration, let us suppose that a particular ore is not either gold or copper. If, then, we were to argue, 'This specimen of ore is either gold or silver or copper or antimony; it is not either gold or copper, and therefore it is either silver or antimony,' we have fallen clearly into a conclusion which is not at

all necessarily true, for the specimen in question might be none of those enumerated in the major premise. Such a conclusion is erroneous because all the species of the genus to which the particular specimen belongs were not enumerated. But, some one may say, we cannot always meet this requirement of complete enumeration, and are we, therefore, precluded from arguing validly to a disjunctive conclusion? The answer is that we must express the indeterminate state of our knowledge about objects in a complete enumeration if we are to reach valid conclusions, and we may do this without necessarily possessing an exhaustive knowledge of the subject to which the argument refers. Thus, to state the above argument in a form from which a valid conclusion may be drawn we may say, 'This specimen of ore is either gold or silver or copper or antimony or some other species,' a statement which does not claim complete knowledge of the subject-matter of the argument, but does rest upon a complete determination of the knowledge that is possessed about it.

The fallacy of ambiguous disjunction occurs when we proceed to a conclusion on the ground of a major premise in which the presented alternatives are not precisely determined with respect to their exclusion. We have seen that any proposition of the form 'S is either A or B' strictly interpreted means 'S is either A or else B,' but we also saw that the alternatives do not necessarily preclude the possibility that 'S may be both A and B.' Unless, therefore, we take special pains to make clear, in particular instances,

in which of these two ways the disjunctive major is to be interpreted, we are likely to be led to a conclusion which leaves us in doubt as to the meaning of that conclusion. If we argue, for instance, that 'Because all male students of the university either drill, or play football, or baseball or tennis, and because X drills, therefore, he does not play football or baseball or tennis,' we are stating a conclusion of the individual which may be true, but it is not true for the reason assigned; for there is nothing in the universe of physical education as conducted by the university that makes an exclusive interpretation of the enumerated alternatives necessary. If, then, we give an exclusive interpretation to a non-exclusive series of alternatives, or a non-exclusive interpretation to an exclusive series, we are guilty of the fallacy of ambiguous disjunction.

REFERENCES

SEE THE END OF THE NEXT CHAPTER

QUESTIONS

1. What is the name and character of Aristotle's treatise on fallacies?
2. In what senses is the term fallacy used at the present time?
3. In what two main groups did Aristotle classify the fallacies?
4. To what distinctions drawn in the text do Aristotle's main groups correspond?
5. Describe the chief features of each of the four main classes into which the text divides fallacies, and by what names are they distinguished?

6. Into what subdivisions, and according to what principle, may the fallacies of interpretation be divided?
7. What do we understand by verbal ambiguity, and how does it differ from mere indeterminateness in the meaning of words?
8. Describe and illustrate the fallacy of equivocation.
9. What is the fallacy of accent, and in what respect does the corresponding Greek fallacy differ from it?
10. In what several ways may one fall into a fallacy of accent? Illustrate.
11. Describe and illustrate the fallacies of composition and division.
12. What is the Latin name for the fallacy of accident? Illustrate the fallacy.
13. Give an illustration of the converse fallacy of accident, and by what Latin name is it known?
14. Describe the character of structural ambiguity.
15. What is the fallacy of amphiboly, and from what other kinds of ambiguity must it be distinguished?
16. State and illustrate the fallacy of false parenthesis.
17. What do you understand by fallacies of implication, and into what two classes may they be divided?
18. What is a fallacy of opposition?
19. To what fallacies are the following logical relations particularly liable: contrariety, subcontrariety, subalternations?
20. What is a fallacy of education?
21. State what is meant by a fallacy of obversion.
22. What is a fallacy of conversion, and in respect of what propositions, and why, is it likely to occur?
23. What are fallacies of inference, and into what two classes may they be divided?
24. What is a deductive fallacy, and how are such fallacies classified?
25. What is a categorical fallacy?
26. What is meant by a fallacy of four terms, and state in what two ways such fallacies arise

27. Classify and describe each of the fallacies of four terms.
28. What are fallacies of insufficient premises, and what rules of the syllogism do they break?
29. Show, in relation to the nature of syllogistic reasoning, how the fallacies of particular and of negative premises occur.
30. Describe as fully as you can the character of an hypothetical fallacy.
31. State and illustrate the two hypothetical fallacies.
32. How does the fact that natural events are complex affect our interpretation of hypothetical statements?
33. What do you understand by a disjunctive fallacy?
34. Into what classes, and according to what principle, may disjunctive fallacies be divided?
35. State and illustrate the fallacy of false disjunction.
36. Describe the fallacy of incomplete disjunction. Illustrate.
37. What is the fallacy of ambiguous disjunction? Illustrate.

EXERCISES

Examine the following arguments and state in what respect, if any, they are fallacious, the technical name of the fallacy of which each is guilty, and the class to which the fallacy belongs:

- (1) It is sometimes argued that the communication of an infectious disease to another person should be punished by law, since the law exists for the purpose of protecting people from harm.
- (2) I have made thee free a slave.
- (3) How can you argue that it is wrong for me to assist my friend to obtain the office by offering business or financial considerations to the voters of my acquaintance if you agree that it is right for a man to do what he can for his friend's good?
- (4) A commercial traveller wrote to his chief: Dear Sir, On Wednesday next I want you to allow me

the day off. My wife having lost her mother is being buried on that date and I should like to attend the funeral.

- (5) A, B, C, are all the known instances of the class X, and since they are all found to possess the characteristic p we may conclude that All X is p.
- (6) Large numbers, though not all, of the instances of the class X have invariably been found to exhibit the character p, therefore, All X is p.
- (7) If it is false that no men are immortal, it must certainly be true that all men are mortal, since there is no middle position between mortal and immortal.
- (8) If you perform an unfriendly act, I may certainly conclude that your attitude is hostile.
- (9) He who is content with what he has is truly rich, but the covetous man is not content with what he has, no covetous man, therefore, is truly rich.
- (10) All Parisians are Frenchmen indeed, when the only reason you have to advance for that statement is that all Parisians and all Frenchmen are Europeans!
- (11) What is not compound is an element, gold is not compound, and is, therefore, not an element.
- (12) If a substance is a metal, it is a good conductor of heat and electricity, but this substance does not conduct either well, and so it cannot be a metal.
- (13) If man were not capable of progress, he would not differ from the brutes, but since he does differ he must be regarded as capable of progress.
- (14) What you speak of must be either blue or green or yellow, but since you say that it is neither of these it must be devoid of colour.
- (15) That happened on Monday or Thursday or in April, but it was not in April for I remember that it was Harvest; it must, therefore, have happened on Monday or Thursday.

CHAPTER XIX

FALLACIES (CONTINUED)

Inductive Fallacies. By fallacies of induction, as was stated above, we understand those errors to which thought is exposed in its endeavour to determine the principle of connection between a number of particular facts. Now, since induction requires, as a condition of the explanation of the particulars of experience, that the facts of experience be carefully observed, it is obvious that we are exposed mainly to two classes of inductive error, to those, namely, which arise in connection with our observation of facts, and to those which arise in connection with our explanation of facts. But since language is an indispensable instrument both for recording what we observe, and for stating the methods and results of the explanations that we have to offer of what is observed, we may include in the present discussion a brief statement of the nature and sources of the fallacies into which a careless use of language may be the means of leading us. The fallacies of induction will, then, be studied under three heads; first, those which originate in our use of language; second, those which involve errors of observation; and third, those which occur in connection with the so-called inductive inference, the errors, that is to say, which invalidate explanation.

Fallacies of Language. Readers of Plato cannot fail to have remarked that the difficulties that he sometimes experienced in making clear to himself and to others the character of some of the more elementary logical distinctions was due, in great measure, to the fact that the nature of even simple grammatical distinctions was not clearly understood by him and his contemporaries. The fact that Aristotle attributed, as we have seen, a large number of the fallacies to the influence of language, also points to the close connection that there is between thought and human speech. In more modern times, Bacon, Hobbes, Locke, and others, have remarked upon the dangers to which we are liable through the words that we employ to express our ideas, and Mill wrote two chapters in which he discusses the requirements and principles of what he calls a "philosophical language." These references, which might be indefinitely extended, are enough to show that while our acquired speech may be sufficient for the ordinary purposes of life, it must be carefully employed if, in regard to the subjects of scientific investigation, it is not to become a source of embarrassment and error.

Bacon gave the name of Idols of the Market-Place (*Idola fori*) to the fallacies which were due to language, and he puts the sum of the matter as follows: "Men imagine that their reason governs words whilst, in fact, words react upon the understanding; and this has rendered philosophy and the sciences sophistical and inactive." The same truth

is dwelt upon at length by Locke, and the general nature of the influence that language is capable of exerting prejudicially upon thought, the only point with which we are here concerned, is evident from the following quotation: "Men," he says, "having been accustomed from their cradles to learn words which are easily got and retained, before they knew or had framed the complex ideas to which they were annexed, or which were to be found in the things they were thought to stand for, they usually continue to do so all their lives; and, without taking the pains necessary to settle in their minds determined ideas, they use their words for such unsteady and confused notions as they have, contenting themselves with the same words other people use, as if their very sound necessarily carried with it constantly the same meaning. . . . This inconsistency in men's words when they come to reason concerning either their tenets or interest, manifestly fills their discourse with abundance of empty, unintelligible noise and jargon, especially in moral matters, where words, for the most part, standing for arbitrary and numerous collections of ideas not regularly and permanently united in nature, their bare sounds are often only thought on, or at least very obscure and uncertain notions annexed to them. Men take the words they find in use amongst their neighbours; and, that they may not seem ignorant what they stand for, use them confidently, without much troubling their heads about a certain fixed meaning; whereby, besides the ease of it, they obtain this

advantage: That, as in such discourses they seldom are in the right, so they are as seldom to be convinced that they are in the wrong; it being all one to go about to draw those men out in their mistakes who have no settled notions, as to dispossess a vagrant of his habitation who has no settled abode."

Fallacies of Observation. Observation may be defective chiefly in two respects. It may be insufficient in amount, or incomplete or imperfect in detail. If we consider these two cases separately, it must not be forgotten that, frequently and perhaps usually, the fallacies which depend upon observation occur as the result of both kinds of defect. Induction, as we have seen, depends upon the collection of data sufficient in amount to suggest a law of connection; but, more important than the number of the facts observed, is the degree of thoroughness with which we are able to analyse them. Mere collections of fact, if such a thing were possible, have, as we have seen, no special value for science, and alone can lead to the formulation of no general principles of connection. The particular data which are the material of science are brought under observation by the exercise of discrimination, comparison, synthesis, in a word, judgment; and this differs in no essential respects from the process by which we detect, within collected groups of fact, the features that are common, and the connections that are causal. But while this should be kept clearly in mind, it is nevertheless true that we may be led into an inductive fallacy, either because the amount of

data available does not warrant an inference, or because, although sufficient in amount, our analysis of the data has not been carried out with sufficient care and thoroughness.

Observation, then, is defective, in the first place, when the number of the observed data is not sufficient in amount to guarantee the conclusion that is based upon it. In view of the situations which this statement has in view, we are particularly interested, in logic, to ascertain what are the conditions that lead to insufficient observation. One important condition is the general tendency to neglect the observation and collection of what are called negative instances. By a negative instance is meant an instance in which a given phenomenon is observed not to occur. If we take, for example, the popular superstitions, — such, for instance, that it is unlucky to begin any new task or undertake any new enterprise on Friday, for people to be associated together in groups of thirteen, or to walk under a ladder that is inclined against a wall, — we are seriously expected, by those who entertain them, to share these superstitions in view of the instances that these persons are able to enumerate in which some untoward result has occurred. An enumeration of the instances in which no such consequence has followed, it would seem, is sufficient to show that the superstitious conclusion is defective if not fallacious by reason of the failure to observe these negative instances.

To have called our attention to this tendency to neglect the evidence afforded by negative instances is one of the merits of Bacon. The following quotation from the *Novum Organum*, not only states the nature of the fallacy, but indicates also its scope. Bacon writes: "The human understanding, when any proposition has been once laid down (either from general admission and belief or from the pleasure it affords), forces everything else to add fresh support and confirmation; and although most cogent and abundant instances may exist to the contrary, yet either does not observe or despises them, or gets rid of and rejects them by some distinction, with violent and injurious prejudice, rather than sacrifice the authority of its first conclusions. It was well answered by him who was shown in a temple the votive tablets suspended by such as had escaped the peril of shipwreck, and was pressed as to whether he would then recognise the power of the gods: But where are the portraits of those who have perished in spite of their vows? All superstition is much the same, whether it be that of astrology, dreams, omens, retributive judgment, or the like, in all of which the deluded observers observe events which are fulfilled, but neglect and pass over their failure, though it be much more common. But this evil insinuates itself still more craftily in philosophy and the sciences, in which a settled maxim vitiates and governs every other circumstance, though the latter be much more worthy of confidence. Besides, even in the absence of that

eagerness and want of thought (which we have mentioned), it is the peculiar and perpetual error of the human understanding to be more moved and excited by affirmatives than by negatives, whereas it ought duly and regularly to be impartial; nay, in establishing any true axiom the negative instance is the more powerful."

The testimony of Darwin is to the same effect. He says: "The success of the 'Origin' may, I think, be attributed in large part to my having long before written two condensed sketches, and to my having abstracted a much larger manuscript, which was itself an abstract. By this means I was enabled to select the more striking facts and conclusions. I had also, during many years, followed a *golden rule* — namely, whenever a published fact, a new observation or thought, came across me, which was *opposed to* my general results, to make a memorandum of it without fail and at once; for I had found by experience that such facts and thoughts were far more apt to escape from memory than favourable ones. Owing to this habit, very few objections were raised against my views which I had not at least noticed and attempted to answer."

Another condition of insufficient observation that may be mentioned is education. Whatever our conception of the aim of education may be, there is no doubt that a large part of the educative process, as we actually experience it, is taken up with an effort to gain familiarity with a body of knowledge that is regarded as traditionally important and fun-

mental. This statement is true for all grades of education from the grammar school to the university. Now it is not only inconceivable, but contrary to experience, that the acquisition of any considerable body of knowledge should be without its effect upon our mental habits; for acquaintance with any class of facts means, in terms of mental habit, a trained ability to perceive with increasing facility other facts that are the same in kind as these, and a corresponding inability to perceive facts that differ considerably from them in kind. A particular illustration of this general truth is found in the case of those individuals whose education assumes a special character, as it does, for instance, in law, medicine, theology, engineering, etc. And even if we consider the case of those whose common school education is not followed by a university training, but by apprenticeship in some business, it is quite apparent, even then, that the knowledge of the business, of its material, methods, and aims, acts exactly in the same way to sharpen the perception to observe a specialised group of facts, and to limit the facility with which facts dissimilar to these are perceived. The truth is that all of us perceive what we are educated to perceive, and that we are limited in respect to the kinds of things that we perceive by the education that we have received.

Bacon referred to the errors into which we are led as the result of the limitations that education and inherited beliefs may have upon the range of

our perceptions as Idols of the Theatre (*Idola Theatri*). Doubtless, as is shown by his illustrations, he had in mind the cramping effect of the traditions of the Schoolmen upon the new learning in his day. Bacon lived at the dawn of the modern scientific era, and he was acquainted with some of the obstacles that it had to encounter on account of theological prejudice (*odium theologicum*) of the times. But if we have passed beyond the stage where the prejudices which come from a one-sided theological position and training are allowed to interfere with the freedom of thought and investigation, it is still possible that prejudices the same in kind, though different in form, may hamper the human mind in its search for truth. The fact that these hindrances are not officially administered, but are grounded in that specialisation, upon which our modern education lays so much importance, would make the tendency to which we are referring the more insidious, and hence more difficult to detect and avoid or overcome. "The success of modern scientific methods," as Creighton remarks, "has sometimes led investigators to despise and belittle the work of those who do not carry on their investigations in laboratories, or do not weigh and measure everything." Thus the mechanical conceptions which were worked out successfully in the realm of physics and astronomy, have been extended to psychology, ethics, and political science, without any consideration being given to the differences in the material which is the subject-matter of this latter group of sciences. In our

own day, the evolutionary hypothesis has provided us with a point of view for looking at all the details of human experience; but it is not impossible that the limitations of this point of view and the errors of observation to which it has led, may, at some future time, be as clear as today are the insufficiencies of the notion of mechanism as employed by the thinkers of the eighteenth century.

In the second place, observation may be defective either because it is incomplete or imperfect in detail. The instances of fallacious inference which belong in this group are those which arise from the fact that the collected data, upon which the inferences rest, have not been examined with sufficient thoroughness. Defects of the kind here in question may be due either to the fact that only part of the material has been studied, or to the fact that, while all the material may have been studied, this has not been done for all or parts of the material with sufficient minuteness. In either case, the inference will be erroneous on account of a failure to carry our analysis of the phenomenon under observation into sufficient detail.

A number of conditions may operate to arrest our analysis of observed data at a point which falls short of that at which it is possible to determine with some degree of accuracy the law of their connection, but two are, from the logical point of view, of sufficient interest to require brief mention. There is no doubt, in the first place, that great

differences exist in the ability of different investigators to perform the task of analysing a group of collected facts. Some individuals, it is found, are splendid collectors, and are exceedingly fertile in devising ways and means of bringing under observation specimens that entirely escape the notice of others not gifted as themselves. But it sometimes happens that among these born collectors there are many who can go but a little way toward that complete analysis of their collected material upon which alone any general conclusion can safely be based. The recognition of this fact has led, in practice, to the introduction of a large amount of coöperation between workers in the same line of research, and has tended to make the laboratory a workshop in which there is found employment for a great variety of talent. It is only the exceptional man who, like Darwin, combines the collector's skill with a keen analytic mind that brings to light the common features that lie hidden under a mass of differences in any group of facts. Natural limitations, then, of the sort we are discussing operate frequently to render the observation of details incomplete, and to throw doubt upon any conclusion which rests upon such imperfectly analysed data.

Another condition which tends to make analysis defective is an imperfectly developed technique or method. We have spoken above of the importance of the construction and use of apparatus if the work of scientific observation is to be successfully carried on. Indeed, we may say that the progress of modern

science has been dependent, in large measure, on the success that has attended the development of special methods for bringing the structure of collected data under detailed observation. In the biological sciences, for example, the methods of hardening, embedding, and staining the various tissues have gone a long way toward placing these sciences on the same experimental plane as physics and chemistry. The further progress of these, as of the other, sciences depends upon the adaptation of the methods already in vogue to new material, and the development of new methods to meet new conditions and situations as they arise. But whenever we speak of improvement, and recognise the possibility of new discoveries, we at the same time imply that our present knowledge is defective in just those respects in which advancement may be expected. This, however, is only to say that so long as the technique of any science is capable of improvement, the conclusions of that science must be entertained with caution and subject to certain reservations.

Fallacies of Explanation. There are two sources of error in inductive processes of reasoning to which we shall direct attention: hasty generalisation and false analogy. It is exceedingly difficult to give a concise statement of what is meant by hasty generalisations, since any explanation may be rendered erroneous by undue haste at any of the stages of the inductive process. For instance, an inductive conclusion may be fallacious on account of haste at either the stage of what was called above empirical

generalisation, or at the stage of hypothesis, or at the stage of verification. Whenever, therefore, any part of the task of explanation is not thoroughly performed, our conclusion, resting as it does upon an incomplete or imperfect acquaintance with the details of the phenomenon, may be called a hasty or premature generalisation. The essential nature of the fallacy consists, then, in the absence of an analytic thoroughness requisite for a true explanation, without respect to the part of the explanatory process in which the failure to be sufficiently thorough occurs.

We may remark, however, that we are not guilty of this fallacy when we consciously arrest our analysis of a phenomenon to be explained at a point which falls short of completeness, or when imperfectly observed or analysed data are generalised, but are not offered as an explanation. The fallacy occurs only when an incomplete analysis is presumed to be complete, or when a preliminary generalisation is mistaken for a final one. As has been said, the value of logic consists in calling attention to the fact that, not only must conclusions be true, but the reasons for the truth of conclusions must be definitely ascertained, if science is to exist. Science, as we have seen, is a consciously controlled process, a process of arriving at a certain conclusion, by means of a method which is definitely regulated at all the stages through which it passes from the earliest observation to the finally accepted explanation. When the control of the method by which conclu-

sions are reached gets out of hand, when we become careless in the use of the methods we adopt for the examination of phenomena, our explanation of such phenomena is erroneous, because it involves, in the sense in which the term has been explained, a hasty generalisation. Such conclusions are fallacious because they do not rest upon a sufficiently detailed basis of fact.

Analogy may give rise to erroneous conclusions when it is supposed that the phenomenon under observation has all the characteristics of the instance which provides us with the analogical resemblance. We have seen that analogy is a useful method, or step in the method, of scientific explanation, but unless it is strictly guarded it is likely to arouse all sorts of distracting associations which lead to misunderstanding and error. When considered as a possible source of error, analogy has certain affinities with figurative language, on the one hand, and with the disregard of negative instances, on the other. With respect to its relation to figurative language, analogy leads us astray when the illustration which provides us with the analogical resemblance engages the whole attention, and we are thereby led to predicate of the phenomenon under investigation qualities that have been actually observed only in the illustration. It was with the dangers of analogical reasoning in mind that a professor was wont to say to his students that they should not try to make the parables of the New Testament "go on all fours," that is, they must not try to find

a point for point resemblance between the truth of the parable and the story which provided the basis of the analogy. It is neglect of such counsel that leads in the sciences to the fallacy of false analogy.

With respect to its relation to negative instances, analogy is likely to lead us into error if the points of difference between a phenomenon and its illustration are lost sight of, and only the resemblances are retained. "For instance," to borrow an illustration from Sidgwick, "during the recent Boer War various analogies were appealed to by various people as helping to throw light on the situation. It was compared to the operations of America in the Philippines, to the Armenian massacres, to the American Civil War, and even to the high-handed action of Spain in the Netherlands in the time of Elizabeth. Our action in South Africa was supposed (by different people) to be analogous to those and other historical precedents, and the analogies were used to support the various judgments passed by these various people on the rights of the war and its probable outcome. It is easy to see that in all these comparisons there is some likeness and also some difference, and that the justice of the analogy depends upon the relevance of the likeness and the difference." But if the difference is a relatively important one, and we proceed nevertheless to build up an explanation on the ground of the likeness alone, we are guilty of the fallacy of false analogy. The fallacy assumes the converse form if we are led to underestimate the value of a resemblance because

it is accompanied by striking differences. In any case, the fallacy arises when we forget that analogical reasoning depends upon what Sidgwick calls "the relevance of the likeness and difference."

Fallacies of the Context. The fallacies that we have considered in the foregoing pages arise out of a failure to determine precisely the nature and relations of the subject-matter which constitutes the contents of particular arguments. The group of fallacies to which we have now to direct attention arises from the fact that the meaning of particular arguments, as we have shown, is determined in part by their relation to a context which they imply with varying degrees of clearness. Now when we say that the context supplies part of the meaning that an argument may rightly be said to have, it is important to bear in mind that the context may be said to refer, not only to the undetermined aspects of the subject-matter under discussion, but also to the interests or purposes which sustain, as we have seen, our inquiries into truth and our search for knowledge. Any discussion of what knowledge or truth is cannot fail to be inadequate, as we have shown, if attention is not given to the part played by the complex character of the material about which knowledge is desired, and also to the character of the interest which knowledge is intended to satisfy. Because, therefore, of its relation to a context, every argument is liable to errors which do not arise directly out of the facts and relations which are expressed by its constituent propositions. There

may be imported, that is to say, into the meaning of arguments elements from their contexts which lead to conclusions which are at variance with what the premises, logically interpreted, really support. Whenever this happens, whenever, that is, we accept arguments on the ground of suggestions, derived from their contexts, which are not related to the truth character of the arguments themselves, there ensues a fallacy of the context. A fallacy of the context may be defined, therefore, as an error in reasoning which is due to the influence that some feature or features of a context, not related to the truth character of the argument, has upon our interpretation of the subject-matter of the argument, such influence inducing us to accept a statement about the subject-matter of the argument as true for reasons which, apart from the influence referred to, would fail wholly or in part, to justify its acceptance.

The nature of these fallacies may be made still clearer, if we observe that in arguments of the kind under consideration the context may contribute irrelevant elements derived from either of the factors by which, as was pointed out, human knowledge is determined, — from the world of relatively independent objects on the one hand, and from the world of our interests, on the other. An imperfect determination of the universe within which an argument is intended to be considered leads to a group of fallacies of which *Petitio Principii*, *Ignoratio Elenchi*, *Non Sequitur*, and *Complex Question* are the chief;

and a failure to confine the interests by which we are guided in our search for truth to those which have a direct relation to that end leads to another group of which *Post Hoc*, *Argumenta ad Hominem*, *ad Populum*, *ad Verecundiam*, *ad Misericordiam*, may be taken as the most important. In the former group, we are considering how the relations which a given object has to other objects may become a source of error in our knowledge of the world of relatively objective things, and to these we may give the name, fallacies of assumption. In the latter, we are studying the ways in which knowledge may be deflected by reason of the operation of certain natural and acquired characteristics of the mental life; and to these we may give the name, fallacies of interest. In both cases the fallacy is due to the admission into our reasoning of elements which have no direct bearing upon the truth character of the judgments in which our knowledge is asserted.

Fallacies of Assumption are those errors in reasoning which occur when the assumptions on which an argument rests are not clearly distinguished from the judgments of which the argument consists. An assumption, in this connection, is anything we take for granted, but do not assert, about the subject-matter of an argument. It is the equivalent of what we have had occasion hitherto to speak of as the universe of discourse. Interpreted from this point of view, it is readily seen that an assumption is not an assertion, and forms no part of the asserted contents of an argument, although, as we have seen, it

has a relation to the argument, a relation which is indicated with sufficient clearness by saying that it points out the sphere of reference in which what is asserted may or not be accepted. Now it is a misinterpretation of the relation between what an argument assumes and what it asserts that lies at the foundation of the fallacies that we have here to consider. Thus, when what is taken for granted or assumed is allowed to function in any part of an argument as an assertion or judgment, or when the assumption on which an argument proceeds is ambiguous, the resulting fallacy is one of assumption.

Petitio Principii is the name of an argument which assumes the conclusion that is to be proved. It is, as Sidgwick says, "the surreptitious assumption of a truth you are pretending to prove." Since, then, the fallacy is one of assumption, it is evident that its source must be found, not in what is definitely asserted, but in the world of reality or existence in which what is asserted has a definite meaning or fulfillment, that is to say, in the universe of discourse from the standpoint of which the argument is interpreted. The nature of the fallacy is seen in the confusion that exists, for the person guilty of it, between the asserted grounds of a conclusion, and the assumptions that thought postulates with respect to the subject-matter of an argument. Whenever it exists, the fallacy directs attention to the fact that the truth of what an argument asserts depends in part upon what assumptions the argument makes; and, in view of the nature of an argu-

ment, it follows that when assumptions are put forward as reasons we necessarily fail to establish a conclusion, and fall into the merest dogmatism unless we are willing to have these assumptions called into question. "When," for instance, "Galileo maintained that the earth goes round the sun, he fully believed it and yet was willing to treat it as a disputable question; but when Pope Urban VIII and his cardinals maintained that the earth stood still, they shut their eyes to the arguments, and said their view was only disputable by any one who chose to be burnt alive." Now, with respect to the right to call into question any of the grounds on which conclusions are supposed to rest, we can only say that this right must be conceded, or that the fallacy we are now considering can have no logical standing. But further, if we cannot go behind the premises, nor consider the rights of assertions to stand as premises, but must accept any set of assertions at their face value as grounds of argument, then the whole of logic has been reduced to a mere formal science, and its processes are valuable only as they enable us to secure consistency of statement. Either this, or we must admit that there is a real difference between assertions and assumptions, between judgments and postulates, that the human mind is likely at times to fail to keep the distinction clearly before it, and, consequently, that the one may sometimes slip, in the course of argument, into the place of the other. Now, when this happens, when in the course of argument assumptions take the place of reasoned

judgments, the argument is fallacious because, for the reason assigned, it involves a *petitio principii*.

The English language, as Whately pointed out, is peculiarly "suitable for the fallacy of *Petitio Principii*, from its being formed from two distinct languages, and thus abounding in synonymous expressions which have no resemblance in sound, and no connection in etymology, so that a Sophist may bring forth a proposition expressed in words of Saxon origin, and give as a reason for it the very same proposition stated in words of Norman origin; e. g. 'To allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the State; for it is highly conducive to the interests of the community, that each individual should enjoy a liberty perfectly unlimited of expressing his sentiments.' "

When the fallacy of *petitio principii* is committed in a single step it is called ὕστερον πρότερον (*hysteron proteron*); and when it involves more than a single step it is called *circulus in probando* or reasoning in a circle.

Closely connected with the foregoing is the fallacy of the Complex Question. By a complex question, in the broadest meaning of that term, is meant one that suggests its own answer. Any question, for instance, that forces us to select, and assert in our answer to it, one of the elements of the question itself, while some other possibility is really open, is complex in the sense in which that term is here employed. If, for example, one were to ask whether

you were going to New York or London, or if your favourite colour were red or blue, or if you had given up a particular bad habit, he would be guilty of the fallacy of the complex question, if, in each case, the alternatives, as a matter of fact, were more numerous than, or were in any way different from, those stated in the question. Any leading question which complicates an issue by over simplification is fallacious for the same reason. Now, in the light of what we have said with respect to the *petitio principii*, it is not difficult to see that the fallacy of the complex question is occasioned by the character of the assumption on which the question rests. In the *petitio principii* an assumption with respect to the subject-matter of an argument functions as a premise, in the complex question it is a similar assumption that shuts out some of the material possibilities of a situation and confines an issue within too narrow limits. As in the former case, so here, the only way of meeting the difficulty is to raise the previous question, that is, to call the assumption which lies back of the fallacy into question.

Ignoratio Elenchi, according to Aristotle, is a fallacy which arises from "ignorance of the nature of refutation." In order to refute an assertion, Aristotle says we must prove its contradictory; the proof, consequently, of a proposition which stood in any other relation than that to the original, would be an *ignoratio elenchi*.

Since Aristotle, the scope of the fallacy has been extended to include all cases of proving the wrong

point. Thus, when H. Spencer, in arguing against the advocates of a classical education, says that "Throughout his after career, a boy, in nine cases out of ten, applies his Latin and Greek to no practical purposes," he is guilty of an *ignoratio elenchi*, for the argument, as Welton points out, "ignores the fact that the advocates of a classical education do not claim that Latin and Greek are of direct use in practical life. What they do urge is that the study of the classics furnishes an unrivalled mental training; and it is this proposition which a true ἐλέγχος (elenchus) must disprove." With this interpretation of the fallacy Whately is in agreement. He writes: "I am required to prove a certain conclusion; I prove, not that, but one which is likely to be mistaken for it; in that lies the fallacy. . . For instance, instead of proving that 'this person has committed an atrocious fraud,' you prove that 'this fraud he is accused of is atrocious;' instead of proving, as in the well-known tale of Cyrus and the two coats, that 'the taller boy had a right to force the other boy to exchange coats with him,' you prove that 'the exchange would have been advantageous to both;' instead of proving that 'a man has not a right to educate his children or dispose of his property in the way *he thinks best*,' you prove that 'the way in which he educates his children or disposes of his property is not *really the best*;' instead of proving that 'the poor ought to be relieved in this way,' you prove that 'they *ought to be relieved*.'" The fallacy usually occurs, as Jevons

remarks, "in the course of long harangues, when the multitude of words and figures leaves room for confusion and forgetfulness." Hence, an *ignoratio elenchi* has been called by Aikins the fallacy of the forgotten issue. Thus it is frequently necessary, for example in debate, to call the discussion back to the point at issue. For example, in a speech in the House of Commons, Canning is reported to have said, in reply to Mr. Percival, that "The question is not, as assumed by my opponent, whether we shall continue the war in the Peninsula, but whether it is essential to our success in the war that our present system of currency remain unchanged."

The nature of the fallacy, then, consists in substituting for a certain issue another which is more or less closely related to it, and arguing the substituted issue. The fallacy does not take into account whether the arguments do or do not really support the substituted issue, it only calls attention to the fact that they do not constitute a proof of the original one. Students are quite often guilty of an *ignoratio elenchi* when in reply to questions they substitute some other though related question and allow their answers to be guided by it. It is a particularly prevalent and subtle fallacy and it assumes a great variety of forms. But whenever it occurs and whatever form it takes, it is brought about by an assumption that leads the person guilty of it to substitute for a definite subject of inquiry another which is in close relation with it. In the *petitio principii* the fallacy may be described as an assump-

tion of the premises; in the complex question, as an assumption of the answer; and in the *ignoratio elenchi*, as an assumption of the question at issue.

Whenever, for any reason, a conclusion does not follow from the assigned premises, we have, in the literal sense of the term, a Non Sequitur. When used in this sense to include all the errors in reasoning which leads to erroneous conclusions, the term is generic, and must be understood as synonymous with 'fallacy.' Any kind of loose or inconsequential argument, from this point of view, would be a *non sequitur*.

More particularly considered, the fallacy of *non sequitur*, as was indicated by Aristotle, is the name we apply to the erroneous conclusion of a hypothetical syllogism. Aristotle regarded it as an "argument from the affirmation of the consequent," and for this reason it is sometimes called the fallacy of the consequent. If, for example, we argue that a particular individual who has no visible means of support must be a professional thief, because "if a man is a professional thief, he will have no visible means of support," the argument will be fallacious because, as we have seen, no conclusion can be reached by affirming the consequent of hypothetical propositions. The term *non sequitur* may also be applied to the other hypothetical fallacy of denying the antecedent. In both the cases to which the term has been said to apply, the assumption is made that the relation between antecedent and consequent of the major premise is causal, an assumption which

we have no right to make in view of the complex interconnection of the facts with which our human knowledge has always to deal. But if it is made, the only way we can avoid a possible fallacy is to call into question the assumption, and thereby to raise definitely the question whether antecedent and consequent are causally connected or not.

The attempt has sometimes been made to apply the name *non sequitur* to the fallacies which occur in connection with categorical arguments. This cannot be considered a useful employment of the term. For if the term applies to any and all of these errors, it must be regarded, as we have said above, as a generic term, and loses all particular significance, and if it is used of a particular kind of categorical fallacy, it immediately takes its place as a synonym of the name of that fallacy. Some logicians, however, do not take this view of the case, and they assign the term to a group of categorical arguments of which the following may be said to be fairly representative: 'Episcopacy is of scriptural origin, the Church of England is the only established church in England, therefore the church established in England is the only church that should be supported.' Again, 'Pennsylvania contains rich coal and iron mines, Pennsylvania has no sea coast, therefore the battle of Gettysburg was fought in that state.' These examples are so flimsy, as examples of argument, that the only sensible course to take with regard to the class of reasoning

for which they stand is to deny that it can have any logical recognition at all. Logic presupposes at the least some development of common sense on the part of those to whom it addresses itself, and one can hardly be expected to take so low an estimate of logic as to think that it can be of any possible service to the individual upon whom such, so-called arguments would have the least chance to impose. The logician has absolutely nothing to do with such an individual who, in the interests of society, had much better be placed in the care of an alienist.

We conclude, therefore, that *non sequitur* is a name that can be applied only to fallacies of the kind that occur in the course of hypothetical arguments. The fallacy consists in misinterpreting the implications of the relation of antecedent and consequent stated in the major premise of hypothetical syllogisms.

Fallacies of Interest. Knowledge, as we have shown, depends for one of its conditions upon what we call interest. But knowledge is not secured and advanced by any or any kind of interest. It is only when, as we have seen, the interest is directed toward, and is accompanied by a desire to ascertain, the nature and relations of the objects by which it is stimulated that it can be considered to be of service in the establishment of knowledge in the strict sense of that term. The interest that must be recognised as an indispensable condition of all knowledge whatever is specifically an interest that points toward and terminates in the establishment

of the knowledge relation. Whenever any other interest than this is allowed, consciously or unconsciously, to influence the steps we take toward knowledge, or the conclusions we accept, the process and the conclusion are for that reason fallacious. Instances of the kind of argument just indicated will be considered in the following paragraphs, and they are grouped together because they all imply that some other interest has been substituted for the one which alone, as we have said, can be of service in the attainment of truth.

Perhaps the most reputable of the fallacies of the kind in question is that which is known as *post hoc, ergo propter hoc*. In discussing the problems connected with the establishment of causal relations, we have seen that a succession of events is to be interpreted always as containing the suggestion of a possible causal connection between the events in question, or with other with which they are constantly associated; but we saw also that the succession itself is never to be taken as evidence of such connection. If, therefore, neglecting this latter consideration, we should argue from an observed succession of events to the causal connection of the events, the conclusion would be fallacious because the only reason that could be assigned for the inference, namely, that, since the one event had followed the other, it was, therefore, caused by that other, is never sufficient to establish a causal relation. The fallacy arises, then, from neglect of the negative criterion of causal relations to which we have just

referred, namely, that mere succession is never a sufficient reason for a causal inference.

The relative prevalence of this fallacy, and the charitable attitude most of us take toward those who are guilty of it, suggest that the error which it implies lies somewhat deeper in the mental life than some others that have come before us. If this is true, the explanation and full exposition of the nature of the fallacy belong to psychology rather than to logic. In a general way, however, we can see that whenever the scientific interest, which seeks to safeguard conclusions from possible sources of error, is not strictly emphasised, or when it is for some reason temporarily weakened, it is easy for the human mind to slip into one of the more usual or conventional attitudes toward the objects of its experience, and thus to arrive at conclusions which, to the extent that this has taken place, are fallacious. In these fallacies, we observe a tendency to confuse associative combinations with causal connections, a tendency which is emphasised whenever the practical takes the place of the theoretical interest in determining our relations to the objects of our experience. An example of this fallacy, frequently met with in educational circles, is the contention that because students who are handicapped by the necessity of earning their livelihood usually manage to pass their examinations, while others who are not so handicapped sometimes fail, the handicap is part of the cause of their success. In similar fashion, we have heard it argued that because Spur-

geon, who never went to college, was a great preacher, that this was due to the fact that he never attended college. There is more common sense and logic in Spurgeon's remark, referring to the above contention, that the result was accomplished in spite of his lacking a collegiate training.

There are a number of fallacies which are due still more obviously to the way in which arguments connect themselves with interests which are distinct from the one with which our search for truth is more particularly associated, and in which the conclusion assumes a character in conformity with these interests. When, for instance, an argument makes an appeal to the character, principles, profession, or interests of an individual, and it is accepted or rejected for that reason, there results a fallacy which is called an *argumentum ad hominem*. If, for example, the evidence for the movement of the earth round the sun is rejected because it is inconsistent with one's religious convictions, the rejection is fallacious because the argument reaches its conclusion by the substitution of the religious for the scientific interest. If, again, the fact that one owns stock in a brewery is allowed to influence his judgment as to what conclusion, with respect to the sale of alcoholic beverages, is warranted by the scientific evidence that is available on the physiological effects of alcohol on the human system, he is guilty of the fallacy under consideration. When, once more, the demands of religion upon the individual are rejected because preachers or church

members are not free from obvious faults, or when they are accepted because thereby business will be helped, the position in each case involves an *ad hominem* fallacy.

The admission, by courts of law, of testimony bearing upon the character of witnesses, requires a word of explanation, because it seems to be in contravention of the recognised fallacious nature of the arguments we are now considering. The difficulty created by this instance is removed when we observe that courts of law do not admit testimony as to the character of witnesses for the purpose of deciding whether the verdict shall be for or against the plaintiff, but for the purpose of eliminating, from the evidence that comes before it, testimony that is irrelevant to the issue, that is, testimony which, because it is prejudicially affected by the character of the individual who gives it, involves an *argumentum ad hominem*. Prejudiced or perjured evidence is fallacious in exactly the sense in which any *ad hominem* argument is fallacious.

When an argument is accepted because it enlists on one side or other of the question at issue, our feelings, passions or prejudices, it is called an *argumentum ad populum*. This is essentially the fallacy of public address. Any cause, as we all know, that can bring impressive oratory and brilliant metaphor to its aid is very likely to secure the support of many upon whom the cause itself and the arguments which really support it, would have little or no influence.

Conspicuous examples of fallacious reasoning of this kind may be found on almost every page of the voluminous writings of the late Robert Ingersoll. A somewhat similar fallacy is that which is known as *argumentum ad misericordiam*. In this case, our judgment is determined by the appeal that is made to our pity or sympathy. Whenever, for example, we recommend a cause by emphasising the plight of those whom it is intended to benefit, or give our support because of our interest in the individuals who advocate it, or give alms to a beggar because he is in rags, we are guilty of letting our sympathy interfere with our judgment, and this is exactly what occurs in any *argumentum ad misericordiam*.

The influence of a great name or of long established customs is pretty generally recognised as serving a useful purpose in the development of the individual and the race. When, however, our reverence for individuals or institutions leads us to accept, without consideration of the arguments which may be advanced for and against them, positions which receive the support of or are in conformity with any established authority, we are said to be guilty of the fallacy, *argumentum ad verecundiam*. This is a fallacy to which the conservatives among us are particularly prone. Our interest in order, in things as they are, unless balanced by a due regard for change, may lead us to accept a position merely because it is in agreement with some established custom, or to reject another merely because it requires the rejection of some accepted authority. A

peculiarly aggravated form of the fallacy comes to light in those instances where an individual's right to speak with authority in one field of knowledge is taken as the ground of our acceptance of that individual's opinion in another field with respect to which he has no special acquaintance. "The prestige of a great name," as Creighton remarks, "is thus irrelevantly invoked when no significance properly belongs to it. Thus, for example, a successful general is sometimes supposed to speak with authority upon problems of statescraft, and the opinions of prominent clergymen are quoted regarding the latest scientific or political theories."

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QUESTIONS

1. What is an inductive fallacy, and into what classes may such fallacies be divided.
2. What testimony can you quote showing the importance of a careful and accurate use of language for logical thinking?
3. What did Bacon mean by Idols of the Market Place?
4. How does Locke explain the fact that language often renders thought fallacious?
5. In what two ways may the observation of facts be fallacious?
6. State and illustrate the fallacy of insufficient observation.
7. What do you understand by a negative instance?
8. What is the testimony of Bacon and Darwin as to the importance of negative instances?
9. In what respect does education predispose one to the neglect of negative instances?
10. What does Bacon mean by Idols of the Theatre?
11. In what form is the fallacy described by Bacon in these terms likely to appear at the present day?
12. What is the fallacy of incomplete or imperfect observation?
13. Explain how the personal factor may lead to incomplete or imperfect observation.
14. In what way does technique affect the thoroughness of observation?
15. What are the two fallacies of explanation considered in the text?
16. Explain what is meant by a hasty generalisation.
17. To what consideration of scientific importance does the possibility of hasty generalisations point?
18. What is the fallacy of false analogy?

19. In what respect is the fallacy of false analogy similar to the fallacy of figurative language?
20. In what respect does false analogy resemble the fallacy of negative instances?
21. What is the general character of the fallacies of the context?
22. Into what two classes may fallacies of the context be divided? Describe each class, naming the particular fallacies that belong therein.
23. What do we mean by a fallacy of assumption?
24. Describe the essential character of a *petitio principii*.
25. How may we proceed to break down a *petitio principii*?
26. How may a ὕστερον πρότερον be distinguished from a *circulus in probando*, and how are they related to a *petitio principii*?
27. What is the fallacy of the complex question?
28. What, according to Aristotle, is an *ignoratio elenchi*?
29. What is an *ignoratio elenchi* according to modern logic?
30. What is Aikin's term for an *ignoratio elenchi*, and discuss its appropriateness.
31. From the standpoint of the assumptions involved, how do the *petitio principii*, the complex question, and the *ignoratio elenchi* differ from one another?
32. What is the generic meaning of the *non sequitur*?
33. What, according to Aristotle, is a *non sequitur*, and what, accordingly, is another name for the fallacy?
34. With what meaning is the term *non sequitur* now used in modern logic?
35. Discuss the relation of the *non sequitur* to categorical arguments.
36. What are the fallacies of interest?
37. Describe the *post hoc, ergo propter hoc* fallacy.
38. What is understood by an *argumentum ad hominem*?
39. Does the admission of evidence as to the character of accused persons, or of those giving evidence, by courts of law involve the verdict in the *ad hominem* fallacy? Explain.

40. What is the *argumentum ad populum*? Illustrate.
41. State and illustrate the *argumentum ad misericordiam*.
42. Show by means of illustrations the nature of the *argumentum ad verecundiam*.

EXERCISES

Examine the following arguments, and state in what respects, if any, they are fallacious, the technical name of the fallacy of which each is guilty, and the class to which the fallacy belongs:

- (1) In the act of landing on the African coast, Cæsar accidentally stumbled, and, as he did so, he exclaimed, 'Africa! I embrace thee.' Against what fallacy on the part of his followers was this exclamation intended to guard?
- (2) It is a dictum of the Cartesian philosophy that whatever can be clearly conceived exists. Is this fallacious, and if so, why?
- (3) The opponents of the Copernican system argued that if the earth moved, a stone let fall from the top of a tower would not reach the ground at the foot of the tower, but at a point some distance from it, in a direction opposite to the earth's movement. Such, however, is not the fact.
- (4) It is reported by travelers that the negroes among whom coral is worn as an amulet affirm that it is always affected by the state of health of the wearer, becoming pale in disease. This is not true.
- (5) Women, as a class, are supposed not to have hitherto been equal in intellect to men, therefore they are necessarily inferior.
- (6) The twenty-five years of Republican tariff legislation have been a record of unparalleled prosperity. Why kill the goose that lays the golden egg?

- (7) The metropolis of a country may be compared to the heart of a living body, hence we may infer that any considerable increase in size of the metropolis is evidence of a diseased condition of the body politic.
- (8) The early Teutonic societies were held together by ties of kinship, because all societies were so held together originally.
- (9) It is sometimes contended that because punishment can only intimidate it is brutalising, and therefore, it should not be employed, but that only shows that there are persons who are so brutalised that they can be influenced only through fear.
- (10) The apologists of Charles I., McCaulay complains, defend the king by urging that he was a good judge of pictures and indulgent to his wife.
- (11) This is a party measure and therefore we must vote for it.
- (12) A religion which elevates the soul above the body can survive persecution, and since the Christian religion has survived persecution, it enables its devotees to prefer the interests of the soul to those of the body.
- (13) It is frequently assumed, because the animals and men native to countries of inclement climates, where the conditions of life are severe, are usually robust, that the hardships they are forced to undergo in youth are the cause of their hardiness.
- (14) I do not know that you will approve it, but I gave him a pair of shoes, an old coat, and something to eat. I felt so sorry for him.

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